

Topic 2 Part 1

Consumer Choice (Chapter 4)

Recall

- Consumer Preference (CH3) tells us about whether a consumer likes one particular bundle more than another.
- However, it does not mention anything about the bundle that the consumer will actually buy.
- The choice the consumer will actually make depends on many factors, such as income and prices, which will be covered in this chapter of Consumer Choice (CH4).

The Budget Constraint

Assume only two goods available: X and Y.

- Price of X is denoted by P_x
- Price of Y is denoted by P_y
- Income is denoted by I

Total expenditure on basket (X,Y): $P_xX + P_yY$

*The Basket is **Affordable** if total expenditure does not exceed total Income:*

$$P_xX + P_yY \leq I$$

The Budget Constraint

Budget Constraint:

- The set of bundles that a consumer can purchase with a limited amount of income:

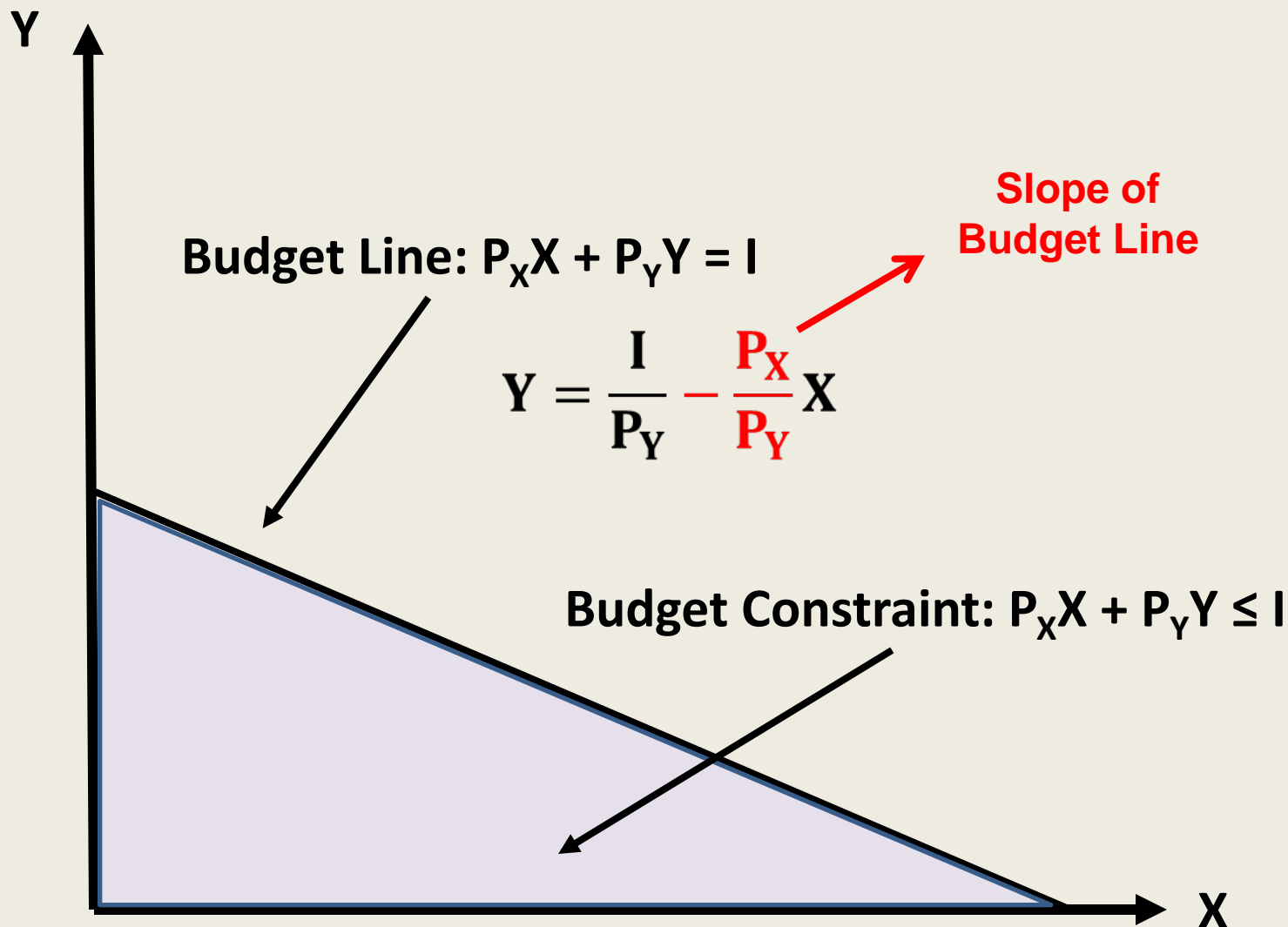
$$P_X X + P_Y Y \leq I$$

Budget Line:

