Prospect Theory and Stock Returns: An Empirical Test

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We test the hypothesis that, when thinking about allocating money to a stock, investors mentally represent the stock by the distribution of its past returns and then evaluate this distribution in the way described by prospect theory. In a simple model of asset prices in which some investors think in this way, a stock whose past return distribution has a high (low) prospect theory value earns a low (high) subsequent return, on average. We find empirical support for this prediction in the cross-section of stock returns in the U.S. market, and also in a majority of forty-six other national stock markets. (*JEL* D03)

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A crucial ingredient in any model of asset prices is an assumption about how investors evaluate risk. Most of the available models assume that investors evaluate risk according to the expected utility framework, and models based on this assumption have been helpful for thinking about a number of empirical facts. Nonetheless, a large body of research shows that, at least in laboratory settings, attitudes to risk can depart significantly from the predictions of expected utility and that an alternative theory – "prospect theory," due to Kahneman and Tversky (1979) and Tversky and Kahneman (1992) – captures these attitudes more accurately. This raises an obvious question: can models

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in which some investors evaluate risk according to prospect theory help us make more sense of the data on asset prices and asset returns? In this paper, we present new evidence on this question. We derive the predictions, for the cross-section of stock returns, of a simple prospect theory-based model and test these predictions in both U.S. and international data.

Applying prospect theory outside the laboratory presents a challenge for researchers. To see why, it is helpful to think of decision making under prospect theory as involving two steps: "representation" and "valuation." First, for any risk that an individual is considering, he forms a mental representation of that risk. Specifically, since, under prospect theory, people are assumed to derive utility from gains and losses, the individual forms a mental representation of the gains and losses he associates with taking the risk. Second, the individual evaluates this representation – this distribution of gains and losses – to see if it is appealing.

Valuation, the second step, is straightforward: Tversky and Kahneman (1992) provide detailed formulas that specify the value that prospect theory would assign to any given distribution of gains and losses. The difficult step, for the researcher, is the first one, namely, representation. How does an individual mentally represent a risk that he is considering? In experimental settings, the answer is clear: participants in laboratory studies are typically *provided* with a representation for any risk they are asked to consider – a 50:50 bet to win $110 or lose $100, say. Outside the laboratory, however, the answer is less clear: how do investors who are thinking about a stock represent that stock in their minds?[1]

We suggest that, for many investors, their mental representation of a stock is the *distribution of the stock's past returns*. The most obvious reason why people might adopt this representation is because they see the past return distribution as a good and easily accessible proxy for the object they are truly interested in, namely, the distribution of the stock's *future* returns. This belief may be mistaken: a stock with a high mean return over the past few years typically has a low subsequent return (De Bondt and Thaler 1985); and a stock with highly skewed past returns does not necessarily exhibit high skewness in its future returns. Nonetheless, many investors may *think* that a stock's past return distribution is a good approximation of its future return distribution, and therefore adopt the past return distribution as their mental representation of the stock.

In this paper, we test the pricing implications of the joint hypothesis laid out above: that some investors in the economy think about stocks in terms of their historical return distributions; and that they evaluate these distributions

[1] Representation plays less of a role in the expected utility framework because of the strong convention that the argument of the utility function is the final wealth level. In prospect theory, by contrast, individuals derive utility from "gains and losses." Kahneman and Tversky (1979) offer little guidance on how these gains and losses should be defined. As a result, the question of representation becomes important.

according to prospect theory. To understand the implications of this hypothesis, we construct a simple model of asset prices in which some investors allocate money across stocks in the following way. For each stock in the cross-section, they take the stock's historical return distribution and compute the prospect theory value of this distribution. If the prospect theory value is high, they tilt toward the stock in their portfolios; by assumption, the stock is appealing to these investors. Conversely, if the prospect theory value is low, they tilt away from the stock; by assumption, the stock is unappealing to these investors. The model makes a simple prediction which we test in our empirical work: that stocks with high prospect theory values will have low subsequent returns, on average, while stocks with low prospect theory values will have high subsequent returns. The intuition is clear: stocks with high prospect theory values are appealing to some investors; these investors tilt toward these stocks in their portfolios, causing the stocks to become overvalued and to earn low subsequent returns.

We expect our prediction about returns to hold more strongly among stocks more heavily traded by individual investors, for example, among small-cap stocks. This is because the behavior that underlies our prediction is relatively unsophisticated and is therefore likely a better description of what individual investors do than of what institutional investors do. For example, the investors we describe engage in "narrow framing:" when thinking about a stock, they evaluate the return distribution of the stock itself; more sophisticated investors would evaluate the return distribution of the overall portfolio that results from tilting toward the stock. Moreover, the investors in our framework evaluate the stock's past returns; more sophisticated investors would try to forecast the stock's future returns, and would evaluate those.

