

Question 0: (Three-variable optimization)

Suppose that there are three firms. Each firm produces one single product that is imperfectly substitutable to the other two products, i.e. differentiated products. Each of the firm *strategically* competes with each other, and faces the following market demand equations given by,

$$Q_1 = 80 - 2P_1 + P_{23}$$

$$Q_2 = 80 - 2P_2 + P_{13}$$

$$Q_3 = 80 - 2P_3 + P_{12}$$

where P_{23} is the average of the prices charged by firms 2 and 3, P_{13} is the average of the prices charged by firms 2, P_{12} is the average of the prices charged by firms 1 and 2 (e.g., $P_{12} = 0.5(P_1 + P_2)$). Suppose that each of the three firms, as indexed by j , has the cost function given by $C^j(Q_j) = c_j Q_j$.

Consider the following problems.

- Set up the profit function of each individual firm, and derive the profit-maximizing condition of each of the three firms when each of them *optimally* sets for the level of price that maximizes its own profit. (Hint: Derive the best response function of each firm, i.e. optimal contingent plan or reaction function.)
- Suppose that Based on (a), what would be the *equilibrium* level of price that each of the firms choose. Calculate the level of profit that each firm yields. (Hint: Solve for the Bertrand equilibrium with differentiated product)
- How does the optimal price vary with respect to the size of marginal cost?

(a)

$$\pi_1 = TR_1 - TC_1$$

$$= P_1 Q_1 - C_1 Q_1$$

$$= P_1 (80 - 2P_1 + P_{23}) - C_1 (80 - 2P_1 + P_{23})$$

$$\frac{d\pi_1}{dP_1} = P_1(-2) + (80 - 2P_1 + P_{23}) + 2C_1 = 0$$

$$80 - 4P_1 + P_{23} + 2C_1 = 0$$

$$P_1 = 80 + P_{23} + 2C_1$$

$$BR_1(P_2, P_3) \quad P_1 = \frac{80 + 0.5(P_2 + P_3) + 2C_1}{4}$$

$$\pi_3 = TR_3 - TC_3$$

$$= P_3 Q_3 - C_3 Q_3$$

$$= P_3 (80 - 2P_3 + P_{12}) - C_3 (80 - 2P_3 + P_{12})$$

$$\frac{d\pi_3}{dP_3} = P_3(-2) + (80 - 2P_3 + P_{12}) + 2C_3 = 0$$

$$80 - 4P_3 + P_{12} + 2C_3 = 0$$

$$P_3 = 80 + P_{12} + 2C_3$$

$$BR_3(P_1, P_2) \quad P_3 = \frac{80 + 0.5(P_1 + P_2) + 2C_3}{4}$$

$$\begin{bmatrix} 4 & -0.5 & -0.5 \\ -0.5 & 4 & -0.5 \\ -0.5 & -0.5 & 4 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix} = \begin{bmatrix} 80 + 2C_1 \\ 80 + 2C_2 \\ 80 + 2C_3 \end{bmatrix}$$

$$adj(A) = [cof(A)]^T$$

$$cof(A) = \begin{bmatrix} (-1)^2(16 - 0.25) & (-1)^4(0.25 + 2) & (-1)^4(0.25 + 2) \\ (-1)^3(-2 - 0.25) & (-1)^3(-2 - 0.25) & (-1)^4(16 - 0.25) \\ (-1)^4(0.25 + 2) & (-1)^4(16 - 0.25) & (-1)^3(-2 - 0.25) \end{bmatrix} = \begin{bmatrix} 15.75 & 2.25 & 2.25 \\ 2.25 & 15.75 & 2.25 \\ 2.25 & 2.25 & 15.75 \end{bmatrix}$$

$$[cof(A)]^T = \begin{bmatrix} 15.75 & 2.25 & 2.25 \\ 2.25 & 15.75 & 2.25 \\ 2.25 & 2.25 & 15.75 \end{bmatrix}$$

$$\det(A) = (64 + .125 + .125) - (1 + 1 + 1) = 61.25 \quad A^{-1} = \frac{1}{61.25} \begin{bmatrix} 15.75 & 2.25 & 2.25 \\ 2.25 & 15.75 & 2.25 \\ 2.25 & 2.25 & 15.75 \end{bmatrix}$$