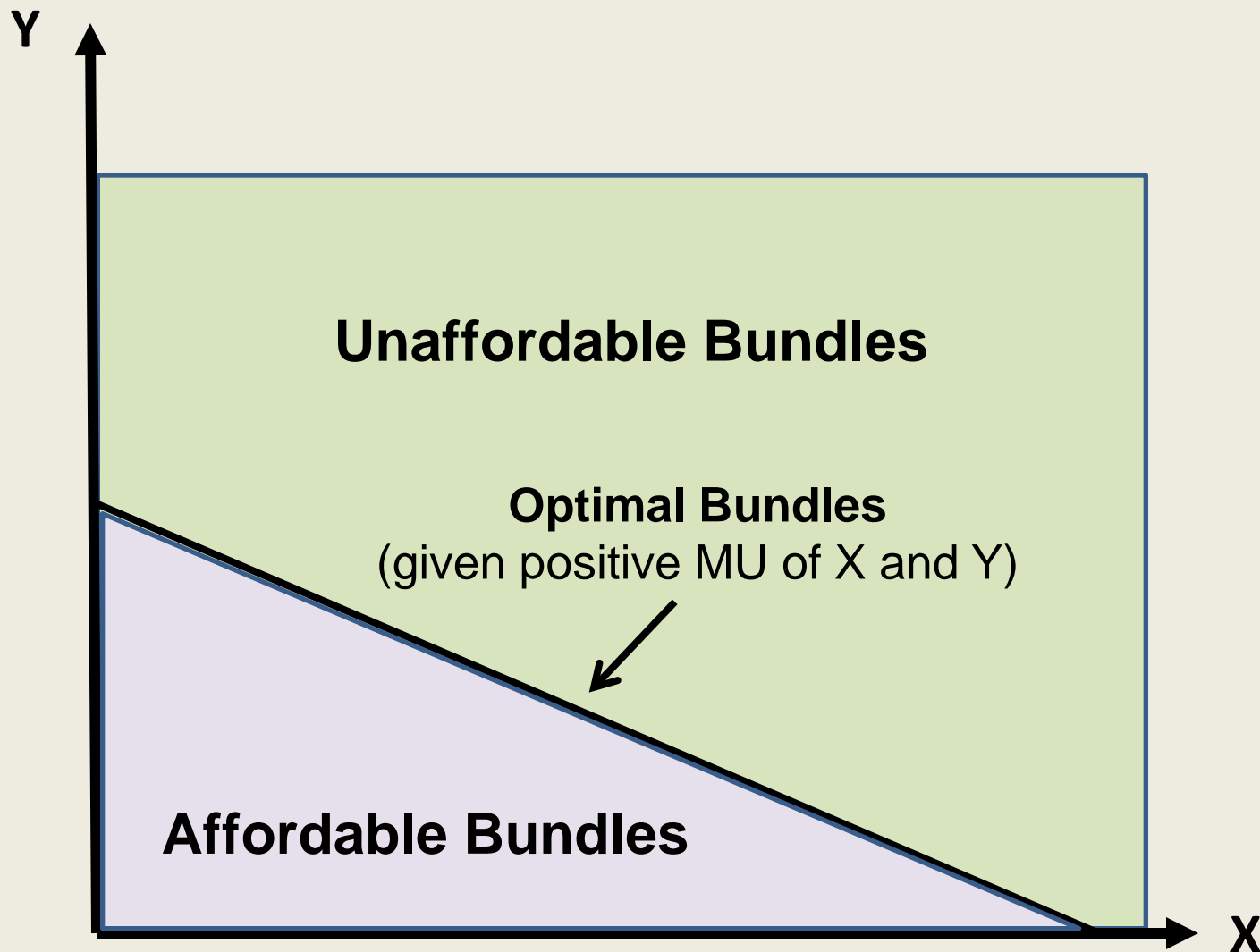
- The set of bundles that a consumer can purchase when spending all his/her income:

$$P_X X + P_Y Y = I$$

The Budget Constraint



The Budget Constraint



The Budget Constraint – Example

Two goods available: X and Y

$$I = \$10$$

$$P_x = \$1$$

$$P_y = \$2$$

All income spent on X
→ I/P_x units of X bought

All income spent on Y
→ I/P_y units of Y bought

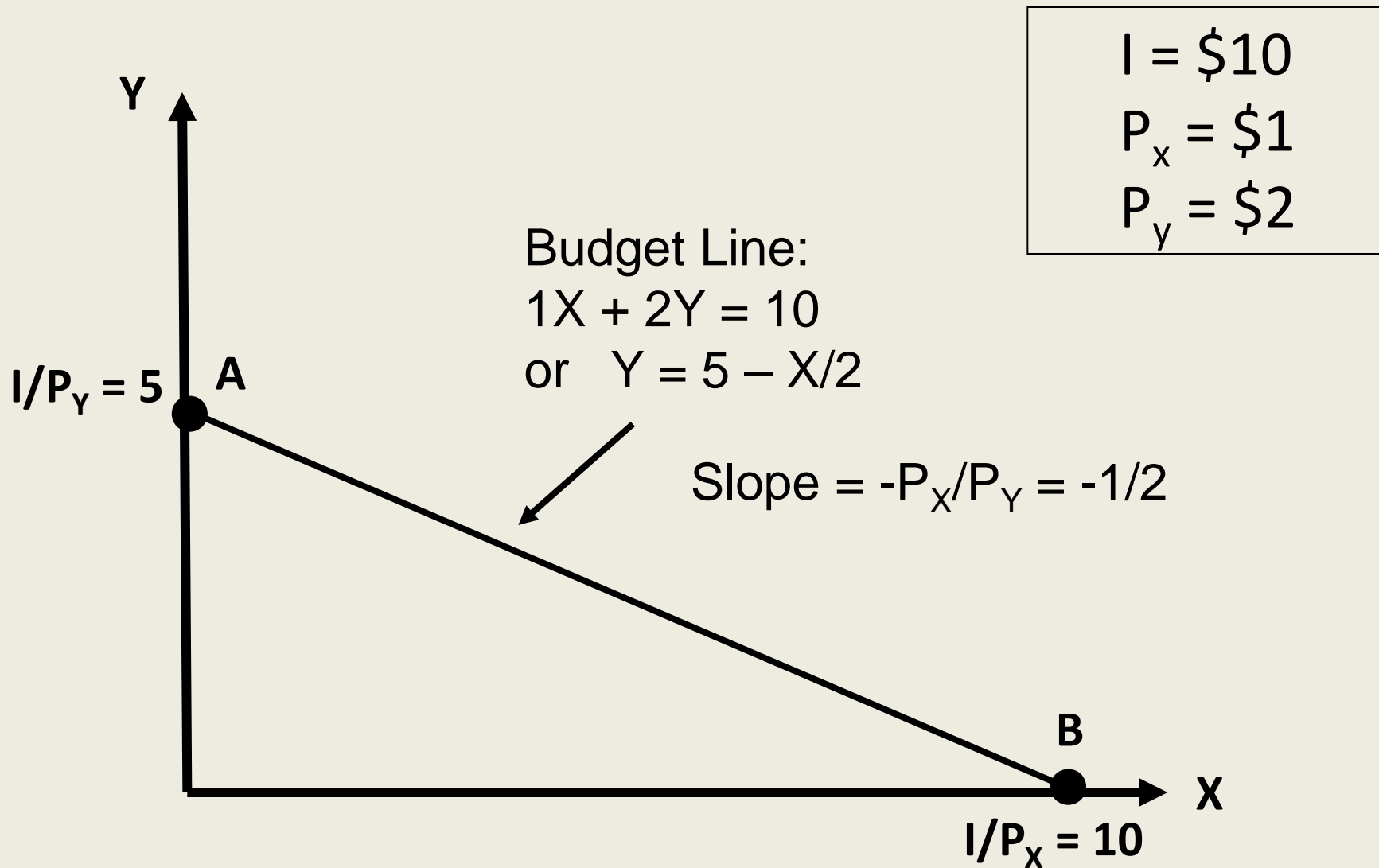
Budget Line:

