EE320 (1/2017)

INTRODUCTORY MATHEMATICAL ECONOMICS

EQUILIBRIUM and COMPARATIVE STATIC ANALYSIS

(Part 2)

Topics

- Equilibrium in Macroeconomic Models
  - Partial Market Equilibrium: Keynesian National-Income Model
  - General Equilibrium: IS-LM Model
- Comparative statics
- Policy effectiveness

Simplest Keynesian National-Income Model

- Consider a *closed* economy (i.e. no trade).
- The equilibrium is: $Y = C + I_0 + G_0 = AE$

  where $C = a + bY$ $(a > 0, 0 < b < 1)$

➢ At equilibrium, $Y = a + bY + I_0 + G_0$

➔ $$Y^* = \frac{a + I_0 + G_0}{1 - b}$$

➢ Corresponding consumption: $C^* = a + bY^*$

➔ $$C^* = \frac{a + b(I_0 + G_0)}{1 - b}$$

Closed Economy with Proportional Income Tax

- Equilibrium condition: $Y = C + I_0 + G_0 = AE$

  where $C = a + bY_d \quad (a > 0, 0 < b < 1)$

  $Y_d = Y - T$

  $T = tY \quad (0 < t < 1)$

- At equilibrium, $Y = a + bY_d + I_0 + G_0$

  $Y = a + b(1-t)Y + I_0 + G_0$

  ➔ $$Y^* = \frac{a + I_0 + G_0}{1 - b(1-t)}$$

  ➔ $$C^* = \frac{a + b(1-t)(I_0 + G_0)}{1 - b(1-t)}$$

Open Economy with Proportional Income Tax

- Equilibrium condition: $Y = C + I_0 + G_0 + X_0 - M = AE$

  where $C = a + bY_d \quad (a > 0, 0 < b < 1)$

  $T = tY \quad (0 < t < 1)$

  $M = mY$

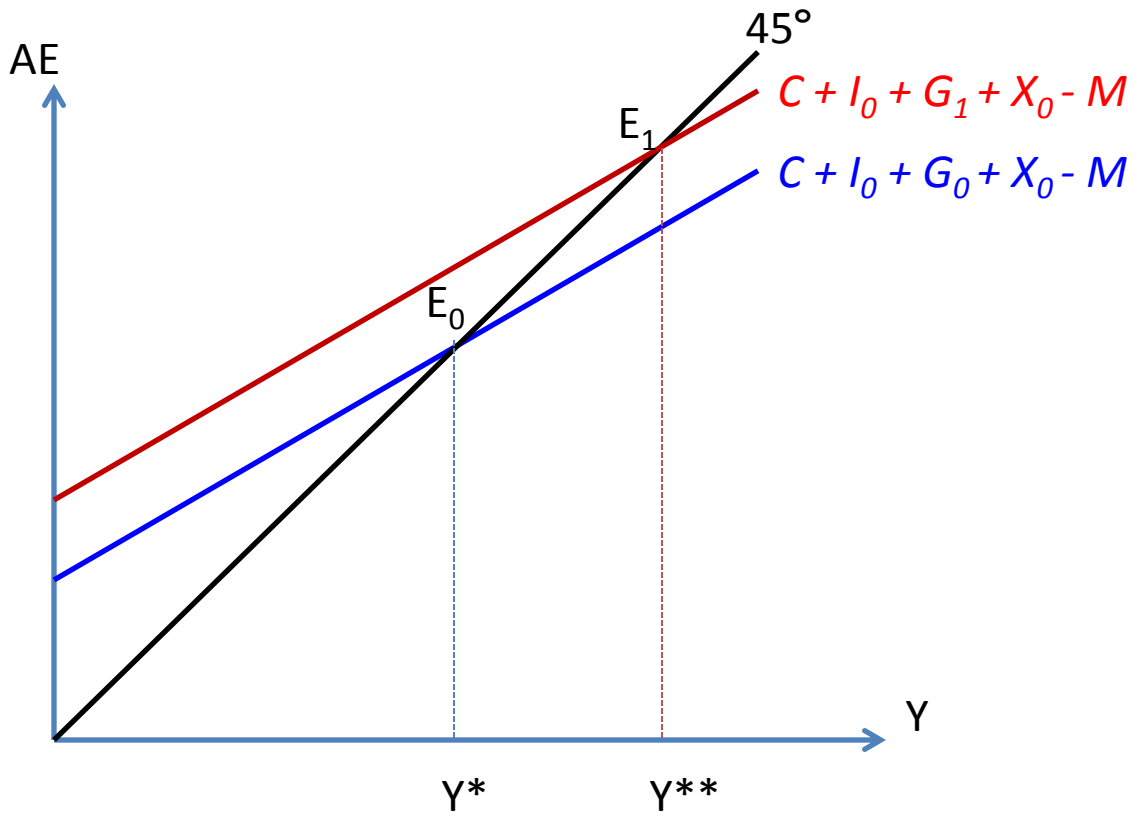
- At equilibrium, $Y = a + b(1-t)Y + I_0 + G_0 + X_0 - mY$

