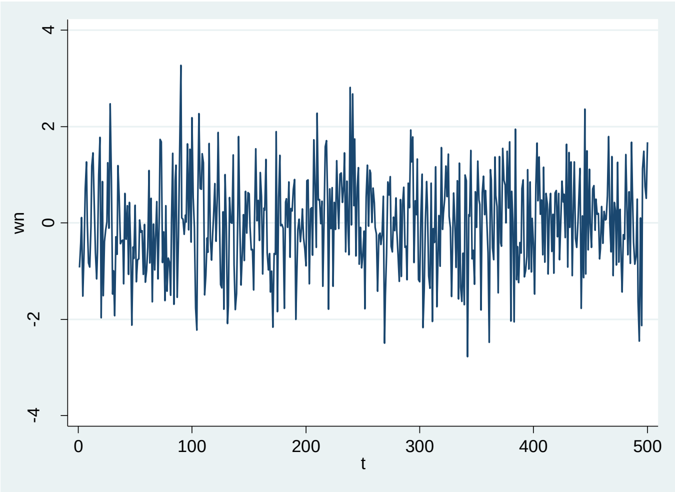
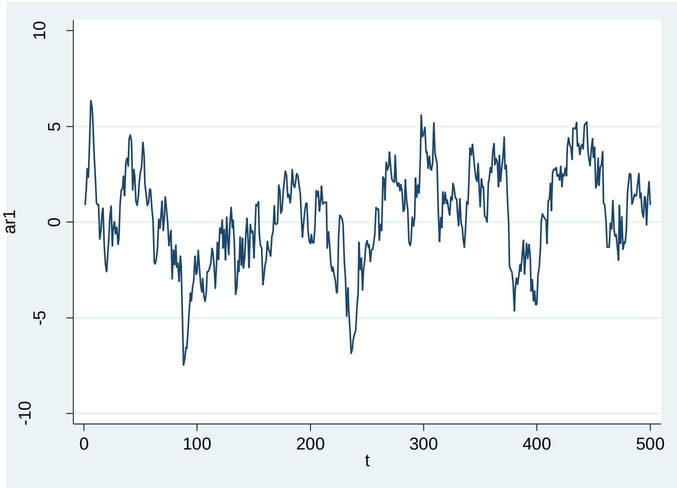
Time Series Models

Simulated AR Time Series

```
. set obs 500
obs was 0, now 500

. g t=_n

. tsset t
        time variable:  t, 1 to 500
                delta:  1 unit

. set seed 1234

. g wn=rnormal(0,1)

. line wn t
```

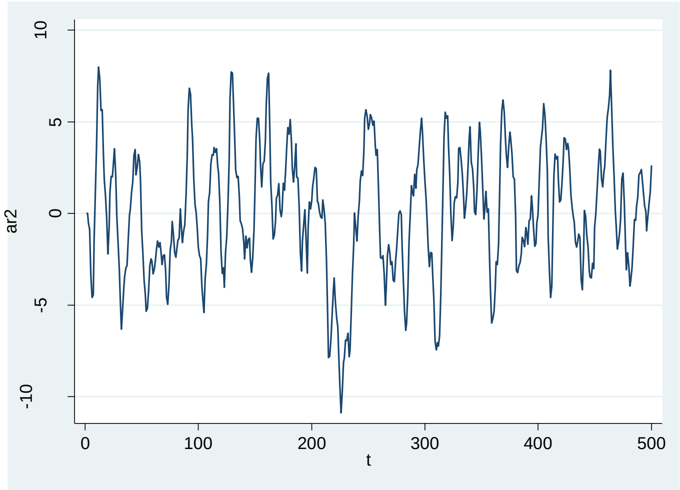



```
. g ar1=0.9 in 1
(499 missing values generated)

. replace ar1=0.9*l.ar1+rnormal(0,1) if t>1
(499 real changes made)

. line ar1 t
```




```
. g ar2=0 if t<3
(498 missing values generated)
```

```
. replace ar2=1.5*l.ar2-0.6*l2.ar2+rnormal(0,1) if t>2
(498 real changes made)

. line ar2 t
```