$$1X + 2Y = 10$$

$$\text{Or } Y = 5 - X/2$$

$$\text{Slope of Budget Line} = -P_x/P_y = -1/2$$

The Budget Constraint – Example



The Budget Constraint – Slope

The Slope of the Budget Line (Ratio of Prices)

- It tells us how many units of the good on the vertical axis a consumer must give up to obtain an additional unit of the good on the horizontal axis.
- For example, Slope = $-P_X/P_Y = -1/2$.

This means the consumer has to give up $\frac{1}{2}$ units of Y to obtain 1 unit of X, since $P_Y = 2$ and $P_X = 1$.

The Budget Constraint – Income

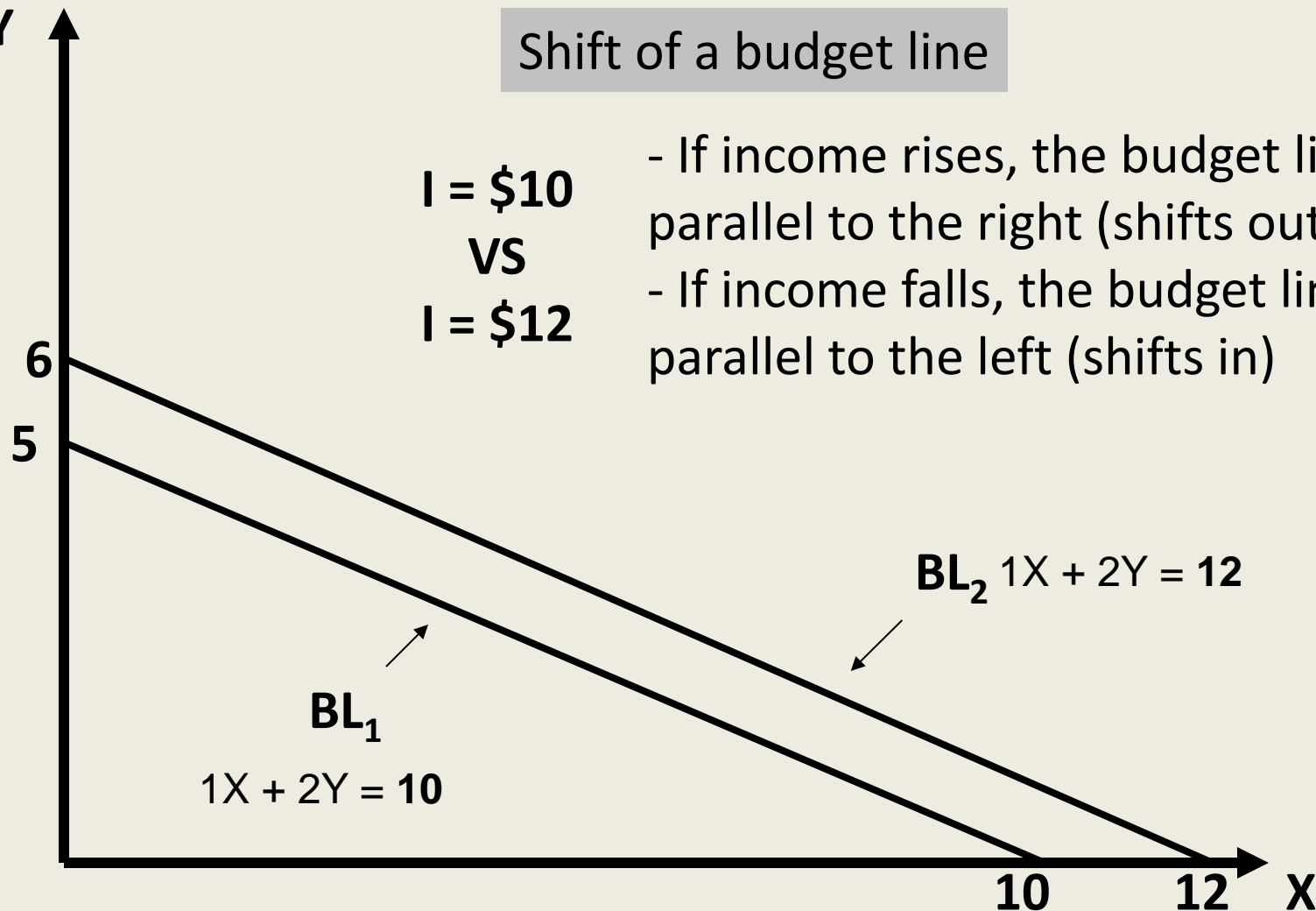
- **Increase in Income will shift the budget line to the right.**
 - More of each product becomes affordable.
- **Decrease in Income will shift the budget line to the left.**
 - Less of each product becomes affordable.

