Example 3.J: Excess burden *formula under linear model & Tax-Revenue-maxizing tax rate*

Demand: $p^d = a - bQ^d \quad ; \ a \geq 0, \quad b \leq 0.$

Supply : $p^s = c + dQ^s \quad ; \ d \geq 0.$

- Solve for quantity and prices equilibrium when the unit tax is imposed. Analyze the result

tax on producer $\longrightarrow p^s = p^d - t$

tax on consumer $\longrightarrow p^d = p^s + t$

Equilibrium (after tax) will not changing (remain the same)

$\overbrace{p^s + t}^{p^d} = a - b \cdot Q^d$

$p^s = a - t - b \cdot Q^d$

$p^s = c + dQ^s$

$Q^d = Q^s \longrightarrow$ equilibrium

$p^s = p^s \rightarrow a - t - b \cdot Q = c + d \cdot Q$

$a - t - c = (b+d)Q$

$Q^* = \dfrac{a-t-c}{b+d}$

$= \dfrac{a-c}{b+d} - \dfrac{t}{b+d}$

Derive t

$t = 0 \rightarrow Q = \dfrac{a-c}{b+d}$

$t > 0 ; Q = \dfrac{a-c}{b+d} - \dfrac{t}{b+d}$

$\Delta Q = Q_{\text{After tax}} - Q_{\text{before tax}}$

$= \dfrac{-t}{b+d}$

So, $\dfrac{\partial Q}{\partial t} ; \dfrac{\Delta Q}{\Delta t} = \dfrac{-1}{b+d}$

$\rightarrow$ when $Q \downarrow$ then $t \uparrow$

- Derive the excess burden formula for buyers and sellers

$$P^s = c + dQ^s$$

$$= c + d\left(\frac{a-c}{a-d} - \frac{t}{b+d}\right)$$

$$= c + \frac{d(a-c)}{b+d} - \frac{dt}{b+d}$$

$$= \frac{cb+cd+da-dc}{b+d} - \frac{dt}{b+d}$$

$$P^{s*}_{(t)} = \underbrace{\frac{cb+da}{b+d}}_{p^s(t=0)} - \underbrace{\frac{dt}{b+d}}_{\Downarrow\, 0}$$

$$p^d = p^s + t$$

$$\to p^d = \frac{cb+da}{b+d} - \frac{dt}{b+d} + t$$

$$p^{d*} = \frac{cb-da}{b+d} + \underbrace{\left|\frac{b}{b+d}\cdot t\right|}_{\Downarrow\, 0}$$

P^s, P^d depends on tax

$$t=0 \to t>0 \to \left.\begin{aligned} |\Delta P^s| &= \frac{dt}{b+d} \\ |\Delta P^d| &= \frac{bt}{b+d} \end{aligned}\right\} = t$$

$$t^{old} > 0 \to t^{new} > 0$$

initial P^e before tax

$$\left|\frac{\Delta P^s}{\Delta t}\right| = \frac{d}{b+d} = \frac{\frac{\Delta P}{\Delta Q^s}}{\frac{\Delta P}{\Delta Q^d} + \frac{\Delta P}{\Delta Q^s}} = \frac{\frac{1}{\frac{\Delta Q^s}{\Delta P}}\cdot\frac{1}{\frac{P^e}{Q^e}}}{\frac{1}{\frac{\Delta Q^d}{\Delta P}}\cdot\frac{1}{\frac{P^e}{Q^e}} + \frac{1}{\frac{\Delta Q^s}{\Delta P}}\cdot\frac{1}{\frac{P^e}{Q^e}}}$$

$$= \frac{\frac{1}{\varepsilon^s}}{\frac{1}{\varepsilon^d} + \frac{1}{\varepsilon^s}} = \frac{\frac{1}{\varepsilon^s}}{\frac{\varepsilon^d+\varepsilon^d}{\varepsilon^d\cdot\varepsilon^s}} = \frac{\varepsilon^d}{\varepsilon^d+\varepsilon^s}$$

Consumer burden

$$\left|\frac{\Delta P^d}{\Delta t}\right| = \frac{b}{b+d} \downarrow = \frac{\varepsilon^s}{\varepsilon^s+\varepsilon^d}$$

$$\frac{\Delta\ \text{consumer burden}}{\Delta \varepsilon^d} = \frac{-\varepsilon^s}{(\varepsilon^s+\varepsilon^d)^2}$$

$$\frac{\Delta\ \text{firm burden}}{\Delta \varepsilon^s} = \frac{-\varepsilon^d}{(\varepsilon^s+\varepsilon^d)^2}$$

$\therefore\ \varepsilon^s\downarrow \to \uparrow$ firm budget

$\therefore\ \varepsilon^d\uparrow \to$ burden $\downarrow$

$\varepsilon^d\downarrow \to$ Burden $\uparrow$

- Calculate the tax rate that maximizes the tax revenue of government.

change Q to component. a, b, c, d, t

$$T = t \cdot Q$$

$$T = t \cdot \left(\frac{a-c}{b+d} - \frac{t}{b+d} \right)$$

Note: $\frac{2t}{b+d} = \frac{a-c}{b+d}$

$$T(t) = \left(\frac{a-c}{b+d} \right) \cdot t - \frac{t^2}{b+d} = \frac{a-c}{b+d} - \frac{2t}{b+d}$$

t^* maximize $T(t)$

$$t^* = \frac{-\left(\frac{a-c}{b+d} \right)}{2\left(-\frac{1}{b+d} \right)} = \frac{a-c}{2}$$