Random Walk without Drift:

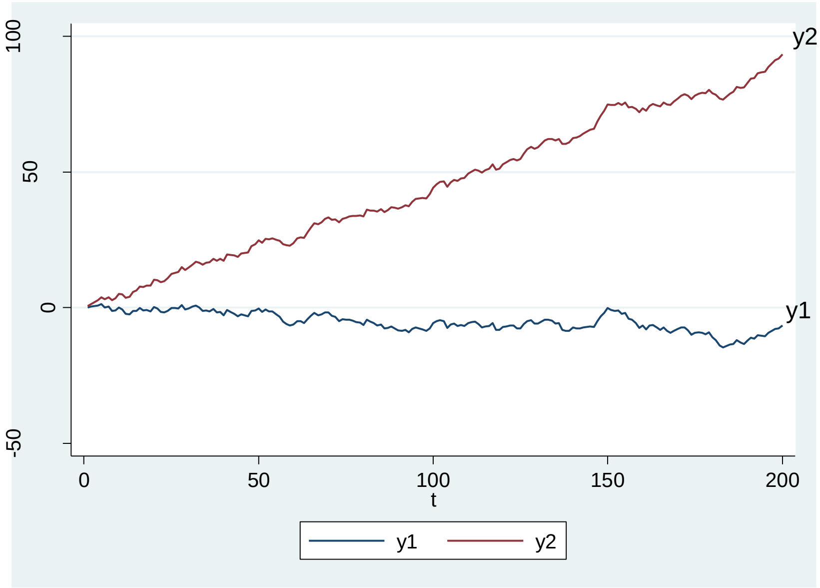
$$Y_{1t} = Y_{1t-1} + u_t$$

where: u_t is white noise error term with mean = 0 and variance = σ^2

Random Walk with Drift:

$$Y_{2t} = \delta + Y_{2t-1} + u_t$$

```
. set obs 200
obs was 0, now 200

. gener t=_n

. tsset t
        time variable:  t, 1 to 200

. gener u= rnormal(0,1)

. gener y1=0 in 1
(199 missing values generated)

. gener y2=0.5 in 1
(199 missing values generated)

. replace y1=l.y1 +u if t>1
(199 real changes made)

. replace y2=0.5+l.y2 + u if t>1
(199 real changes made)

. line y1 y2 t
```