The Budget Constraint – Income

Shift of a budget line

$I = \$10$
VS
 $I = \$12$

- If income rises, the budget line shifts parallel to the right (shifts out)
- If income falls, the budget line shifts parallel to the left (shifts in)



The Budget Constraint – Prices

- **Increase in Price of Y will pivot the budget line inwards around the horizontal intercept.**
 - Less of Y can be bought.
- **Decrease in Price of Y will pivot the budget line outwards around the horizontal intercept.**
 - More of Y can be bought.

The Budget Constraint – Prices

Pivot of a budget line

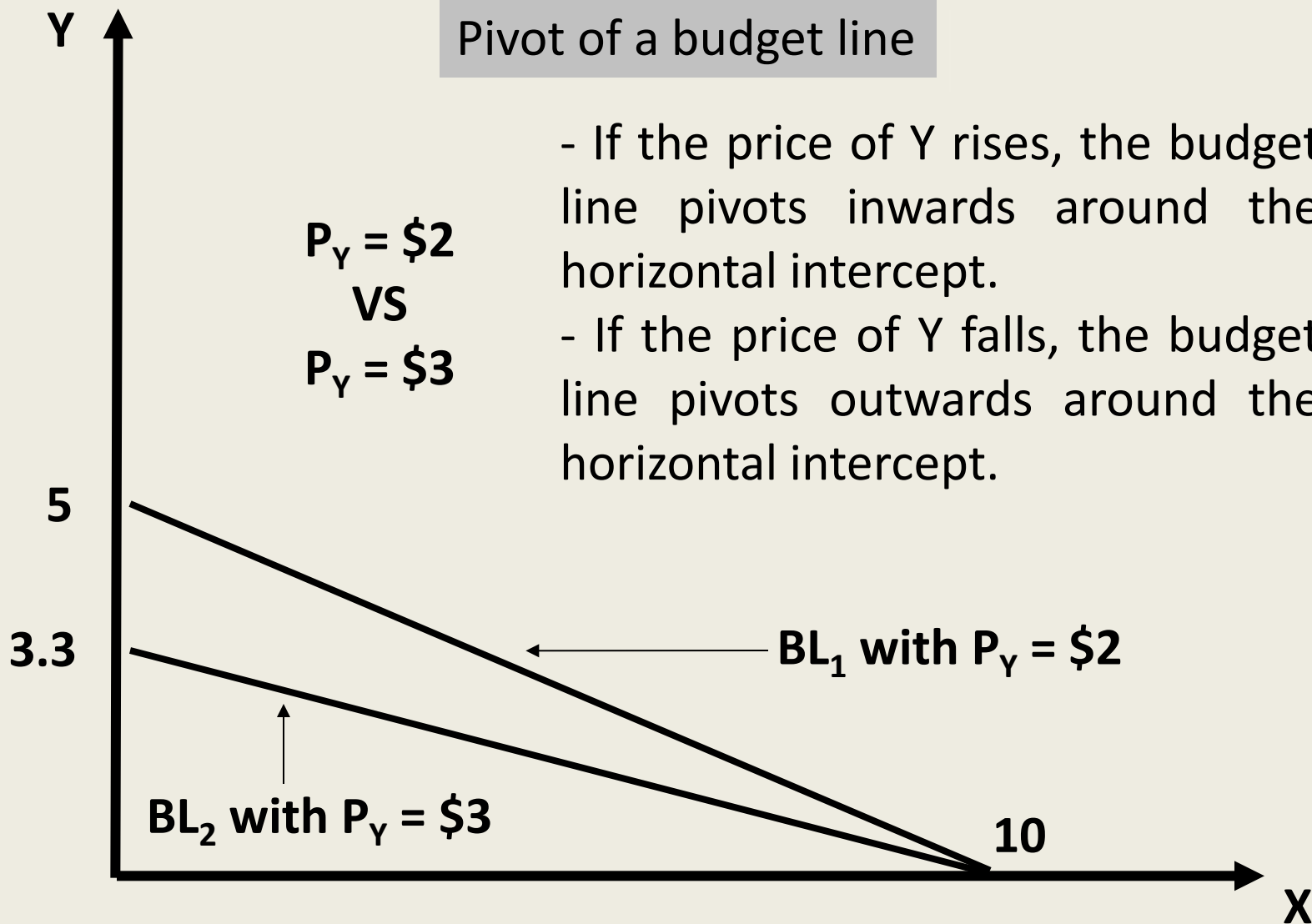
$P_Y = \$2$

VS

$P_Y = \$3$

- If the price of Y rises, the budget line pivots inwards around the horizontal intercept.

- If the price of Y falls, the budget line pivots outwards around the horizontal intercept.