To test our prediction – that the prospect theory value of a stock's past return distribution predicts the stock's subsequent return with a negative sign – we need to define "past return distribution." The easiest way for investors to learn about a stock's past return distribution is to look at a chart of the stock's past price movements – specifically, at the chart that usually appears front and center when they look up information about the stock. In defining "past return distribution," we therefore take guidance from the typical format of these charts. In the Internet era, these charts come in a variety of formats. Most of our data are drawn from the pre-Internet era, however, and during this period, the main sources of information about stocks for many investors were so-called investment handbooks, such as the Value Line Investment Survey. These handbooks feature charts prominently and present them using a fairly standard format. Based on a review of these sources, we suggest that a natural mental representation of a stock's past return distribution is the distribution of its monthly returns over the previous five years.

In summary, our main empirical prediction is that stocks whose historical return distributions have high (low) prospect theory values will have low (high) subsequent returns. We expect this prediction to hold primarily among small-cap stocks, in other words, among stocks for which individual investors play a more important role.

In our empirical analysis of U.S. stock returns, we find support for this prediction. We conduct a variety of tests, but it is easiest to understand our main result in a Fama-MacBeth framework. Each month we compute, for each stock in the cross-section, the stock's prospect theory value – the prospect theory value of the distribution of the stock's monthly returns over the previous five years. For each month in the sample, we then run a cross-sectional regression of subsequent stock returns on this prospect theory value, including the important known predictors of returns as controls. Consistent with our hypothesis, we find that the coefficient on the stock's prospect theory value, averaged across all the monthly regressions, is significantly negative: stocks with higher prospect theory values have lower subsequent returns. We also find, again consistent with our framework, that this result is stronger among small-cap stocks.

Further analysis provides additional support for our hypothesis. We show that the predictive power of prospect theory value for subsequent stock returns is stronger among stocks less subject to arbitrage – for example, among illiquid stocks and stocks with high idiosyncratic volatility. And in an important out-of-sample test, we repeat our analysis in each of forty-six international stock markets covered by Datastream. We find support for our main predictions in a large majority of these markets as well.

In our final set of results, we try to understand exactly what it is about a high prospect theory value stock that might be appealing to investors, and what it is about a low prospect theory value stock that might be aversive. We find that a significant part of prospect theory value's predictive power for returns comes from the "probability weighting" component of prospect theory. Under probability weighting, an individual overweights the *tails* of a return distribution, a device that, among other things, captures the widespread preference for lottery-like gambles. The fact that probability weighting plays an important role in our results suggests – and we confirm this in the data – that a high prospect theory value stock is a stock whose past returns are very positively skewed. Part of what may be driving our results, then, is that when investors observe the stock's past return distribution, perhaps by looking at a price chart, they see the skewness, which, in turn, leads them to think of the stock as a lottery-like gamble and hence to find it appealing. By tilting toward the stock in their portfolios, they cause it to become overvalued and to earn a low subsequent return.

The trading behavior we propose in this paper has an important precedent in Benartzi and Thaler's (1995) influential work on the equity premium puzzle. Benartzi and Thaler suggest that people evaluate the stock market by computing the prospect theory value of its historical return distribution and, similarly, that they evaluate the bond market by computing the prospect theory value of *its* historical return distribution. The individuals in our framework think in a similar way: they evaluate a stock by computing the prospect theory value of its historical return distribution. In this sense, our analysis can be thought of as the stock-level analog of Benartzi and Thaler (1995), one that, surprisingly, has not yet been investigated.

Our research is also related to prior work that uses prospect theory to think about the cross-section of average returns. Barberis and Huang (2008) study asset prices in a one-period economy in which investors derive prospect theory utility from the change in their wealth over the course of the period. This framework generates a new prediction, one that does not emerge from the traditional analysis based on expected utility, namely, that a security's expected future skewness – including even idiosyncratic skewness – will be priced: a stock whose future returns are expected to be positively skewed will be "overpriced" and will earn a lower average return. Over the past few years, several papers, using various measures of expected skewness, have presented evidence in support of this prediction (Kumar 2009; Boyer, Mitton, and Vorkink 2010; Bali, Cakici, and Whitelaw 2011; Conrad, Dittmar, and Ghysels 2013). Moreover, the idea that expected skewness is priced has been used to make sense of a variety of empirical facts, including the low average returns of IPO stocks, distressed stocks, high volatility stocks, stocks sold in over-the-counter markets, and out-of-the-money options (all of these assets have positively skewed returns); the diversification discount; and the lack of diversification in many household portfolios.[2]