$$= \begin{bmatrix} 0.2571 & 0.0367 & 0.0367 \\ 0.0367 & 0.2571 & 0.0367 \\ 0.0367 & 0.0367 & 0.2571 \end{bmatrix}$$

$$A^{-1}D = \begin{bmatrix} 0.2571 & 0.0367 & 0.0367 \\ 0.0367 & 0.2571 & 0.0367 \\ 0.0367 & 0.0367 & 0.2571 \end{bmatrix} \begin{bmatrix} 80 + 2C_1 \\ 80 + 2C_2 \\ 80 + 2C_3 \end{bmatrix} = \begin{bmatrix} 20.568 + 0.5142C_1 + 2.936 + 0.0734C_2 + 2.936 + 0.0734C_3 \\ 2.936 + 0.0734C_1 + 20.568 + 0.5142C_2 + 2.936 + 0.0734C_3 \\ 2.936 + 0.0734C_1 + 2.936 + 0.0734C_2 + 20.568 + 0.5142C_3 \end{bmatrix}$$

$$= \begin{bmatrix} 26.44 + 0.5142C_1 + 0.0734C_2 + 0.0734C_3 \\ 26.44 + 0.0734C_1 + 0.5142C_2 + 0.0734C_3 \\ 26.44 + 0.0734C_1 + 0.0734C_2 + 0.5142C_3 \end{bmatrix}$$

$$P_1^* = 26.44 + 0.5142C_1 + 0.0734C_2 + 0.0734C_3$$

$$P_2^* = 26.44 + 0.0734C_1 + 0.5142C_2 + 0.0734C_3$$

$$P_3^* = 26.44 + 0.0734C_1 + 0.0734C_2 + 0.5142C_3$$

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b)

$$P_1^* = 26.44 + 0.5142C_1 + 0.0734C_2 + 0.0734C_3$$

$$P_2^* = 26.44 + 0.0734C_1 + 0.5142C_2 + 0.0734C_3$$

$$P_3^* = 26.44 + 0.0734C_1 + 0.0734C_2 + 0.5142C_3$$

$$\pi_1^* = (26.44 + 0.5142C_1 + 0.0734C_2 + 0.0734C_3)(80 - 2P_1 + P_2)$$

$$= (26.44 - 0.4858C_1 + 0.0734C_2 + 0.0734C_3)(80 - 2P_1 + P_2)$$

$$\pi_2^* = (26.44 + 0.0734C_1 + 0.5142C_2 + 0.0734C_3)(80 - 2P_2 + P_3)$$

$$= (26.44 + 0.0734C_1 - 0.4858C_2 + 0.0734C_3)(80 - 2P_2 + P_3)$$

$$\pi_3^* = (26.44 + 0.0734C_1 + 0.0734C_2 + 0.5142C_3)(80 - 2P_3 + P_2)$$

$$= (26.44 + 0.0734C_1 + 0.0734C_2 - 0.4858C_3)(80 - 2P_3 + P_2)$$

c)

$$\frac{\partial P_1^*}{\partial c} = 0.5142$$

$$\frac{\partial P_2^*}{\partial c} = 0.5142$$

$$\frac{\partial P_3^*}{\partial c} = 0.5142$$

∴ when the marginal cost rises by one unit each of the firms optimal price increases by 0.5142 unit

Now suppose that all the three firms agree to operate under a cartel (collusive) agreement. That is, they consider a joint pricing scheme that maximizes the joint profit function of the three firms combined. Consider the following problems.

- d) Construct the joint profit function.
- e) Given that $c_j = c, \forall j$. Calculate the level of optimal price setting that each of the firm will set under the joint profit function problem. Confirm your result with the second-order derivative test.
- f) Is the Cartel agreement self-sustaining? Why?