The Budget Constraint – Example

Two goods available: X and Y

$$I = \$800$$

$$P_x = \$20$$

$$P_y = \$40$$

All income spent on X $\rightarrow I/P_x$
units of X bought

All income spent on Y $\rightarrow I/P_y$
units of Y bought

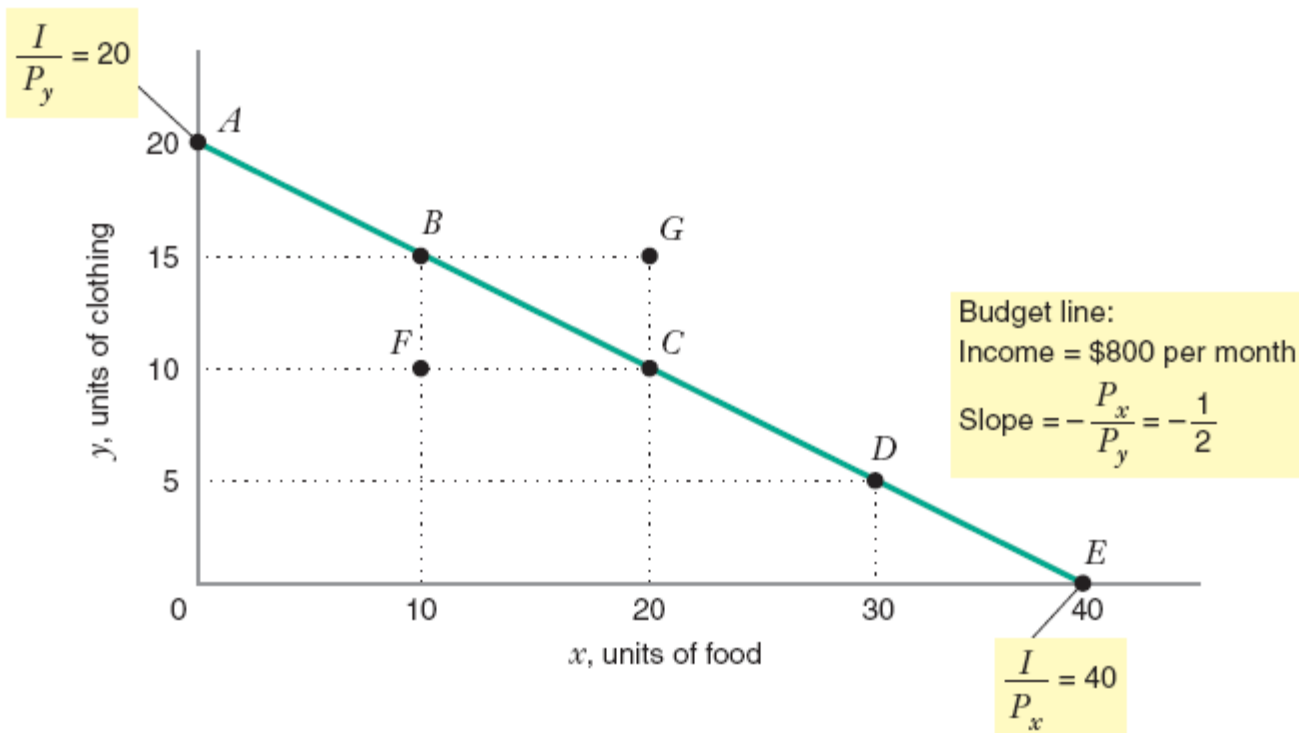
Budget Line 1:
 $20X + 40Y = 800$

Or

$$Y = 20 - X/2$$

Slope of Budget Line = $-P_x/P_y = -1/2$

The Budget Constraint – Example





LEARNING-BY-DOING EXERCISE 4.1

Good News/Bad News and the Budget Line

Suppose that a consumer's income (I) doubles and that the prices (P_x and P_y) of both goods in his basket also double. He views the doubling of income as good news because it increases his purchasing power. However, the doubling of prices is bad news because it decreases his purchasing power.

Problem What is the net effect of the good and bad news?

Consumer Choice

The optimal choice problem for the consumer is expressed as follows:

Consumer's Problem:

$$\text{Max}_{(X,Y)} U(X,Y) \quad \text{subject to} \quad P_x X + P_y Y \leq I$$

i.e. choose X and Y to maximize $U(X,Y)$ subject to the constraint that total expenditure does not exceed total income.

Consumer Choice

Optimal Choice:

consumer choice of a bundle that

- 1) maximizes utility, and
- 2) allows the consumer to live within the budget constraint (i.e. affordable).

Consumer Optimum

- 1) The Optimal Choice is a bundle that maximizes utility.**
- This implies that the optimal choice must be on the highest indifference curve.

Consumer Optimum

2) The Optimal Choice is a bundle that is affordable.

- This implies that if the consumer likes both goods (MU_x and MU_y are positive), the optimal bundle must lie on the budget line.
- Otherwise, the unspent income could be used to increase utility by purchasing more goods.