In this paper, we examine the cross-section of average stock returns using a different implementation of prospect theory, one that makes a different assumption about the representation of gains and losses that investors have in their minds when thinking about a stock. In Barberis and Huang's (2008) framework, investors apply prospect theory to gains and losses in the value of their overall *portfolios*, and the portfolio gains and losses they are thinking about are *future* gains and losses. By contrast, in our framework, investors apply prospect theory to *stock*-level gains and losses (narrow framing) and specifically to *past* stock-level gains and losses. The differing assumptions of the two frameworks lead to distinct empirical predictions.

1. Conceptual Framework

In our framework, the amount of money that some investors allocate to a stock depends, in part, on the prospect theory value of the stock's historical return distribution. In this section, we discuss our framework in more detail. Specifically, in Section 1.1, we review the mechanics of prospect theory. In Section 1.2, we discuss how "historical return distribution" should be defined. And in Section 1.3, we present a simple model that formalizes our main

2 For further discussion of these applications, see Mitton and Vorkink (2007), Boyer, Mitton, and Vorkink (2010), Ilmanen (2012), Boyer and Vorkink (2014), and Eraker and Ready (2015).

empirical prediction – that, in the cross-section, a stock's prospect theory value will predict the stock's subsequent return with a negative sign.

1.1 Prospect theory

In this section, we review the elements of prospect theory. Readers already familiar with this material may prefer to jump to Section 1.2.

The original version of prospect theory is described in Kahneman and Tversky (1979). While this paper contains all of the theory's essential insights, the specific model it presents has some limitations: it can be applied only to gambles with at most two nonzero outcomes, and it predicts that people will sometimes choose dominated gambles. Tversky and Kahneman (1992) propose a modified version of the theory, known as cumulative prospect theory, that resolves these problems. This is the version typically used in economic analysis and is the version we adopt in this paper.[3]

To see how cumulative prospect theory works, consider the gamble

$$(x_{-m}, p_{-m}; \ldots; x_{-1}, p_{-1}; x_0, p_0; x_1, p_1; \ldots; x_n, p_n), \tag{1}$$

which should be read as "gain or lose x_{-m} with probability p_{-m}, x_{-m+1} with probability p_{-m+1}, and so on," where $x_i < x_j$ for $i < j$ and where $x_0 = 0$, so that x_{-m} through x_{-1} are losses and x_1 through x_n are gains, and where $\sum_{i=-m}^{n} p_i = 1$. For example, a 50:50 bet to win \$110 or lose \$100 would be written as $(-\$100, \frac{1}{2}; \$110, \frac{1}{2})$. In the expected utility framework, an individual with utility function $U(\cdot)$ evaluates the gamble in (1) by computing

$$\sum_{i=-m}^{n} p_i U(W + x_i), \tag{2}$$

where W is his current wealth. A cumulative prospect theory individual, by contrast, assigns the gamble the value

$$\sum_{i=-m}^{n} \pi_i v(x_i), \tag{3}$$

where

$$\pi_i = \begin{cases} w^+(p_i + \ldots + p_n) - w^+(p_{i+1} + \ldots + p_n) & \text{for} \quad 0 \le i \le n \\ w^-(p_{-m} + \ldots + p_i) - w^-(p_{-m} + \ldots + p_{i-1}) & \phantom{\text{for}} \quad -m \le i < 0 \end{cases}, \tag{4}$$

and where $v(\cdot)$ is known as the value function and $w^+(\cdot)$ and $w^-(\cdot)$ as probability weighting functions.[4] Tversky and Kahneman (1992) propose the functional

[3] While our analysis is based on cumulative prospect theory, we often abbreviate this to "prospect theory."

[4] When $i = n$ or $i = -m$, Equation (4) reduces to $\pi_n = w^+(p_n)$ and $\pi_{-m} = w^-(p_{-m})$, respectively.

