

① Firm 1:

$$\pi_1 = TR_1 - TC_1$$

$$\pi_1 = (a - bq_1 - bq_2 - bq_3)q_1 - C_1$$

$$\frac{\partial \pi_1}{\partial q_1} = a - 2bq_1 - bq_2 - bq_3 = 0$$

$$a - bq_2 - bq_3 = 2bq_1$$

$$q_1 = \frac{a - bq_2 - bq_3}{2b}$$

$$\text{sub } \textcircled{a} \rightarrow q_1 = \frac{a - \left(\frac{a - q_3 b}{3b}\right)b - q_3 b}{2b}$$

$$q_1 = \frac{3a - a + q_3 b - 3q_3 b}{6b}$$

$$q_1 = \frac{a - q_3 b}{3b} \quad \text{--- } \textcircled{1}$$

$$\text{sub } \textcircled{b} \rightarrow q_1 = \frac{a - \left(\frac{a}{4b}\right)b}{3b}$$

$$q_1 = \frac{4a - a}{12b} = \frac{a}{4b}$$

Firm 2:

$$\pi_2 = TR_2 - TC_2$$

$$\pi_2 = (a - bq_1 - bq_2 - bq_3)q_2 - C_2$$

$$\frac{\partial \pi_2}{\partial q_2} = a - bq_1 - 2bq_2 - bq_3 = 0$$

$$a - bq_1 - bq_3 = 2bq_2$$

$$\text{sub ①} \rightarrow 2q_2 b = a - \left( \frac{a - q_2 b - q_3 b}{2b} \right) b - q_3 b$$

$$2q_2 b = \frac{2a - a + q_2 b + q_3 b - 2q_3 b}{2}$$

$$4q_2 b = a + q_2 b - q_3 b$$

$$q_2 = \frac{a - q_3 b}{3b} \quad \text{--- ②}$$

$$\text{sub ③} \rightarrow q_2 = \frac{a - \left( \frac{a}{4b} \right) b}{3b}$$

$$q_2 = \frac{4a - a}{12b} = \frac{a}{4b}$$

Firm 3:

$$\pi_3 = TR_3 - TC_3$$

$$\pi_3 = (a - bq_1 - bq_2 - bq_3)q_3 - c_3$$

$$\frac{\partial \pi_3}{\partial q_3} = a - bq_1 - bq_2 - 2bq_3 = 0$$

$$a - bq_1 - bq_2 = 2bq_3$$

$$q_3 = \frac{a - bq_1 - bq_2}{2b}$$

$$\text{sub ① \& ②} \rightarrow q_3 = \frac{a - b \left( \frac{a - q_3 b}{3b} \right) - b \left( \frac{a - q_3 b}{3b} \right)}{2b}$$