Consumer Optimum

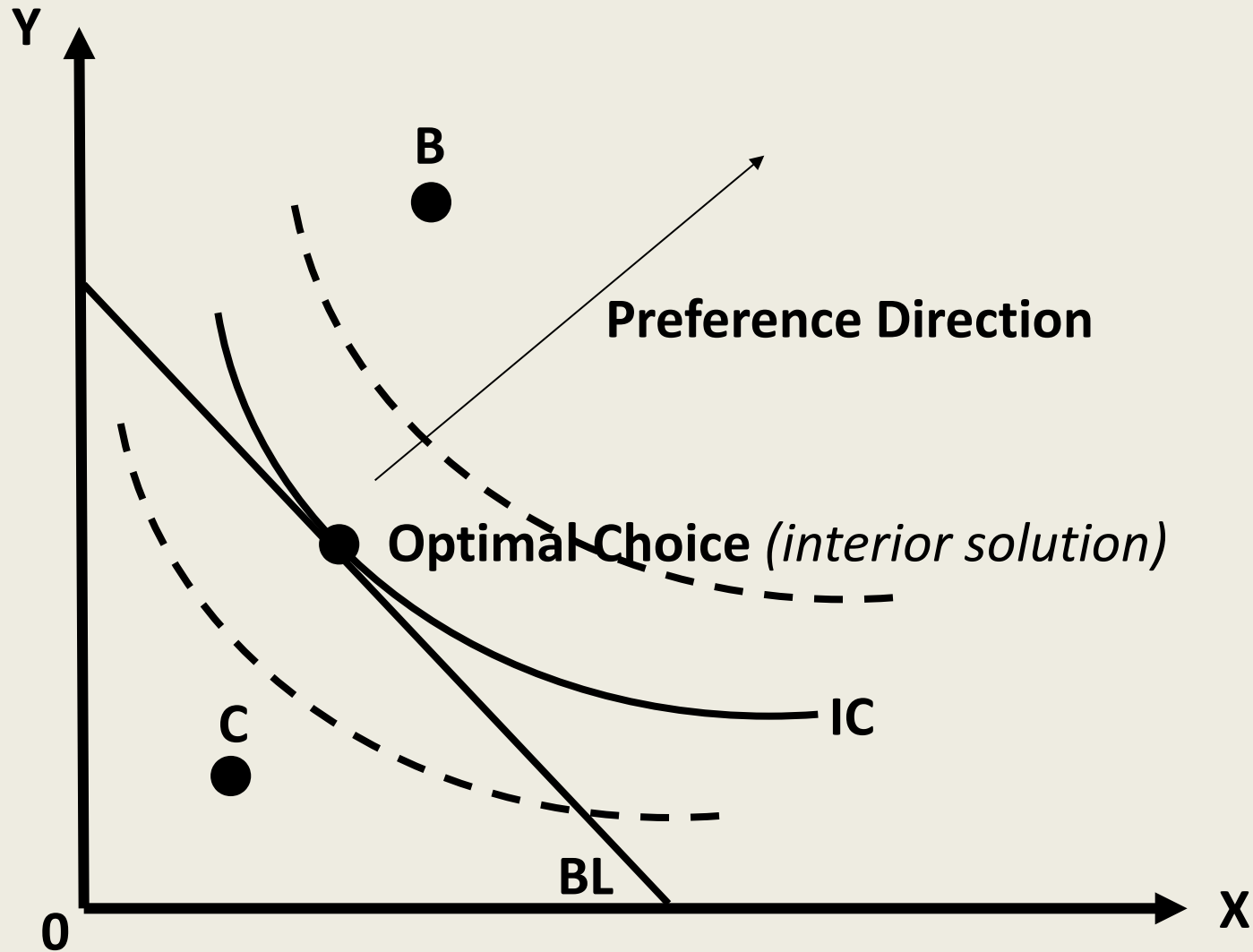
Hence, given the definition of optimal choice, we are looking for the bundle that is

- 1) on the highest indifference curve, AND
- 2) on the budget line.

There are two cases to consider:

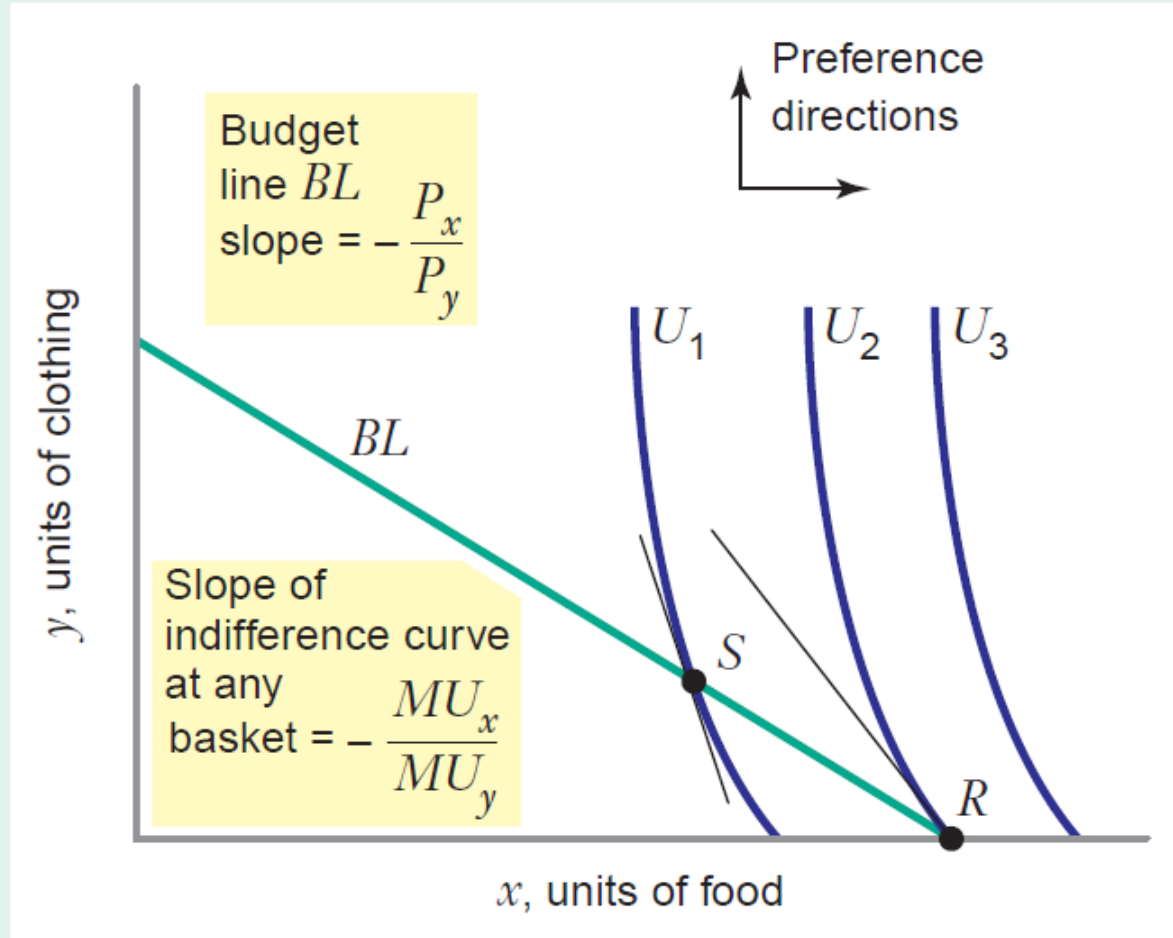
- 1) Interior Solution**
- 2) Corner Solution**