d)

$$\pi_T = P_1Q_1 + P_2Q_2 + P_3Q_3 - c_1Q_1 - c_2Q_2 - c_3Q_3$$

$$= P_1(80 - 2P_1 + P_2) + P_2(80 - 2P_2 + P_3) + P_3(80 - 2P_3 + P_2) - C_1(80 - 2P_1 + P_2) - C_2(80 - 2P_2 + P_3) - C_3(80 - 2P_3 + P_2)$$

$$= (80 - 2P_1 + P_2)(P_1 - C_1) + (80 - 2P_2 + P_3)(P_2 - C_2) + (80 - 2P_3 + P_2)(P_3 - C_3) \neq$$

e)

$$\frac{d\pi_T}{dP_1} = (80 - 2P_1 + P_2) - (P_1 - C_1)(-2) + (P_2 - C_2)(0.5) + (P_3 - C_3)(0.5) = 0$$

$$80 - 2P_1 + 0.5P_2 + 0.5P_3 - 2P_1 + 2C_1 + 0.5P_2 - 0.5C_2 + 0.5P_3 - 0.5C_3 = 0$$

$$80 - 4P_1 + P_2 + P_3 + C_1 = 0$$

$$4P_1 - P_2 - P_3 = 80 + C_1 \quad \text{--- (1)}$$

$$\frac{d\pi_T}{dP_2} = (P_1 - C_1)(0.5) + (80 - 2P_2 + P_3) + (P_2 - C_2)(-2) + (P_3 - C_3)(0.5) = 0$$

$$0.5P_1 - 0.5C_1 + C_2 + 80 - 2P_2 + 0.5P_1 + 0.5P_3 - 2P_2 + 2C_2 + 0.5P_3 - 0.5C_3 = 0$$

$$80 + P_1 - 4P_2 + P_3 + C_2 = 0$$

$$-P_1 + 4P_2 - P_3 = 80 + C_2 \quad \text{--- (2)}$$

$$\frac{d\pi_T}{dP_3} = (P_1 - C_1)(0.5) + (P_2 - C_2)(0.5) + (80 - 2P_3 + P_2) + (P_3 - C_3)(-2) = 0$$

$$0.5P_1 - 0.5C_1 + 0.5P_2 - 0.5C_2 + (80 - 2P_3 + 0.5P_1 + 0.5P_2) - 2P_3 + 2C_3 = 0$$

$$80 + P_1 + P_2 - 4P_3 + C_3 = 0$$

$$-P_1 - P_2 + 4P_3 = 80 + C_3 \quad \text{--- (3)}$$

$$-4P_1 - P_2 - P_3 = 80 + 2C_1 - 0.5C_2 - 0.5C_3$$

$$-P_1 + 4P_2 - P_3 = 80 - 0.5C_1 + 2C_2 - 0.5C_3$$

$$-P_1 - P_2 + 4P_3 = 80 - 0.5C_1 - 0.5C_2 + 2C_3$$

$$\textcircled{1} = \textcircled{2}$$

Plug $P_1 = P_2$ into $\textcircled{1}$ & $\textcircled{3}$

$$4P_1 - P_2 - P_3 = -P_1 + 4P_2 - P_3$$

$$4P_1 - P_1 - P_3 = -P_1 - P_1 + 4P_3$$

$$5P_1 = 5P_2$$

$$3P_1 = -2P_1 + 5P_3$$

$$P_1 = P_2$$

$$P_1 = P_3$$

Plug $P_1 = P_2 = P_3$ into $\textcircled{1}$

$$4P_1 - P_1 - P_1 = 80 - C$$

$$P_1^* = P_2^* = P_3^* = \frac{80 - C}{3}$$

Second Order Derivative Test

$$H = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix} = \begin{bmatrix} -4 & 1 & 1 \\ 1 & -4 & 1 \\ 1 & 1 & -4 \end{bmatrix} \quad \begin{matrix} -4 & 1 \\ 1 & -4 \\ 1 & 1 \end{matrix}$$

$$|H_1| = -4 \quad ; |H_1| < 0 \quad ; \forall P_1, \forall P_2, \forall P_3$$

$$|H_2| = 16 - 1 = 15 \quad ; |H_2| > 0 \quad ; \forall P_1, \forall P_2, \forall P_3$$

$$|H_3| = (-64 + 1 + 1) - (-4 - 4 - 4) = -62 + 12 = -50$$

$\therefore H$ is negative definite $\rightarrow \det H < 0$

The function is globally concave

P_1^*, P_2^* and P_3^* are thus the global solutions

f) Cartel agreement is not self-sustaining even though all three products of the firms are different.