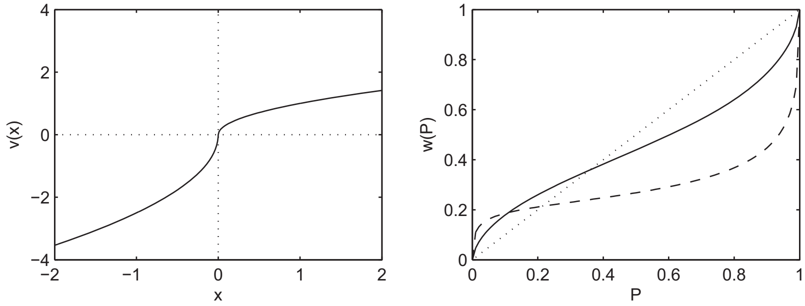

Figure 1
The prospect theory value function and probability weighting function
The left panel plots the value function proposed by Tversky and Kahneman (1992) as part of their cumulative prospect theory, namely, $v(x)=x^{\alpha}$ for $x \geq 0$ and $v(x)=-\lambda(-x)^{\alpha}$ for $x<0$, for $\alpha=0.5$ and $\lambda=2.5$. The right panel plots the probability weighting function they propose, namely, $w(P)=P^{\delta}/(P^{\delta}+(1-P)^{\delta})^{1/\delta}$, for three different values of δ. The dashed line corresponds to $\delta=0.4$, the solid line to $\delta=0.65$, and the dotted line to $\delta=1$.

forms

$$v(x)=\begin{cases} x^{\alpha} & \text{for} \quad x \geq 0 \\ -\lambda(-x)^{\alpha} & \phantom{\text{for}} \quad x<0 \end{cases} \tag{5}$$

and

$$w^{+}(P)=\frac{P^{\gamma}}{(P^{\gamma}+(1-P)^{\gamma})^{1/\gamma}}, \qquad w^{-}(P)=\frac{P^{\delta}}{(P^{\delta}+(1-P)^{\delta})^{1/\delta}}, \tag{6}$$

where α, γ, $\delta \in (0,1)$ and $\lambda>1$. The left panel in Figure 1 plots the value function in (5) for $\alpha=0.5$ and $\lambda=2.5$. The right panel in the figure plots the weighting function $w^{-}(P)$ in (6) for $\delta=0.4$ (the dashed line), for $\delta=0.65$ (the solid line), and for $\delta=1$, which corresponds to no probability weighting (the dotted line). Note that $v(0)=0$, $w^{+}(0)=w^{-}(0)=0$, and $w^{+}(1)=w^{-}(1)=1$.

There are four important differences between (2) and (3). First, the carriers of value in prospect theory are gains and losses, not final wealth levels: the argument of $v(\cdot)$ in (3) is x_i, not $W+x_i$. Second, while $U(\cdot)$ is typically differentiable everywhere, the value function $v(\cdot)$ is kinked at the origin, as shown in Figure 1, so that the individual is more sensitive to losses – even small losses – than to gains of the same magnitude. This element of prospect theory is known as loss aversion and is designed to capture the widespread aversion to bets such as $(-\$100, \frac{1}{2}; \$110, \frac{1}{2})$. The severity of the kink is determined by the parameter λ; a higher value of λ implies a greater relative sensitivity to losses. Tversky and Kahneman (1992) estimate $\lambda=2.25$ for their median subject.

Third, while $U(\cdot)$ is typically concave everywhere, $v(\cdot)$ is concave only over gains; over losses, it is convex. This pattern can be seen in Figure 1. The concavity/convexity plays a very minor role in our results, in part because the curvature estimated by Tversky and Kahneman (1992) is mild: using experimental data, they estimate $\alpha = 0.88$. To a first approximation, then, $v(\cdot)$ is piecewise-linear.

Finally, under cumulative prospect theory, the individual does not use objective probabilities when evaluating a gamble, but rather uses transformed probabilities obtained from objective probabilities via the weighting functions $w^{+}(\cdot)$ and $w^{-}(\cdot)$. Equation (4) shows that, to obtain the probability weight π_i for an outcome $x_i \geq 0$, we take the total probability of all outcomes equal to or better than x_i, namely $p_i + \ldots + p_n$, the total probability of all outcomes strictly better than x_i, namely $p_{i+1} + \ldots + p_n$, apply the weighting function $w^{+}(\cdot)$ to each, and compute the difference. To obtain the probability weight for an outcome $x_i < 0$, we take the total probability of all outcomes equal to or worse than x_i, the total probability of all outcomes strictly worse than x_i, apply the weighting function $w^{-}(\cdot)$ to each, and compute the difference.

