Optimization without Constraint : The Case of More than One Choice Variable III

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CW Ch.11

Outline

- Economic Applications
- Comparative-Static Aspects of Optimization*

Economic Applications: Example 1: Problem of Multiproduct Firm

- A two-product firm is in a perfect competitive environment.
- With perfect competition, the prices of both products must be determined exogenously (from the viewpoint of the firm). These prices will be denoted by P_{10} and P_{20}, respectively.
- The firm (total) revenue function is

$$R = P_{10}Q_1 + P_{20}Q_2,$$

  where Q_i represents the output level of the ith product.
- The firm's (total) cost function is assumed to be

$$C = 2Q_1^2 + Q_1Q_2 + 2Q_2^2.$$

Economic Applications: Example 1: Problem of Multiproduct Firm

- The profit function of this firm can be written as

$$\pi = R - C = P_{10}Q_1 + P_{20}Q_2 - 2Q_1^2 - Q_1Q_2 - 2Q_2^2.$$

- To maximize profit,

FOC

$$\pi_1 = P_{10} - 4Q_1 - Q_2 = 0 \qquad (P_1 - MC_1 = 0)$$

$$\pi_2 = P_{20} - Q_1 - 4Q_2 = 0, \qquad (P_2 - MC_2 = 0)$$

which yield the unique solution

$$Q_1^* = \frac{4P_{10} - P_{20}}{15} \text{ and } Q_2^* = \frac{4P_{20} - P_{10}}{15}.$$

Economic Applications: Example 1: Problem of Multiproduct Firm

- SOC $\quad |H| = \begin{vmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{vmatrix} = \begin{vmatrix} -4 & -1 \\ -1 & -4 \end{vmatrix}$

  Since $|H_1| = -4 < 0$ and $|H_2| = 15 > 0$, $d^2\pi$ is negative definite and the solution does maximize profit.

- In fact, since the signs of the leading principal minors do not depend on the values of Q_1 and Q_2, $d^2\pi$ is *everywhere* negative definite. The maximum point found here is actually the unique absolute maximum.

Exercise 11.6 No.1

- Suppose a firm supplies two products to competitive markets. Let P_{10} and P_{20} be competitive prices of the products. The firm's cost function is $c = 2Q_1^2 + 2Q_2^2$. Figure out the maximum profit of the firm.

Economic Applications: Example 2: Monopolist with Multiproduct

- A two-product firm is the monopolist in both markets.
- Suppose the monopolist faces the demands as follows

$$Q_1 = 40 - 2P_1 + P_2$$
$$Q_2 = 15 + P_1 - P_2.$$

Observe that the two products are substitute goods.

- First, we need to figure out the prices of the two products as functions of the two quantities. From the demand functions above, we have

$$-2P_1 + P_2 = Q_1 - 40$$
$$P_1 - P_2 = Q_2 - 15.$$

Solving the two equations, considering Q_1 and Q_2 as parameters, we get

$$P_1 = 55 - Q_1 - Q_2$$
$$P_2 = 70 - Q_1 - 2Q_2.$$

These are the desired average revenue functions.

Economic Applications: Example 2: Monopolist with Multiproduct

- Consequently, the firm's total revenue function can be written as

$$\begin{aligned} R &= P_1Q_1 + P_2Q_2 \\ &= (55 - Q_1 - Q_2)Q_1 + (70 - Q_1 - 2Q_2)Q_2 \\ &= 55Q_1 + 70Q_2 - 2Q_1Q_2 - Q_1^2 - 2Q_2^2. \end{aligned}$$

- Assume the total cost function be

$$C = Q_1^2 + Q_1Q_2 + Q_2^2.$$

- The profit function of the monopolist can be written as

$$\pi = R - C = 55Q_1 + 70Q_2 - 3Q_1Q_2 - 2Q_1^2 - 3Q_2^2.$$

Economic Applications: Example 2: Monopolist with Multiproduct

- To maximize profit,

  FOC $\pi_1 = 55 - 3Q_2 - 4Q_1 = 0$

  $\pi_2 = 70 - 3Q_1 - 6Q_2 = 0,$

  which yields the solution output levels $Q_1^* = 8$ and $Q_2^* = 7\frac{2}{3}$.

  SOC $|H| = \begin{vmatrix} -4 & -3 \\ -3 & -6 \end{vmatrix}$.

  Since $|H_1| = -4 < 0$ and $|H_2| = 15 > 0$, $d^2\pi$ is negative definite and the solution does maximize profit.

- By substituting Q_1^* and Q_2^* into the relevant equations, we find that

$$P_1^* = 39\frac{1}{3}, P_2^* = 46\frac{2}{3}, \text{and } \pi^* = 488\frac{1}{3}.$$

  π^* is the maximum profit that the monopolist can get from producing these two products.