The quantity demand of each good is interdependent on one another. If one of the firms were to set price a little bit higher than it is in the agreement, the quantity demand for other products will drop. Accordingly, the cheated firm will gain relatively higher profit compared to the rest.

g) Revisit question (c) and assume that the marginal costs are varied with respect to firms. What would be the implication of the collusive equilibrium?

The implication of collusive equilibrium would be that if marginal costs are varied with respect to firms, they should set the price of each product based on marginal cost of all three goods. That is, if the price of product I has high marginal cost, holding all other things constant, the firm should sell it at relatively higher price than the product II and product III in order to maximize its profit. At the end, they should satisfy the condition where $MR(Q_1^*) = MR(Q_2^*) = MR(Q_3^*) = MC(Q_{total})$.

Question 2:

Pakorn's utility function depends on the consumption of two commodities, x and y , and it is given by

$$U(x, y) = 2xy$$

Suppose that his income is \$72, and the prices per unit of x and y are \$4 and \$6, respectively. Assume that Pakorn spends all of his income, and the values of x and y are both non-zero.

- Use the Lagrange method to determine the values of x^* and y^* that maximize Pakorn's utility given an income constraint. Verify that the second-order sufficient conditions are satisfied.
- Determine the maximum utility level and the Lagrange multiplier. Interpret the economic interpretation of the Lagrange multiplier.
- Suppose that the income is now \$73. Approximate the new maximum utility level.

$U(x, y) = 2xy$
Objective function

$I = 72 \$$ $P_x = 4 \$$ $P_y = 6 \$$

BL $\Rightarrow 4x + 6y = 72 \rightarrow$ Constraint function $\rightarrow 72 - 4x - 6y = 0$

a) Use Lagrange method to find x^*, y^*

Objective + $\lambda(c)$

$$L(x, y, \lambda) = 2xy + \lambda(72 - 4x - 6y)$$

F.O.C

[x] $\rightarrow \frac{\partial L}{\partial x} = 2y - 4\lambda = 0$ — ①

[y] $\rightarrow \frac{\partial L}{\partial y} = 2x - 6\lambda = 0$ — ②

[λ] $\rightarrow \frac{\partial L}{\partial \lambda} = 72 - 4x - 6y = 0$ — ③

slope BL $\leftarrow \frac{P_x}{P_y} = \frac{MU_x}{MU_y} \rightarrow$ slope IC

① $\Rightarrow \frac{4}{6} = \frac{y}{x}$

$\frac{2}{3} = \frac{y}{x}$

$x = \frac{3y}{2} \Rightarrow y = \frac{2x}{3}$

$72 - 4x - 6y = 0$

$-12y = -72$

$\therefore y = 6^*$ units $\therefore x = 9^*$ units

S.O.C

$$H = \begin{bmatrix} L_{xx} & L_{xy} & L_{x\lambda} \\ L_{xy} & L_{yy} & L_{y\lambda} \\ L_{x\lambda} & L_{y\lambda} & L_{\lambda\lambda} \end{bmatrix} \quad n=2 \rightarrow (n+1)C(n+1)$$

check: Bordered Hessian matrix

$$H = \begin{bmatrix} 0 & -4 & -6 & 0 & -4 \\ -4 & 0 & 2 & 4 & 0 \\ -6 & 2 & 0 & -6 & 2 \end{bmatrix}$$

$|H| = (0 + 48 + 48) - (0) = 96 > 0 \Rightarrow$ max point

$\therefore x = 9, y = 6$ is the absolute maximum of this function.

b. Determine the maximum utility level and the Lagrange multiplier. Interpret the economic interpretation of the Lagrange multiplier.

$$\text{maximum utility level} \Rightarrow UC(x, y) = 2xy = 2(4)(6) = 108$$

Lagrange multiplier $\Rightarrow \lambda^*$ is Lagrange multiplier

$$x^* = 4, y^* = 6$$

From F.O.C

$$[x] \rightarrow \frac{\partial L}{\partial x} = 2y - 4\lambda = 0 \quad \text{--- (1)}$$

$$y \rightarrow \textcircled{1} \Rightarrow 12 - 4\lambda = 0 \\ \therefore \lambda^* = 3$$

$$[y] \rightarrow \frac{\partial L}{\partial y} = 2x - 6\lambda = 0 \quad \text{--- (2)}$$

$$[\lambda] \rightarrow \frac{\partial L}{\partial \lambda} = 72 - 4x - 6y = 0 \quad \text{--- (3)}$$