Unit Root Problem

Autoregressive (Stationary) Series

$$x_{1t} = 0.05 + 0.95\, x_{1t-1} + u_{1t}$$

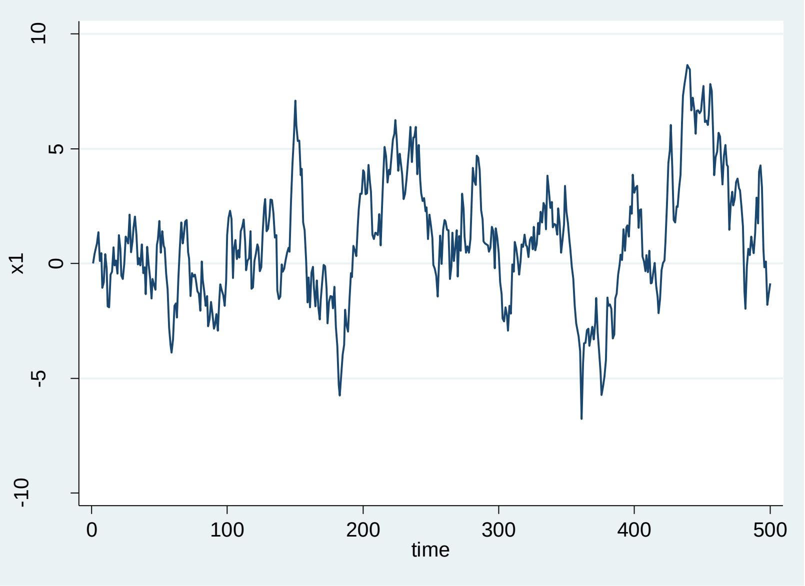
Random Walk with Drift

$$x_{2t} = 0.05 + x_{2t-1} + u_{2t}$$

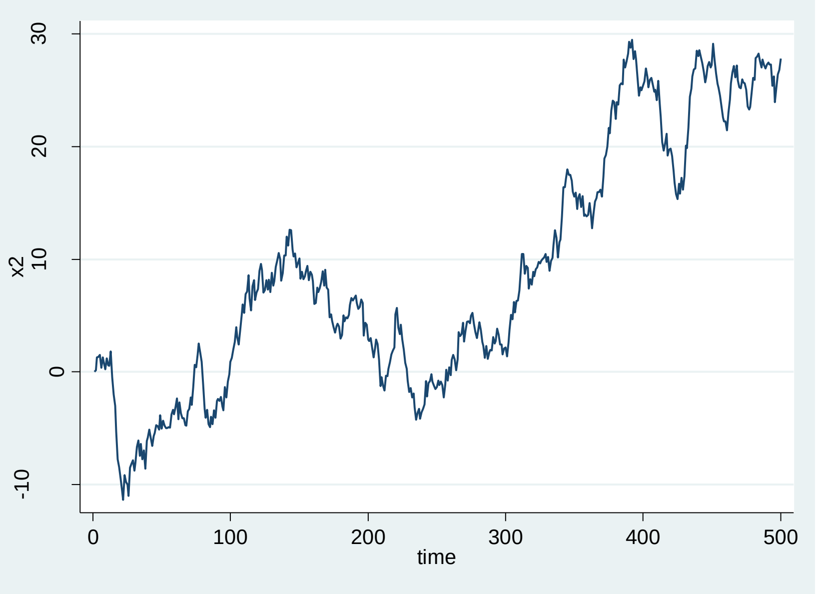
Explosive Series

$$x_{3t} = 0.05 + 1.05\, x_{3t-1} + u_{3t}$$

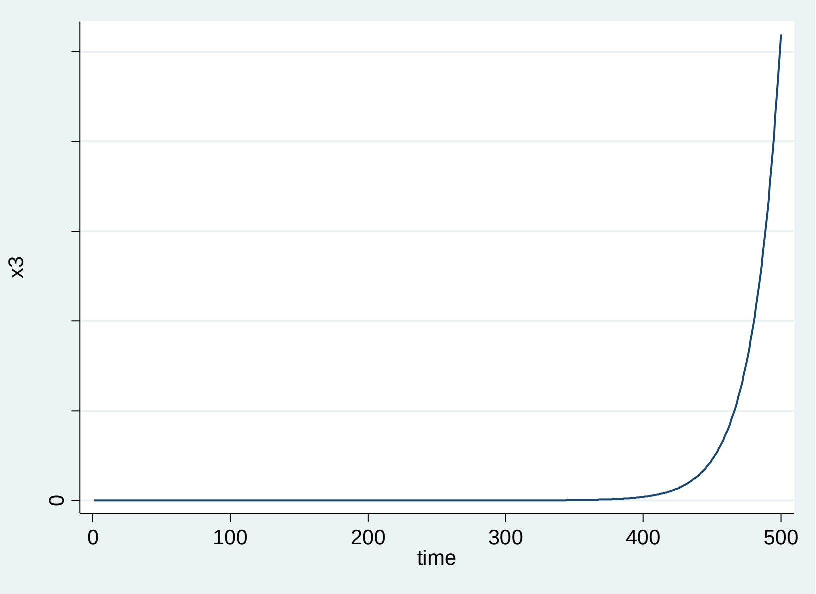
```
. set obs 500
obs was 0, now 500

. gener time=_n

. tsset time
        time variable:  time, 1 to 500

. gener x1=0 in 1
(499 missing values generated)

. gener x2=0 in 1
(499 missing values generated)

. gener x3=0 in 1
(499 missing values generated)

. gener u1= rnormal(0,1)

. gener u2= rnormal(0,1)

. gener u3= rnormal(0,1)

. replace x1=0.05+0.95*l.x1+u1 if time>1
(499 real changes made)

. replace x2=0.05+l.x2+u2 if time>1
(499 real changes made)

. replace x3=0.05+1.05*l.x3+u3 if time>1
(499 real changes made)

. line x1 time

. line x2 time

. line x3 time
```