The main consequence of the probability weighting in (4) and (6) is that the individual overweights the *tails* of any distribution he faces. In Equations (3) and (4), the most extreme outcomes, x_{-m} and x_n, are assigned the probability weights $w^{-}(p_{-m})$ and $w^{+}(p_n)$, respectively. For the functional form in (6) and for $\gamma, \delta \in (0,1)$, $w^{-}(P) > P$ and $w^{+}(P) > P$ for low, positive P; the right panel of Figure 1 illustrates this for $\delta = 0.4$ and $\delta = 0.65$. If p_{-m} and p_n are small, then, we have $w^{-}(p_{-m}) > p_{-m}$ and $w^{+}(p_n) > p_n$, so that the most extreme outcomes – the outcomes in the tails – are overweighted.

The overweighting of tails in (4) and (6) is designed to capture the simultaneous demand many people have for both lotteries and insurance. For example, people typically prefer (\$5000, 0.001) to a certain \$5, but also prefer a certain loss of \$5 to (−\$5000, 0.001).[5] By overweighting the tail probability of 0.001 sufficiently, cumulative prospect theory can capture both of these choices. The degree to which the individual overweights tails is governed by the parameters γ and δ; lower values of these parameters imply more overweighting of tails. Tversky and Kahneman (1992) estimate $\gamma = 0.61$ and $\delta = 0.69$ for their median subject.

1.2 Construction of return distributions

Our assumption in this paper is that, when thinking about a stock, many investors mentally represent it by the distribution of its past returns, most likely because they see the past return distribution as a good and easily accessible proxy for the stock's *future* return distribution. Earlier, we noted an implication of this assumption for the cross-section of stock returns, namely, that the prospect theory value of a stock's past return distribution should negatively predict the stock's subsequent return. We formalize this prediction in Section 1.3 and test it in Section 2.

To check whether the prospect theory value of a stock's past return distribution has predictive power for subsequent returns, we need to define

[5] We abbreviate $(x, p; 0, q)$ as (x, p).

"past return distribution." The easiest way for an investor to learn about a stock's past return distribution is to look at a chart that shows the stock's historical price movements. Price charts are ubiquitous in the financial world and usually appear front and center when an investor looks up information about a stock. To define "past return distribution," we therefore take guidance from the typical presentation of these charts.

In the Internet era, investors have a number of different chart formats at their disposal. However, most of the data that we use in our empirical analysis comes from the pre-Internet era, a time when the main reference sources on stocks for many investors were so-called investment handbooks, the most popular of which was the Value Line Investment Survey. The Value Line Survey presents a page of information about each stock. The page is dominated by a chart of historical price fluctuations that goes back several years. The other investment handbooks that we have examined also present charts spanning several years. The average time window across the various sources is, very approximately, five years. On these charts, the daily and weekly fluctuations are not discernible, but the monthly fluctuations are, and they make a clear impression on the viewer. By merely glancing at a chart, the investor obtains a quick sense of the distribution of monthly returns on the stock over the past few years. A large body of evidence in the field of judgment and decision making suggests that people often passively accept the representation that is put in front of them.[6] Under this view, if the monthly return distribution over the past few years is the distribution that jumps out at investors when they look at a chart, it is plausible that this is the representation that they adopt when thinking about the stock. In short, then, when computing the prospect theory value of a stock's past return distribution, we take "past return distribution" to mean the distribution of monthly returns over the past five years.

The final thing we need to specify is whether the monthly returns we use to construct the historical distribution are raw returns, or something else – returns in excess of the risk-free rate, say, or returns in excess of the market return. On the one hand, raw returns are closest to what is depicted in a chart of past price fluctuations. On the other hand, investors looking at a stock chart are likely to have a sense of the performance of the overall market over the period in question, and this may affect their reaction to the chart. For example, if they see a chart showing a decline in the price of a stock, they may react neutrally, rather than negatively, if they know that the market also performed poorly over the same period. In our benchmark results, we therefore use stock returns in excess of the market return. However, we also present results based on raw returns and returns in excess of the risk-free rate; these results are similar to those for the benchmark case.

[6] See, for example, Gneezy and Potters (1997), Thaler et al. (1997), Benartzi and Thaler (1999), and Gneezy, Kapteyn, and Potters (2003).

In summary, when thinking about a stock, some of the investors in our framework mentally represent it as the distribution of its monthly returns in excess of the market over the past five years. To determine their allocation to the stock, they evaluate this distribution according to prospect theory, thereby obtaining the stock's prospect theory value. We now explain how this prospect theory value is computed.