  ➔ $$Y^* = \frac{a + I_0 + G_0 + X_0}{1 - b(1-t) + m}$$

  ➔ $$C^* = \frac{a(1+m) + b(1-t)(I_0 + G_0 + X_0)}{1 - b(1-t) + m}$$

- If $M = mY_d$, $Y^* = ?$ (own practice!)

Graph: Keynesian National-Income Model



Suppose G increases. What is the new value of Y*?

➔ Will show the answer on slide 12.

IS-LM Model

- The previous Keynesian model only deals with the equilibrium in the goods (or commodity) market. It seeks to determine the equilibrium level of national income.
- The IS-LM model deals with not only the commodity market but also the money market. It determines the equilibrium levels of national income and interest rate in the economy.
- The IS curve represents the equilibrium in the commodity market.
  - Planned "investment" = Planned "savings"
- The LM curve represents the equilibrium in the money market.
  - Liquidity preference (money demand) = Money supply

IS-LM Model: Closed Economy & No Tax

- Commodity Market:

  $Y = C + I + G_0$

  $C = a + bY$ $(a > 0, 0 < b < 1)$

  $I = I_0 - ir$ (I_0, $i > 0$, and r is interest rate)

  - $Y = a + bY + I_0 - ir + G_0$ ➔ $Y = \frac{(a + I_0 + G_0) - ir}{1 - b}$: IS

- Money Market:

  $M^S = M_0$

  $M^D = kY - hr$ $(k, h > 0)$

  - At equilibrium, $M_0 = kY - hr$ ➔ $Y = \frac{M_0}{k} + \frac{h}{k} r$: LM

- From the two equilibrium conditions, we can solve for Y^* and r^*:

$$Y^* = \frac{(a + I_0 + G_0)h + iM_0}{ik + h(1 - b)}$$ and $$r^* = \frac{(a + I_0 + G_0)k - (1 - b)M_0}{ik + h(1 - b)}$$

IS-LM Model: Closed Economy with Tax

- Commodity Market:

$Y = C + I + G_0$

$C = a + bY_d$ $(a > 0, 0 < b < 1)$

$I = I_0 - ir$ $(I_0, i > 0)$

$Y_d = Y - T$ where $T = tY$ $(0 < t < 1)$

➔ $Y = a + b(1-t)Y + I_0 - ir + G_0$ ➔ $Y = \dfrac{(a + I_0 + G_0) - ir}{1 - b(1-t)}$: IS

- Money Market:

$M^S = M_0$

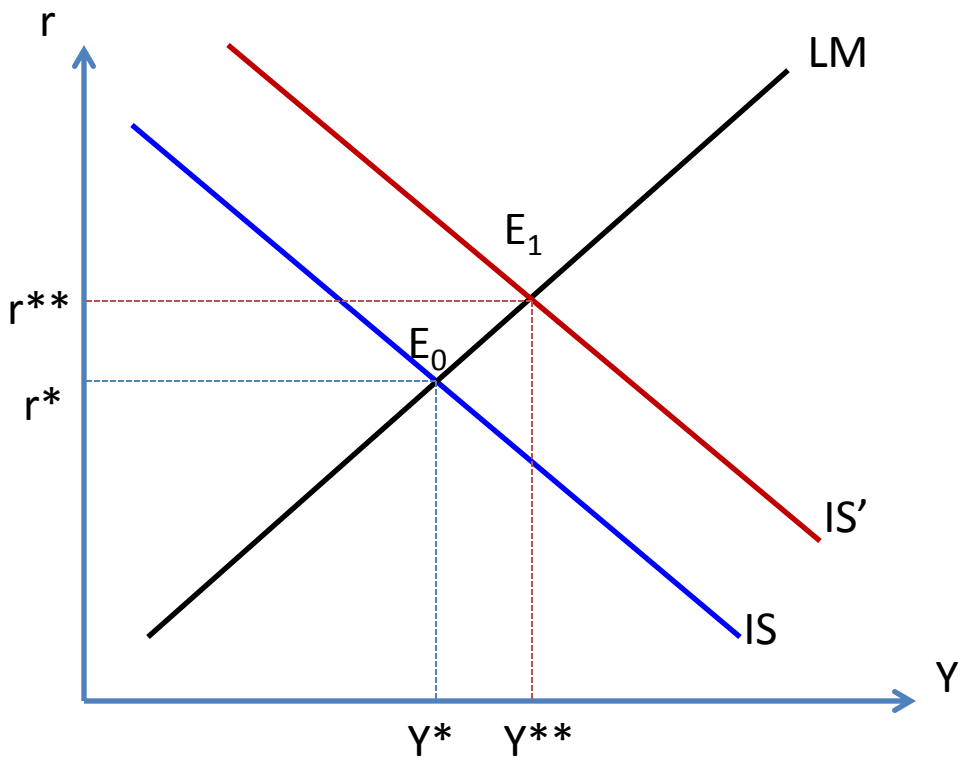
$M^D = kY - hr$ $(k, h > 0)$

➔ $M_0 = kY - hr$ ➔ $Y = \dfrac{M_0}{k} + \dfrac{h}{k} r$: LM

➢ Equilibrium values for Y^* and r^*:

$$Y^* = \frac{(a + I_0 + G_0)h + iM_0}{ik + h[1 - b(1-t)]}$$ and $$r^* = \frac{(a + I_0 + G_0)k - [1 - b(1-t)M_0]}{ik + h[1 - b(1-t)]}$$

Graph: Equilibrium in IS-LM Model



- When the government increases its expenditure, what's the impact on the equilibrium Y and r? (➔ Will show the answer on slide 13.)

Comparative Statics Analysis:
Simple Macroeconomic Model

- Suppose government increases its expenditure, what's the impact on the equilibrium national income (i.e. ΔY*/ΔG=?)
- Recall: $Y^* = \frac{a + I_0 + G_0 + X_0}{1 - b(1-t) + m}$, and suppose $\Delta G = G_1 - G_0$.

➢ $$Y^{*1} - Y^{*0} = \frac{a + I_0 + G_1 + X_0}{1 - b(1-t) + m} - \frac{a + I_0 + G_0 + X_0}{1 - b(1-t) + m}$$

➢ $$\Delta Y^* = Y^{*1} - Y^{*0} = \frac{\Delta G}{1 - b(1-t) + m}$$

➔ $$\boxed{\frac{\Delta Y^*}{\Delta G} = \frac{1}{1 - b(1-t) + m} > 0}$$

Comparative Statics Analysis: IS-LM Model

- In a closed economy with tax, suppose G increases ($\Delta G = G_1 - G_0$), $\Delta Y^*/\Delta G = ?$ and $\Delta r^*/\Delta G = ?$
- Recall: $Y^* = \dfrac{(a + I_0 + G_0)h + iM_0}{ik + h[1 - b(1-t)]}$ and $r^* = \dfrac{(a + I_0 + G_0)k - [1 - b(1-t)]M_0}{ik + h[1 - b(1-t)]}$

$$\Delta Y^* = Y^{*1} - Y^{*0} = \frac{h}{ik + h[1 - b(1-t)]}\Delta G \Rightarrow \boxed{\frac{\Delta Y^*}{\Delta G} = \frac{h}{ik + h[1 - b(1-t)]} > 0}$$

$$\Delta r^* = r^{*1} - r^{*0} = \frac{k}{ik + h[1 - b(1-t)]}\Delta G \Rightarrow \boxed{\frac{\Delta r^*}{\Delta G} = \frac{k}{ik + h[1 - b(1-t)]} > 0}$$