$$q_3 = \frac{3a - a + q_3 b - a + q_3 b}{3b}$$

$$q_3 = \frac{a + 2q_3 b}{3b}$$

$$3bq_3 = a + 2q_3 b$$

$$4bq_3 = a + 2q_3 b$$

$$q_3 = \frac{a}{4b}$$

At equilibrium price;  $p = a - bQ$

$$p = a - b(q_1 + q_2 + q_3)$$

$$p = a - b\left(\frac{q}{4b} + \frac{q}{4b} + \frac{q}{4b}\right)$$

$$p = a - \left(\frac{3q}{4}\right) = \frac{q}{4} = 0.25q$$

$\therefore$  firm 1;  $\pi_1 = pQ - c_1$

$$= \left(0.25q \cdot \frac{q}{4b}\right) - c_1$$

$$\pi_1 = \frac{q^2}{16b} - c_1 \quad \#$$

firm 2;  $\pi_2 = pQ_2 - c_2$

$$= \left(0.25q \cdot \frac{q}{4b}\right) - c_2$$

$$\pi_2 = \frac{q^2}{16b} - c_2 \quad \#$$

firm 3;  $\pi_3 = pQ_3 - c_3$

$$= \left(0.25q \cdot \frac{q}{4b}\right) - c_3$$

$$\pi_3 = \frac{q^2}{16b} - c_3 \quad \#$$

$$\textcircled{2} \text{ Assume: } q_1 + q_2 + \dots + q_n = A$$

$$P = a - b(q_1 + q_2 + \dots + q_n)$$

$$p = a - bq_1 - bq_2 - \dots - bq_n$$

$$\pi_1 = (a - bq_1 - bq_2 - \dots - bq_n) \cdot q_1 - c_1$$

$$\pi_n = (a - bq_1 - bq_2 - \dots - bq_n) \cdot q_n - c_n$$

$$\frac{\partial \pi_1}{\partial q_1} = a - 2bq_1 - bq_2 - \dots - bq_n = 0$$

$$q_1 = \frac{a}{2b} - 0.5(q_2 + q_3 + \dots + q_n)$$

$$q_n = \frac{a}{2b} - 0.5(q_1 + q_2 + \dots + q_{n-1})$$

$$\therefore q_1 - 0.5q_1 = \frac{a}{2b} - 0.5(q_1 + q_2 + \dots + q_n)$$

$$0.5q_1 = \frac{a}{2b} - 0.5A$$

$$q_1 = \frac{a}{b} - A \quad \text{--- (1)}$$

$$q_2 = \frac{a}{b} - A$$

⋮

$$q_n = \frac{a}{b} - A$$

$$\text{Since } A = q_1 + q_2 + \dots + q_n$$

$$\text{so } A = n\left(\frac{a}{b} - A\right)$$

$$= n\frac{a}{b} - nA$$

$$A + nA = n\left(\frac{a}{b}\right)$$

$$A(1+n) = n\left(\frac{a}{b}\right)$$

$$A = \frac{na}{(n+1)b}$$

$$\text{sub into ①; } q_1 = \frac{a}{(n+1)b}$$

$$\therefore q_i = \frac{a}{(n+1)b} \quad \#$$

$$\text{equilibrium price; } p = a - b(A)$$

$$p = a - b\left(\frac{nq}{(n+1)b}\right)$$

$$p = a - \left(\frac{n}{n+1}\right)q$$

$$p = \frac{a(n+1) - nq}{n+1}$$

$$p = \frac{na + a - nq}{n+1} = \frac{a}{n+1}$$

$$\pi_i = pq_i - c_i$$

$$= \left(\frac{a}{n+1} \cdot \frac{a}{(n+1)b}\right) - c_i$$

$$= \frac{a^2}{(n+1)^2 b} - c_i \quad \#$$

③

if  $n \rightarrow \infty$  ;

$q_i = \frac{a}{(n+1)b}$  ; nearly zero and each firms will sell  $q_i$  at nearly zero unit.

$A = nq_i$  ; nearly zero. So,  $Q$  of each firms combined will nearly  $\infty$  unit.

$p = \frac{a}{n+1}$  ; nearly zero. When supply increases, price will decrease nearly zero.

$\pi_i = \frac{a^2}{(n+1)^2 b} - c_i$  ; each firm will lose the profit.

If  $n = 1$  ;

$q_i = \frac{a}{(n+1)b} = \frac{a}{2b}$  Since  $Q = \frac{a}{2b} < Q = \frac{na}{(n-1)b}$ , monopoly will sell less quantity.

$A = nq_i = Q$  Since  $n = 1$ , the firm will be monopoly.

$p = \frac{a}{n+1} = \frac{a}{2}$  since  $P_M = \frac{a}{2} > P = \frac{a}{n-1}$ , monopoly firm will set the higher price.

$\pi_i = \frac{a^2}{(n+1)^2 b} - c_i = \frac{a^2}{4b} - c_i$  which higher than  $\pi_i = \frac{a^2}{(n+1)^2 b}$

As a result, the monopoly will gain more profit.