x1
10
5
0
-5
-10
0
100
200
300
400
500
time



x2
30
20
10
0
-10
0
100
200
300
400
500
time



x3
0
0
100
200
300
400
500
time

Spurious Regression

$$Y_t = Y_{t-1} + u_t$$
$$X_t = X_{t-1} + v_t$$

If X_t and Y_t are uncorrelated $I(1)$ processes, regression $Y_t = \beta_1 + \beta_2 X_t + \varepsilon_t$ can lead to spurious problem.

```
. set obs 500
obs was 0, now 500

. gener time=_n

. tsset time
        time variable:  time, 1 to 500

. gener y=0 in 1
(499 missing values generated)

. gener x=0 in 1
(499 missing values generated)

. gener u= rnormal(0,1)

. gener v= rnormal(0,1)

. replace y=l.y+u if time>1
(499 real changes made)

. replace x=l.x+v if time>1
(499 real changes made)

. regress y x

      Source |       SS       df       MS              Number of obs =     500
-------------+------------------------------           F(  1,   498) =   75.33
       Model |  1563.54402     1  1563.54402           Prob > F      =  0.0000
    Residual |  10336.0273   498  20.7550748           R-squared     =  0.1314
-------------+------------------------------           Adj R-squared =  0.1297
       Total |  11899.5713   499  23.8468362           Root MSE      =  4.5558

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           x |  -.1385236   .0159599    -8.68   0.000    -.1698807   -.1071665
       _cons |   -10.1792     .24019   -42.38   0.000    -10.65111   -9.707289
------------------------------------------------------------------------------

. dwstat

Durbin-Watson d-statistic(  2,   500) =  .0573669
```

Trend Stationary (TS) and Difference Stationary (DS)

Pure random walk $\beta_1 = 0, \beta_2 = 0, \beta_3 = 1$

$$Y_{1t} = Y_{1t-1} + u_t$$

Random walk with drift $\beta_1 \neq 0, \beta_2 = 0, \beta_3 = 1$

$$Y_{2t} = \beta_1 + Y_{2t-1} + u_t$$

Deterministic trend $\beta_1 \neq 0, \beta_2 \neq 0, \beta_3 = 0$

$$Y_{3t} = \beta_1 + \beta_2 t + u_t$$

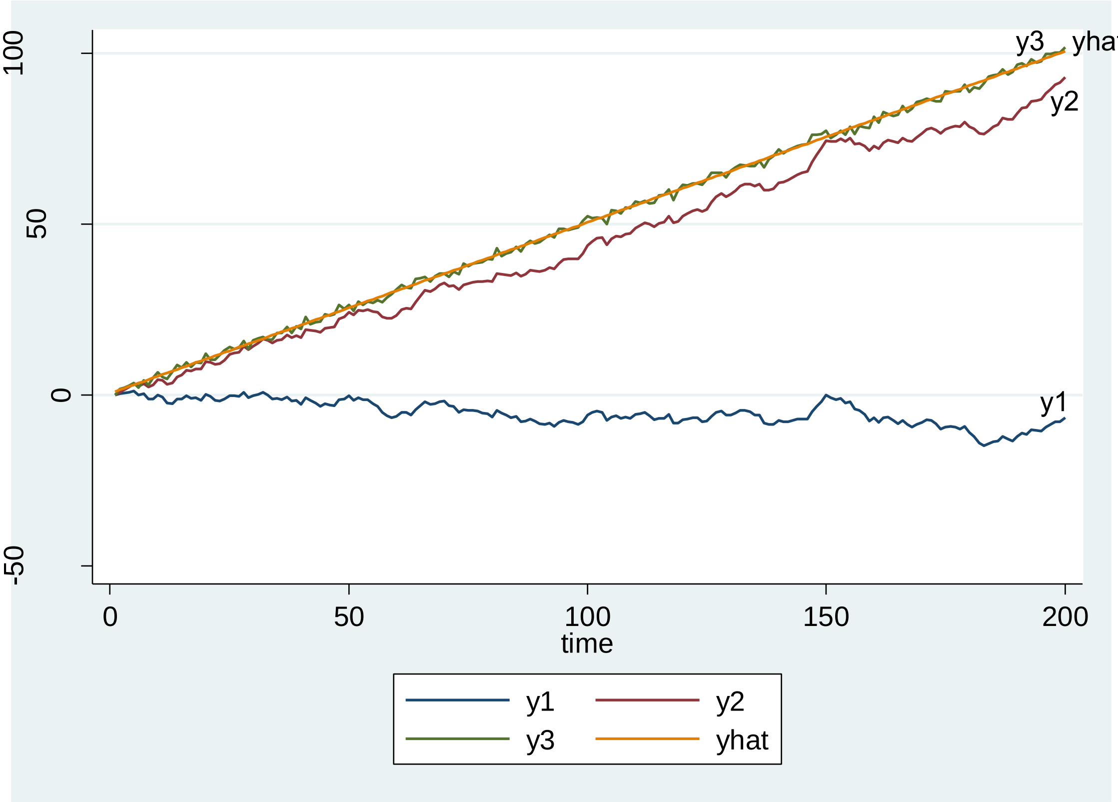
Random walk with drift and deterministic trend $\beta_1 \neq 0, \beta_2 \neq 0, \beta_3 = 1$