Comparative Statics Analysis: IS-LM Model

- In a closed economy with tax, suppose G increases ($\Delta G = G_1 - G_0$), $\Delta Y^*/\Delta G = ?$ and $\Delta r^*/\Delta G = ?$
- Recall: $Y^* = \dfrac{(a + I_0 + G_0)h + iM_0}{ik + h[1 - b(1-t)]}$ and $r^* = \dfrac{(a + I_0 + G_0)k - [1 - b(1-t)]M_0}{ik + h[1 - b(1-t)]}$

$$\Delta Y^* = Y^{*1} - Y^{*0} = \frac{h}{ik + h[1 - b(1-t)]}\Delta G \Rightarrow \boxed{\frac{\Delta Y^*}{\Delta G} = \frac{h}{ik + h[1 - b(1-t)]} > 0}$$

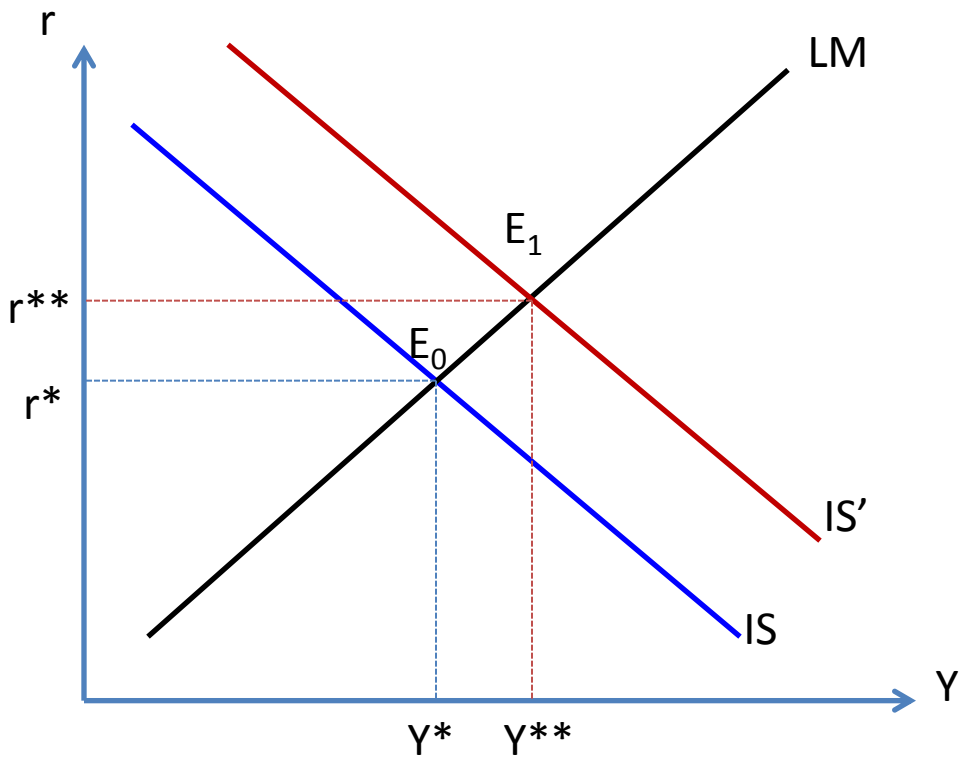
$$\Delta r^* = r^{*1} - r^{*0} = \frac{k}{ik + h[1 - b(1-t)]}\Delta G \Rightarrow \boxed{\frac{\Delta r^*}{\Delta G} = \frac{k}{ik + h[1 - b(1-t)]} > 0}$$

Policy Implications

- **Crowding out effect**
  - An increase in government expenditure (G) can have a side-effect in lowering private investment (I), through higher *r*. As a result, the aggregate expenditure (Y) will not increase as in the case in which *r* is fixed.