Given a specific stock, we record the stock's return in excess of the market in each of the previous sixty months and then sort these sixty excess returns in increasing order, starting with the most negative through to the most positive. Suppose that m of these returns are negative, while $n = 60 - m$ are positive. Consistent with the notation of Section 1.1, we label the most negative return as r_{-m}, the second most negative as r_{-m+1}, and so on, through to r_n, the most positive return, where r is a monthly return in excess of the market. The stock's historical return distribution is then

$$(r_{-m}, \frac{1}{60}; r_{-m+1}, \frac{1}{60}; \ldots; r_{-1}, \frac{1}{60}; r_1, \frac{1}{60}; \ldots; r_{n-1}, \frac{1}{60}; r_n, \frac{1}{60}), \tag{7}$$

in words, the distribution that assigns an equal probability to each of the sixty excess returns that the stock posted over the previous sixty months. From Section 1.1, the prospect theory value of this distribution is

$$\begin{aligned} \text{TK} \equiv & \sum_{i=-m}^{-1} v(r_i) \left[w^{-} \left(\frac{i+m+1}{60} \right) - w^{-} \left(\frac{i+m}{60} \right) \right] \\ & + \sum_{i=1}^{n} v(r_i) \left[w^{+} \left(\frac{n-i+1}{60} \right) - w^{+} \left(\frac{n-i}{60} \right) \right]. \end{aligned} \tag{8}$$

Note that we label a stock's prospect theory value as "TK," which stands for Tversky and Kahneman (1992), the paper that first introduced cumulative prospect theory.

To compute the expression in (8), we need to specify the value function parameters α and λ in Equation (5) and the weighting function parameters γ and δ in (6). We use the parameter estimates obtained by Tversky and Kahneman (1992) from experimental data, namely,

$$\alpha = 0.88, \lambda = 2.25$$

$$\gamma = 0.61, \delta = 0.69.$$

Subsequent to Tversky and Kahneman (1992), several papers have used more sophisticated techniques, in conjunction with new experimental data, to estimate these parameters (Gonzalez and Wu 1999; Abdellaoui 2000). Their estimates are similar to those obtained by Tversky and Kahneman (1992).

We have proposed that the TK variable captures the impression that investors form of a stock after seeing its historical price fluctuations in a chart. Some investors may see this chart only after some delay. The TK measure in (8) may be approximately valid even for these investors. If the chart that investors are looking at is not up to date, they are likely to try to "fill in the gap" by using another source to find out the returns on the stock between the date at which the chart ends and the current date. Indeed, just by looking up the current price of the stock and comparing it to the last recorded price on the chart, they learn the stock's most recent return. If investors act in this way, they will have a sense of the stock's returns right up to the current time, thereby enabling them to form the impression of the stock captured by the TK variable in (8).

One property of TK is that it does not depend on the order in which the sixty past returns occur in time. One justification for this is that, if TK is capturing an investor's quick, passive reaction to a chart, this reaction is likely to be based on the chart as an integral whole, with the early part of the chart affecting the investor just as much as the later part. However, some investors may put less weight on more distant past returns. We therefore also consider a modified TK measure, TK(ρ), which downweights, by a multiplicative factor $\rho \in (0,1)$, the components of TK associated with more distant past returns. Specifically, if $t(i)$ is the number of months ago that return r_i was realized, we define

$$\text{TK}(\rho) \equiv \frac{1}{\varrho} \sum_{i=-m}^{-1} \rho^{t(i)} v(r_i) \left[w^- \left(\frac{i+m+1}{60} \right) - w^- \left(\frac{i+m}{60} \right) \right]$$
$$+ \frac{1}{\varrho} \sum_{i=1}^{n} \rho^{t(i)} v(r_i) \left[w^+ \left(\frac{n-i+1}{60} \right) - w^+ \left(\frac{n-i}{60} \right) \right], \qquad (9)$$

where $\varrho = \rho + \ldots + \rho^{60}$. While our focus is on the TK variable in (8), we will also report some results on the predictive power of TK(ρ).

1.3 Model

We now present a simple model that formalizes our main empirical prediction: that the prospect theory value of a stock's historical return distribution will predict the stock's subsequent return in the cross-section with a negative sign.

We work in a mean-variance framework. There is a risk-free asset with a fixed return of r_f. There are J risky assets, indexed by $j \in \{1, \ldots, J\}$. Asset j has return $\widetilde{r}_j$ whose mean and standard deviation are μ_j and σ_j, respectively. The covariance between the returns on assets i and j is $\sigma_{i,j}$. More generally, given a portfolio p, we use $\widetilde{r}_p$, μ_p, σ_p, and $\sigma_{p,q}$ to denote the portfolio's return, mean, standard deviation, and covariance with portfolio q, respectively.