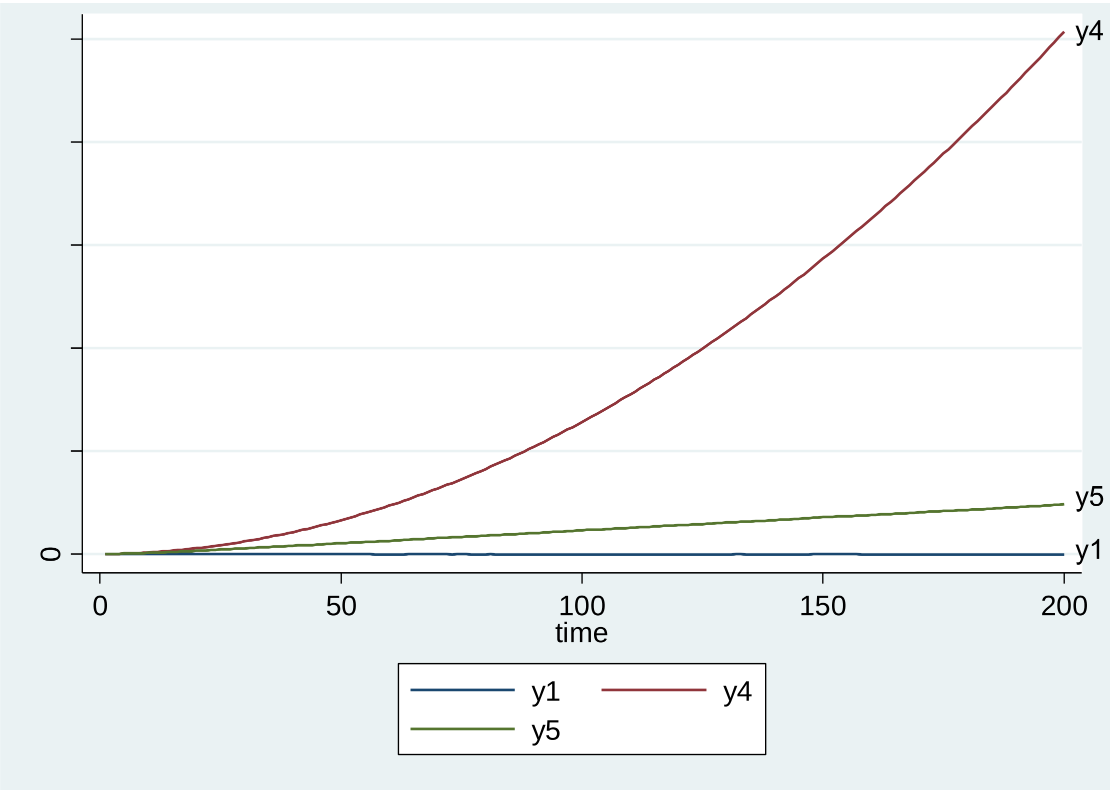
$$Y_{4t} = \beta_1 + \beta_2 t + Y_{4t-1} + u_t$$