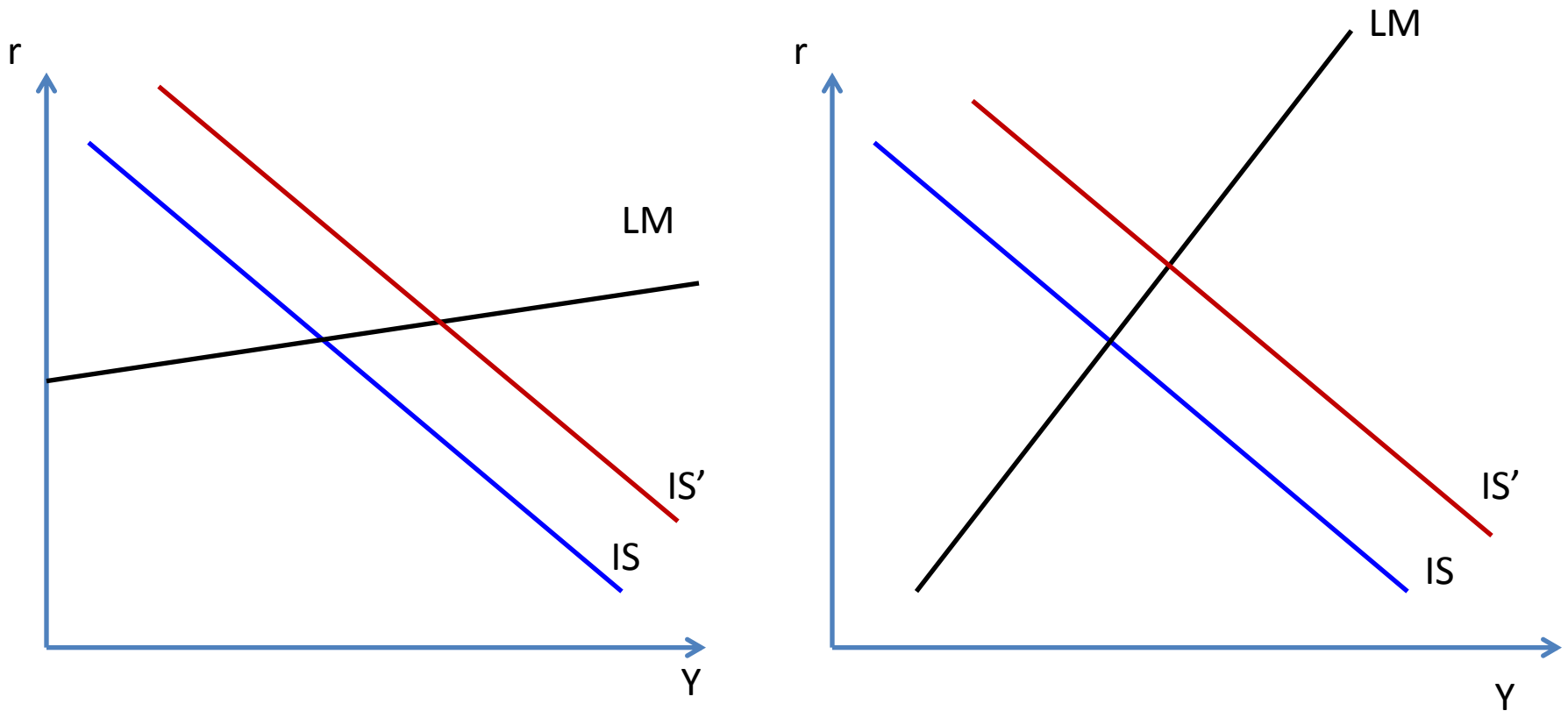
- **Policy effectiveness**
  - Monetary policy is more effective when IS curve is flatter.
  - Fiscal policy is more effective when LM curve is flatter.