Interior Consumer Optimum



Corner Consumer Optimum

FIGURE 4.7 **Corner Point**
At basket S the slope of the indifference curve U_1 is steeper (more negative) than the budget line. This means that the marginal utility per dollar spent on food is higher than on clothing, so the consumer would like to purchase less clothing and more food. He would move along the budget line until he reaches the corner point basket R , where no further substitution is possible because he purchases no clothing at R .



Interior Consumer Optimum

Interior Optimum: The optimal consumption basket is at a point where the indifference curve is just *tangent* to the budget line.

This is where the slopes of two lines are equal.

Optimality Condition:

$$MRS_{xy} = MU_x / MU_y = P_x / P_y$$

“The rate at which the consumer would be willing to exchange X for Y is the same as the rate at which they are exchanged in the marketplace.”

Interior Consumer Optimum

Optimality Condition

can also be written as

$$MU_x/P_x = MU_y/P_y$$

“MU per dollar spent on each good is the same.”

Now, we can solve for X and Y from the following system:

1. $MU_x/P_x = MU_y/P_y$
2. $P_x X + P_y Y = I$

Interior Consumer Optimum

Example

$$I = \$800, P_x = \$20, P_y = \$40$$

Budget Line:

$$I = P_x X + P_y Y$$

$$800 = 20X + 40Y$$

Slope of Budget Line:

$$-P_x/P_y = -1/2$$

$$U(X, Y) = XY$$

$$MU_x = Y$$

$$MU_y = X$$

Slope of Indifference Curve:

$$-MRS_{xy} = -Y/X$$

Interior Consumer Optimum

Example

Optimality Condition:

$$-P_x/P_y = -MRS_{xy} \quad \text{OR} \quad 1/2 = Y/X$$

$$\mathbf{X = 2Y}$$

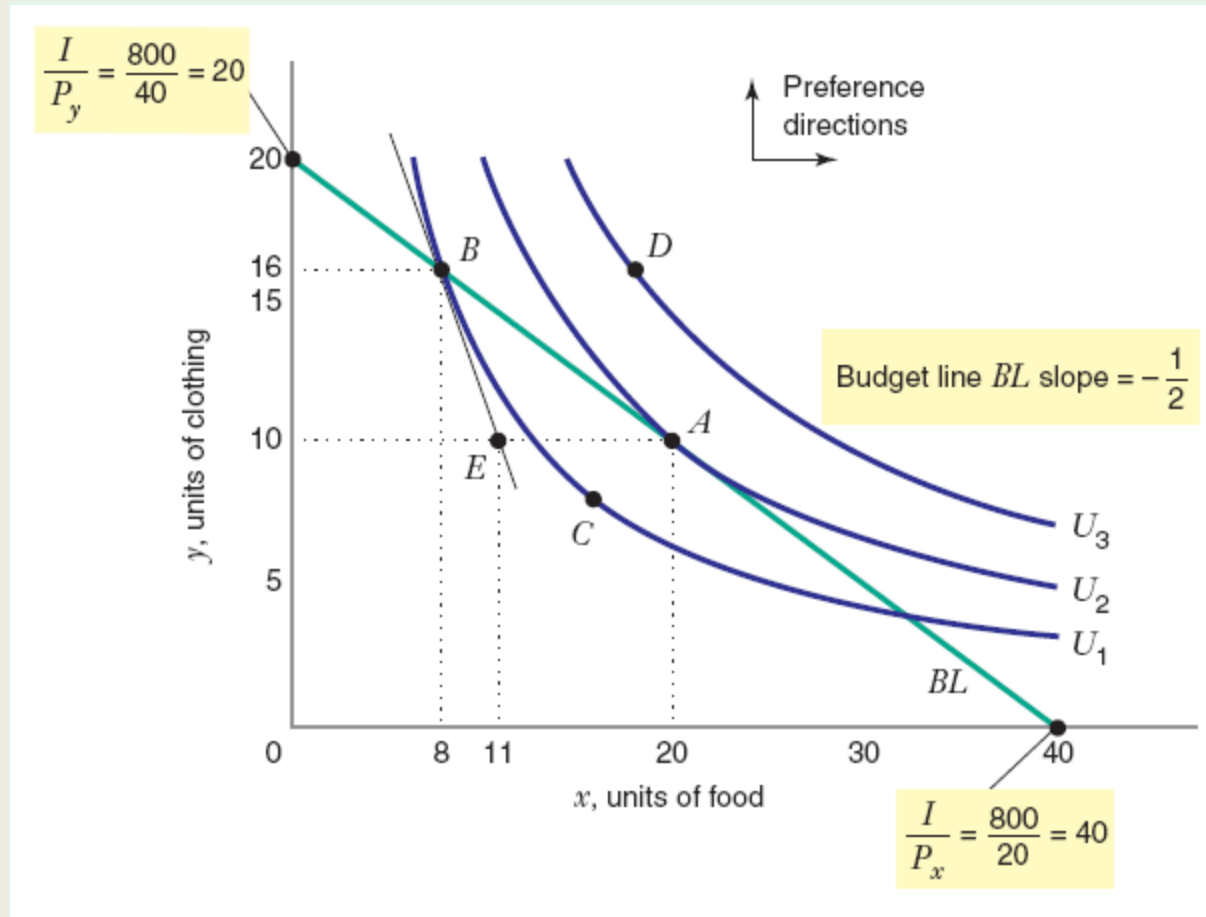
The consumer should consume 2 units of Y for each unit of X.