Deterministic trend with stationary AR(1) component $\beta_1 \neq 0, \beta_2 \neq 0, \beta_3 < 1$

$$Y_{5t} = \beta_1 + \beta_2 t + \beta_3 Y_{5t-1} + u_t$$

```
. set obs 200
obs was 0, now 200

. gener time=_n

. tsset time
        time variable:  time, 1 to 200

. gener y1=0 in 1
(199 missing values generated)

. gener y2=0 in 1
(199 missing values generated)

. gener y3=0 in 1
(199 missing values generated)

. gener y4=0 in 1
(199 missing values generated)

. gener y5=0 in 1
(199 missing values generated)

. gener u = rnormal(0,1)

. gener t=_n

. replace y1=l.y1+u if time>1
(199 real changes made)

. replace y2=0.5+l.y2+u if time>1
(199 real changes made)

. replace y3=0.5+0.5*t+u if time>1
(199 real changes made)

. replace y4=0.5+0.5*t+l.y4+u if time>1
(199 real changes made)

. replace y5=0.5+0.5*t+0.9*l.y5+u if time>1
(199 real changes made)

. gener yhat=0.5+0.5*t

. line y1 y2 y3 yhat time

. line y1 y4 y5 time
```


100
50
0
-50
0
50
100
150
200
time
y3
yhat
y2
y1
y1
y2
y3
yhat


0
0
50
100
150
200
time
y4
y5
y1
y1
y4
y5

Unit Root Test

1. Test with all terms (intercept, trend, and lags):

```
. dfuller y, trend lags(1) regress

Augmented Dickey-Fuller test for unit root         Number of obs   =       498

                              ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -2.049            -3.980            -3.420            -3.130
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.5747

------------------------------------------------------------------------------
D.y          |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |
         L1. |  -.0187253   .0091402    -2.05   0.041    -.0366837   -.0007668
         LD. |  -.1694873   .0445717    -3.80   0.000    -.2570608   -.0819138
      _trend |   .0299962   .0292846     1.02   0.306    -.0275415    .0875339
       _cons |  -14.66152   9.461164    -1.55   0.122    -33.25061    3.927562
------------------------------------------------------------------------------
```

The result shows p-value=0.5747 (fail to reject null hypothesis of unit root). Then, check whether trend is significant and find out that p-value=0.306. Thus, there must be no trend. Go to next step:

2. Test with intercept and lags:

```
. dfuller y,  lags(1) regress

Augmented Dickey-Fuller test for unit root         Number of obs   =       498

                              ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -1.935            -3.440            -2.870            -2.570
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.3156

------------------------------------------------------------------------------
         D.y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |
         L1. |  -.0175501   .0090684    -1.94   0.054    -.0353673    .0002672
         LD. |  -.1685012   .0445635    -3.78   0.000    -.2560582   -.0809442
             |
       _cons |  -6.704068   5.400393    -1.24   0.215    -17.31459    3.906451
------------------------------------------------------------------------------
```

The result shows p-value=0.3156 (fail to reject null hypothesis of unit root). Then, check whether constant term is significant and find out that p-value=0.215. Thus, there must be no constant term. Go to next step:

3. Test with lags – no intercept:

```
. dfuller y, nocon lags(1) regress

Augmented Dickey-Fuller test for unit root         Number of obs   =       498

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(t)             -1.484            -2.580            -1.950            -1.620

------------------------------------------------------------------------------
         D.y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |
         L1. |  -.0104078   .0070134    -1.48   0.138    -.0241875    .0033718
         LD. |  -.1723889   .0444776    -3.88   0.000    -.2597766   -.0850012
------------------------------------------------------------------------------
```

The result shows the test statistic value (`Z(t)`) `= -1.484` that lies inside the 95% confident level (`5% Critical Value = -1.950`) range, therefore, the null hypothesis of Unit-root test is failed to reject, then, the series are non-stationary.