$\lambda^* = \frac{\partial f}{\partial c}$ that is $c \uparrow$ by 1 unit, f increases by 3 units.

c. Suppose that the income is now \$73. Approximate the new maximum utility level.

$$\begin{array}{l} \text{now } c \text{ is } 72 \$ \\ \text{If income change to } 73 \$ \end{array} \left. \vphantom{\begin{array}{l} \text{now } c \text{ is } 72 \$ \\ \text{If income change to } 73 \$ \end{array}} \right\} \Delta c = 1 \\ \therefore \Delta f = 3 \Delta c = 3 \cdot 1 = 3$$

the maximum utility level will increase by 3 unit

$$\text{that is } 108 + 3 = 111 \quad *$$

where r is the interest rate ($0 < r < 1$), and I_1 is the income in period 1.

Answer the following questions.

- a) Write down the utility maximization problem subject to the above budget constraint. Use Lagrange method to derive the optimal values of C_1^* and C_2^* in terms of the parameters θ , β , r , and I_1 .
- b) Based on your answer in (3.1), derive the impact of an increase in the interest rate on the optimal consumption in period 1 ($\frac{\partial C_1^*}{\partial r}$). Interpret its economic meaning.
- c) Verify your answer in (a) by using the second-order sufficient condition.

Question 4:

A consumer has the utility function $U = \ln C + \ln(24 - N)$, where C is consumption and N is labor supply. Her budget constraint is $pC = \bar{M} + wN$, where p is the price of the consumption good, w the wage rate, and \bar{M} the consumer's non-wage income.

- (a) Formulate the problem of utility maximization subject to the budget constraint, and derive the first-order conditions, using the Lagrange multiplier approach and ignoring the nonnegativity constraints.
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- (b) Find the demand function $C = C^*(p, w, \bar{M})$ and the labor supply function $N = N^*(p, w, \bar{M})$ (i.e., express C and N in terms of p , w , and \bar{M}). Show that N^* and C^* are homogeneous of degree zero in (p, w, \bar{M}) .
 - (c) Let $U^* = \ln[C^*(p, w, \bar{M})] + \ln[24 - N^*(p, w, \bar{M})]$. Show that $\partial U^*/\partial \bar{M} > 0$ and $\partial U^*/\partial p < 0$. Show that U^* is concave in \bar{M} and convex in p . What is the relationship between $\partial U^*/\partial \bar{M}$ and the Lagrange multiplier?
-

b) demand function $C^* = \frac{0.5\bar{M} + 12w}{p} = \frac{(0.5\bar{M}) + (12w)}{(1p)}$
 $= \frac{(1') (0.5\bar{M}) + (12w)}{p}$

supply function $N^* = \frac{-\bar{M} + 24w}{2w} = \frac{(-\bar{M}) + (24w)}{(2w)}$
 $= (1') \frac{(-\bar{M} + 24w)}{2w}$

(c) Let $U^* = \ln[C^*(p, w, \bar{M})] + \ln[24 - N^*(p, w, \bar{M})]$. Show that $\partial U^*/\partial \bar{M} > 0$ and $\partial U^*/\partial p < 0$. Show that U^* is concave in \bar{M} and convex in p . What is the relationship between $\partial U^*/\partial \bar{M}$ and the Lagrange multiplier?

$$U^* = \ln\left(\frac{0.5\bar{M} + 12w}{p}\right) + \ln\left(\frac{24 + \bar{M} - 24w}{2w}\right)$$

$$\frac{\partial U^*}{\partial \bar{M}} = \frac{1}{\frac{0.5\bar{M} + 12w}{p}} \cdot \frac{p(0.5) - (0.5\bar{M} + 12w)(0)}{p^2} + \frac{1}{\frac{24w + \bar{M}}{2w}} \left(\frac{2w \cdot 1 - (24w + \bar{M})(0)}{(2w)^2} \right)$$

$$= \frac{p}{0.5\bar{M} + 12w} \cdot \frac{1}{p} + \frac{2w}{24 + \bar{M}} \cdot \frac{1}{2w} = \frac{1}{24 + \bar{M}} > 0 \quad \checkmark$$