There are two types of traders in the economy. Traders of the first type are traditional mean-variance investors who hold the tangency portfolio that, among all combinations of risky assets, has the highest Sharpe ratio. The tangency portfolio has return $\widetilde{r}_t$, and the weights of the J risky assets in the tangency portfolio are given by the $J \times 1$ vector ω_t.

Traders of the second type are "prospect theory" investors. These investors construct their portfolio holdings by taking the tangency portfolio ω_t and adjusting it, increasing their holdings of stocks with high prospect theory values and decreasing their holdings of stocks with low prospect theory values. Formally, they hold a portfolio p whose risky asset weights are given by

$$\omega_p = \omega_t + k\omega_{TK} \tag{10}$$

for some $k > 0$, and where ω^j_{TK}, the j'th element of the $J \times 1$ vector ω_{TK}, is

$$\omega^j_{TK} = \mathrm{TK}_j - \overline{\mathrm{TK}}, \tag{11}$$

where TK_j, defined in (8), is the prospect theory value of stock j's past returns – specifically, as described in Section 1.2, the prospect theory value of the distribution of the sixty past monthly returns on the stock in excess of the market – and where $\overline{\mathrm{TK}} = \sum_{j=1}^{J} \mathrm{TK}_j / J$. In other words, these investors hold a portfolio that, relative to the tangency portfolio, tilts toward stocks with higher prospect theory values than average and away from stocks with lower prospect theory values than average.

If the fraction of traditional mean-variance investors in the overall population is π, so that the fraction of prospect theory investors is $1-\pi$, the market portfolio ω_m can be written

$$\begin{aligned}\omega_m &= \pi\omega_t + (1-\pi)(\omega_t + k\omega_{TK}) \\ &= \omega_t + \eta\omega_{TK},\end{aligned} \tag{12}$$

where $\eta = (1-\pi)k$.

In the Appendix, we prove the following proposition, which guides our empirical work. In the proposition, β_x is the market beta of asset or portfolio x, and "portfolio TK" is the portfolio whose risky asset weights are given by the vector ω_{TK}.

Proposition 1. In the economy described above, the mean return μ_j of asset j is given by

$$\frac{\mu_j - r_f}{\mu_m - r_f} = \beta_j - \frac{\eta s_{j,TK}}{\sigma_m^2(1-\eta\beta_{TK})}, \tag{13}$$

where $s_{j,TK}$ is the covariance between the residuals $\tilde{\varepsilon}_j$ and $\tilde{\varepsilon}_{TK}$ obtained from regressing asset j's excess return and portfolio TK's excess return, respectively, on the market excess return:

$$\tilde{r}_j = r_f + \beta_j(\tilde{r}_m - r_f) + \tilde{\varepsilon}_j, \tag{14}$$

$$\tilde{r}_{TK} = r_f + \beta_{TK}(\tilde{r}_m - r_f) + \tilde{\varepsilon}_{TK}. \tag{15}$$

Under the additional assumption that $Cov(\widetilde{\varepsilon}_i, \widetilde{\varepsilon}_j) = 0$ for distinct $i, j \in \{1, \ldots, J\}$, we obtain

$$\frac{\mu_j - r_f}{\mu_m - r_f} = \beta_j - \frac{\eta \omega^j_{TK} s_j^2}{\sigma_m^2 (1 - \eta \beta_{TK})}$$

$$= \beta_j - \frac{\eta (\mathrm{TK}_j - \overline{\mathrm{TK}}) s_j^2}{\sigma_m^2 (1 - \eta \beta_{TK})}, \qquad (16)$$

where s_j^2 is the variance of the residuals $\widetilde{\varepsilon}_j$ in (14).

Equation (16) captures the prediction we test in the next section: that stocks with higher prospect theory values (higher TK_j) than average will have lower alphas.

A multiperiod extension of our one-period model yields an additional prediction: that the *expected change* in a stock's TK value over the next month should predict the stock's return over the same period with a positive sign. For example, if a stock had a very good return sixty months ago, then an outside observer can predict that the stock's TK value is likely to fall over the next month and therefore that the stock's price is also likely to fall as investors who base their demand on TK become less enthusiastic. However, through simulations, we find that this prediction is not robust to small misspecifications in our assumptions about the behavior of the prospect theory investors – for example, mistakenly assuming that investors construct TK based on four years of data, rather than on five. By contrast, the prediction derived from the one-period model – that a stock's TK value will negatively predict the stock's subsequent return – is more robust to small misspecifications. We therefore focus on the latter prediction throughout the paper.