Since the series are not integrated at order 0, next step is to test whether the series are integrated at order 1 or not by perform unit root test of Δy_t or `D.y`.

Additional Testing Methods
Dickey-Fuller GLS Test

```
. dfgls y, maxlag(10)

DF-GLS for y                                          Number of obs =   489

               DF-GLS tau      1% Critical       5% Critical      10% Critical
  [lags]     Test Statistic        Value             Value             Value
------------------------------------------------------------------------------
    10           -2.206           -3.480            -2.849            -2.563
    9            -2.229           -3.480            -2.852            -2.567
    8            -2.166           -3.480            -2.856            -2.570
    7            -2.508           -3.480            -2.859            -2.572
    6            -2.452           -3.480            -2.862            -2.575
    5            -2.062           -3.480            -2.865            -2.578
    4            -2.044           -3.480            -2.868            -2.581
    3            -1.894           -3.480            -2.871            -2.583
    2            -1.831           -3.480            -2.874            -2.586
    1            -1.698           -3.480            -2.876            -2.588

Opt Lag (Ng-Perron seq t) =  8 with RMSE  90.41123
Min SC   =  9.089339 at lag  1 with RMSE  92.94486
Min MAIC =  9.062736 at lag  8 with RMSE  90.41123

. dfgls d.y, maxlag(10)

DF-GLS for D.y                                        Number of obs =   488

               DF-GLS tau      1% Critical       5% Critical      10% Critical
  [lags]     Test Statistic        Value             Value             Value
------------------------------------------------------------------------------
    10           -4.642           -3.480            -2.849            -2.564
    9            -4.526           -3.480            -2.852            -2.567
    8            -4.790           -3.480            -2.856            -2.570
    7            -5.308           -3.480            -2.859            -2.572
    6            -4.967           -3.480            -2.862            -2.575
    5            -5.404           -3.480            -2.865            -2.578
    4            -6.980           -3.480            -2.868            -2.581
    3            -7.989           -3.480            -2.871            -2.583
    2           -10.145           -3.480            -2.874            -2.586
    1           -13.424           -3.480            -2.876            -2.588

Opt Lag (Ng-Perron seq t) =  7 with RMSE  92.82793
Min SC   =  9.149196 at lag  5 with RMSE  93.36743
Min MAIC =  9.550801 at lag  9 with RMSE  92.51999
```

Phillips-Perron Test

```
. pperron y, trend regress

Phillips-Perron test for unit root                 Number of obs   =       499
                                                   Newey-West lags =         2

                                     ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(rho)           -9.437          -28.898           -21.499           -18.100
 Z(t)             -2.228           -3.980            -3.420            -3.130
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.4742

------------------------------------------------------------------------------
y            |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |
         L1. |   .9779577   .0091986   106.32   0.000     .9598848    .9960306
      _trend |   .0262301   .0295507     0.89   0.375      -.03183    .0842901
       _cons |  -14.93069    9.53107    -1.57   0.118    -33.65693    3.795564
------------------------------------------------------------------------------

. pperron y,  regress
```

```
Phillips-Perron test for unit root                 Number of obs   =       499
                                                   Newey-West lags =         5

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(rho)           -9.754           -20.499           -14.000           -11.200
 Z(t)             -2.223            -3.440            -2.870            -2.570
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.1981

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |
         L1. |   .9789795   .0091243   107.29   0.000     .9610525    .9969064
             |
       _cons |  -7.988098   5.445442    -1.47   0.143    -18.68702    2.710825
------------------------------------------------------------------------------

. pperron y,  nocon regress

Phillips-Perron test for unit root                 Number of obs   =       499
                                                   Newey-West lags =         5

                               ---------- Interpolated Dickey-Fuller ---------
                  Test         1% Critical       5% Critical      10% Critical
               Statistic           Value             Value             Value
------------------------------------------------------------------------------
 Z(rho)           -5.695           -13.700            -8.000            -5.700
 Z(t)             -1.691            -2.580            -1.950            -1.620

------------------------------------------------------------------------------
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |
         L1. |   .9874316   .0070831   139.41   0.000     .9735151    1.001348
------------------------------------------------------------------------------
```