$$\frac{\partial U^*}{\partial p} = \frac{1}{\frac{0.5\bar{M} + 12w}{p}} \cdot \frac{p(0) - (0.5\bar{M} + 12w)(1)}{p^2} + \frac{1}{\frac{24w + \bar{M}}{2w}} \cdot \frac{2w(0) - 24w\bar{M}(0)}{(2w)^2}$$

$$= \frac{p}{0.5\bar{M} + 12w} \cdot \frac{-0.5\bar{M} - 12w}{p^2} = \frac{-0.5\bar{M} - 12w}{0.5\bar{M} + 12w} \cdot \frac{1}{p} = \frac{-1}{p} < 0 \quad \checkmark$$

S.O.C $\Rightarrow \frac{\partial^2 U^*}{\partial \bar{M}^2} = \frac{-1}{(24 + \bar{M})^2} < 0 \Rightarrow$ concave \checkmark

$\frac{\partial^2 U^*}{\partial p^2} = \frac{1}{p^2} > 0 \Rightarrow$ convex \checkmark

relationship of $\frac{\partial v^r}{\partial m^*}$ \rightarrow

$$= \frac{1}{24+m}$$

that is when $m^* \uparrow$ by 1 unit, $v^r \uparrow$ by $\frac{1}{24+m}$ units.

Lagrange multiplier $\rightarrow \lambda^*$

$$c^r = \frac{0.5\bar{m} + 12w}{p}$$

from (2) $\rightarrow \frac{1}{c} - \lambda p = 0$

$$\therefore \lambda^* = \frac{1}{24+m}$$

$$\frac{\partial v}{\partial m} = \frac{1}{24+m}$$

that is $m \uparrow 1 \Rightarrow v \uparrow \frac{1}{24+m}$ units.

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Question 4: Cost minimization problem

Consider a cost minimization problem where firm chooses for optimal combination of capital (K) and labor (L). Suppose that r and w are the prices per unit of capital and labor, respectively. And assume further that the production technology of this firm is given by $\sqrt{K} + L = Q$. Consider the following problem

- a. Solve for the optimal combination of capital and labor.
- b. State the condition under which both types of factor inputs are used by firm.
- c. Derive the cost function.

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min $wL + rK$
 objective

$\sqrt{K} + L = Q$
 constraint $\rightarrow Q - \sqrt{K} - L$

$\mathcal{L}(L, K, \lambda; w, r, Q) = wL + rK + \lambda(Q - \sqrt{K} - L)$

$\frac{\partial \mathcal{L}}{\partial L} = w - \lambda = 0 \quad \text{--- (1)}$

$\frac{\partial \mathcal{L}}{\partial K} = r - \frac{1}{2} \lambda K^{-1/2} = 0 \quad \text{--- (2)}$

$\frac{\partial \mathcal{L}}{\partial \lambda} = Q - \sqrt{K} - L = 0 \quad \text{--- (3)}$

slope isocost \rightarrow
 (1) $\frac{w}{r} = \frac{1}{2} K^{-1/2}$
 $\rightarrow MP_L$

$\frac{w}{r} = 2K^{1/2}$

$\frac{w}{2r} = K^{1/2}$

$\therefore K^* = \frac{w^2}{4r^2}$

$Q - \sqrt{\frac{w^2}{4r^2}} - L = 0$

$Q - \frac{w}{2r} - L = 0$

$L^* = Q - \frac{w}{2r}$

optimal $L^* = Q - \frac{w}{2r}$

optimal $K^* = \frac{w^2}{4r^2}$

c) cost function = $c(L^*, k^*) = wL + rk$

$$= w \left(Q - \frac{w}{2r} \right) + r \left(\frac{w^2}{4r^2} \right)$$

$$= wQ - \frac{w^2}{2r} + \frac{w^2}{4r}$$

$$= wQ$$

∴ cost function = wQ

$x^* = \underline{mc}$ ← $\frac{dc}{dQ} = w$ that is when $Q \uparrow 1$
Total cost will increase by w unit.

b) change in TC will depend on the value of w .

If the value of w is high, the cost will highly increase too.

∴ Firm will choose to minimize w that is also minimize cost too.

Therefore, firm will prefer K more than L . Because w will reduce if firm use less L .