Budget Constraint: $800 = 20X + 40Y$

Solve for Y: $800 = 40Y + 40Y \ggg Y = 10$

Solve for X: $X = 2Y \ggg X = 20$

Interior Consumer Optimum





LEARNING-BY-DOING EXERCISE 4.2

Finding an Interior Optimum

Eric purchases food (measured by x) and clothing (measured by y) and has the utility function $U(x, y) = \sqrt{xy}$.

He has a monthly income of \$800. The price of food is $P_x = \$20$, and the price of clothing is $P_y = \$40$.

Problem Find Eric's optimal consumption bundle.

Corner Consumer Optimum

Corner Solution

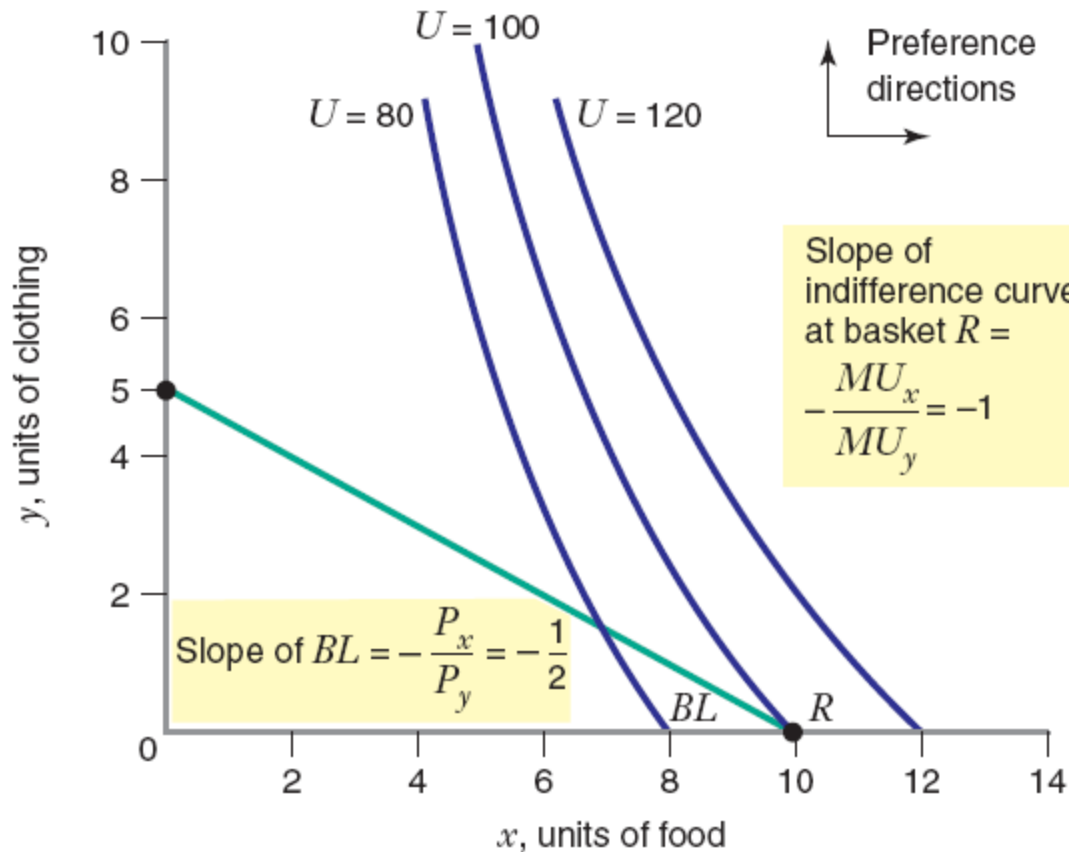
- One good is not being consumed at all.
- Optimal basket lies on the axis.
- For example, some consumers may not spend money on alcohol.

Condition for Corner Solution:

$$MU_x/P_x > MU_y/P_y \text{ for all bundles}$$

In this case, Y will not be consumed at all.

Corner Consumer Optimum



$$MU_x/MU_y > P_x/P_y$$

$$MU_x/P_x > MU_y/P_y$$

MU per dollar spent on X is always higher, so the consumer only consumes X.

Corner Solution: At R, slopes of BL and IC are not equal.



LEARNING-BY-DOING EXERCISE 4.3

Finding a Corner Point Solution

David is considering his purchases of food (x) and clothing (y). He has the utility function $U(x, y) = xy + 10x$, with marginal utilities $MU_x = y + 10$ and $MU_y = x$. His income is $I = 10$. He faces a price of food $P_x = \$1$ and a price of clothing $P_y = \$2$.

Problem What is David's optimal basket?

Duality

The mirror image of the original constrained optimization problem is called the **dual problem**.

$$\text{Min}_{(X,Y)} P_x X + P_y Y \quad \text{subject to: } U(X,Y) = U^*$$

where: U^ is a target level of utility.*

Here, we try to find X and Y that minimize total expenditure, provided that U^* is achieved.

Duality

FIGURE 4.5 Optimal Choice: Minimizing Expenditure to Achieve a Given Utility

Which basket should the consumer choose if he wants to minimize the expenditure necessary to achieve a level of utility U_2 ? He should select basket A, which can be purchased at a monthly expenditure of \$800. Other baskets on U_2 will cost the consumer more than \$800. For example, to purchase R or S (also on U_2), the consumer would need to spend \$1,000 per month (since R and S are on BL_3). Any total expenditure less than \$800 (e.g., \$640, represented by BL_1) will not enable the consumer to reach the indifference curve U_2 .