Graph: Crowding-Out Effect



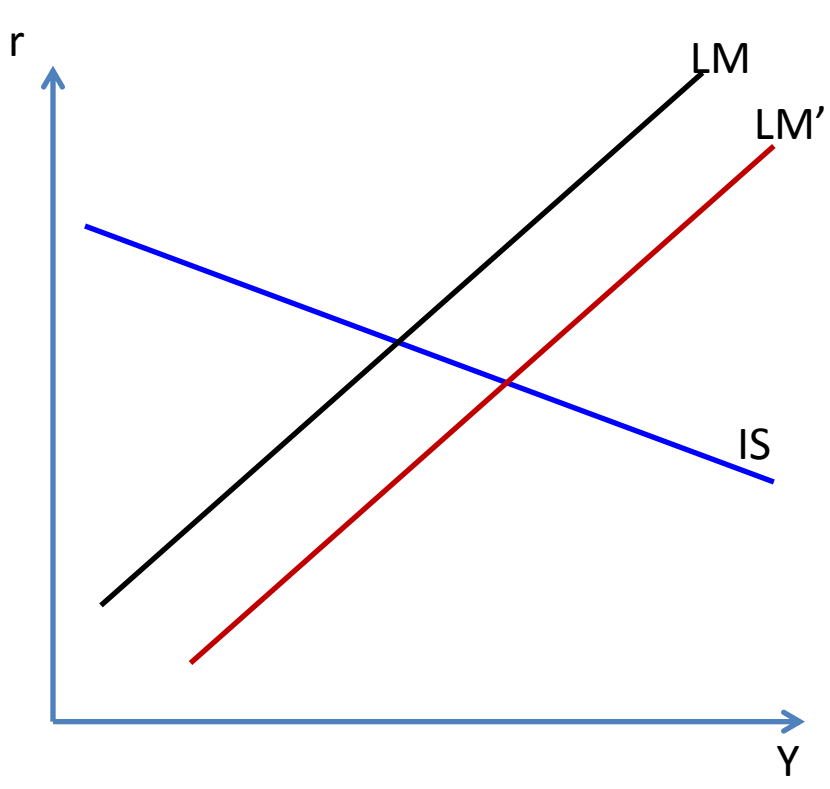
G ↑ ➔ IS curve shifts right ➔ Y↑ ➔ Md ↑➔r ↑ ➔ I ↓➔ AE ↓

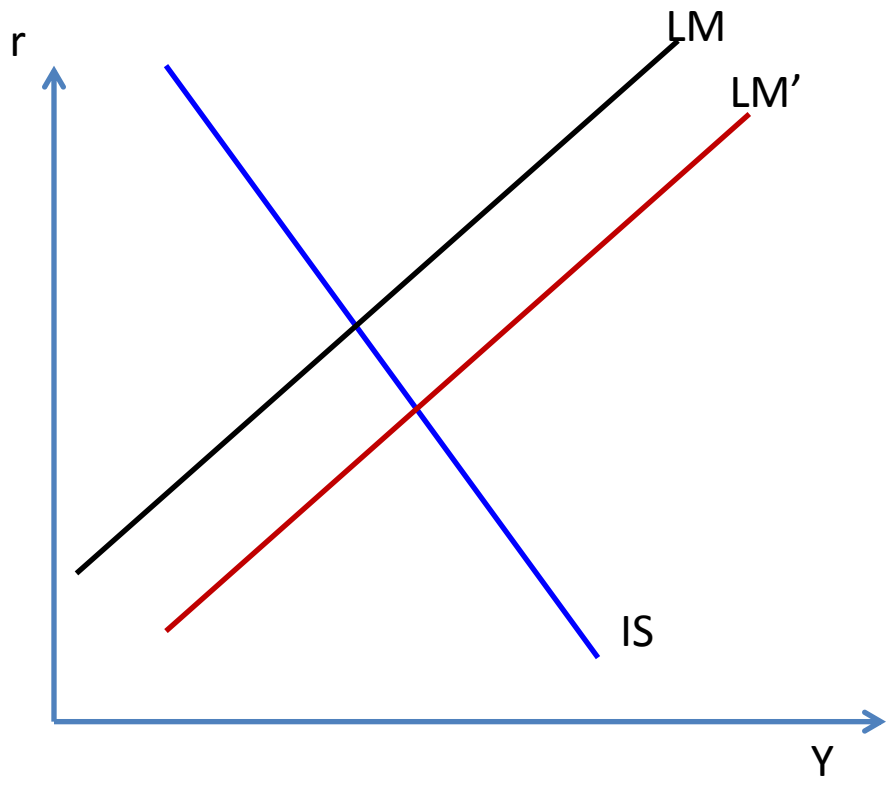
Graph: Policy Effectiveness (1)



The fiscal policy is more effective when the LM curve is relatively flatter.

Graph: Policy Effectiveness (2)





The monetary policy is more effective when the IS curve is relatively flatter.