2. Empirical Analysis

We now test the predictions of the framework laid out in Section 1. Our main prediction is that stocks whose past return distributions have higher prospect theory values – higher values of TK – will subsequently earn lower returns, on average. We expect this prediction to hold primarily for stocks with lower market capitalizations, in other words, for stocks where individual investors play a more important role: individual investors are more likely to make buying and selling decisions based on the thinking we have described.

2.1 Data

Our data come from standard sources. For U.S. firms, the stock price and accounting data are from CRSP and Compustat. Our analysis includes all stocks in the CRSP universe from 1926 to 2010 for which the variable TK can be calculated – in other words, all stocks with at least five years of monthly return

Table 1
Data summary

A. Means and standard deviations

	TK	Beta	Size	Bm	Mom	Rev	Illiq	Lt rev	Ivol	Max	Min	Skew	Eiskew	Coskew
Mean	**−0.05**	1.16	11.03	−0.16	0.15	0.01	0.58	0.80	0.02	0.06	0.05	0.66	0.47	−0.00
SD	**0.03**	0.57	1.82	0.86	0.44	0.12	2.518	1.58	0.02	0.06	0.04	0.80	0.47	0.24

B. Correlations

	TK	Beta	Size	Bm	Mom	Rev	Illiq	Lt rev	Ivol	Max	Min	Skew	Eiskew	Coskew
TK	**1**													
Beta	**−0.03**	1												
Size	**0.36**	−0.13	1											
Bm	**−0.34**	0.05	−0.42	1										
Mom	**0.32**	−0.01	0.11	−0.14	1									
Rev	**0.11**	−0.01	0.04	0.01	0.00	1								
Illiq	**−0.25**	0.08	−0.44	0.25	−0.11	0.02	1							
Lt rev	**0.56**	0.04	0.19	−0.35	−0.02	−0.01	−0.12	1						
Ivol	**−0.31**	0.26	−0.49	0.21	−0.09	0.14	0.58	−0.13	1					
Max	**−0.22**	0.24	−0.37	0.17	−0.07	0.32	0.50	−0.10	0.88	1				
Min	**−0.29**	0.26	−0.42	0.18	−0.06	−0.18	0.49	−0.09	0.79	0.59	1			
Skew	**0.22**	0.22	−0.37	0.11	0.07	0.03	0.20	0.00	0.30	0.25	0.24	1		
Eiskew	**−0.21**	0.20	−0.61	0.24	−0.13	−0.03	0.33	−0.07	0.44	0.35	0.37	0.39	1	
Coskew	**0.04**	0.22	0.07	0.04	−0.02	−0.00	−0.01	−0.05	0.01	0.01	0.02	0.23	−0.04	1

The table presents summary statistics for the variables we use in our analysis: the mean and standard deviation of each variable (panel A) and the correlations between them (panel B). We compute the means, standard deviations, and correlations from the cross-section month by month and report the time-series averages of the monthly statistics. TK is the prospect theory value of a stock's historical return distribution (see Section 1.2). *Beta* is a stock's beta calculated from monthly returns over the previous five years, following Fama and French (1992). *Size* is the log market capitalization at the end of the previous month. *Bm* is the log book-to-market ratio. When the book value of equity is missing from Compustat, we use data from Davis, Fama, and French (2002); observations with negative book value are removed. *Mom* is the cumulative return from the start of month t-12 to the end of month t-2. *Rev* is the return in month t-1. *Illiq* is Amihud's (2002) measure of illiquidity, scaled by 10^5. *Lt rev* is the cumulative return from the start of month t-60 to the end of month t-13. *Ivol* is idiosyncratic return volatility, as in Ang et al. (2006). *Max* and *Min* are the maximum and the negative of the minimum daily return in month t-1, as in Bali, Cakici, and Whitelaw (2011). *Skew* is the skewness of monthly returns over the previous five years. *Eiskew* is expected idiosyncratic skewness, as in Boyer, Mitton, and Vorkink (2010). *Coskew* is coskewness, computed as in Harvey and Siddique (2000) using five years of monthly returns. The sample period runs from July 1931 to December 2010, except in the case of *Eiskew*, where it starts in January 1988 due to data availability.

data. Compustat does not cover the first part of our sample period; for these early years, our data on book equity are from Kenneth French's Web site. Stock price and accounting data for non-U.S. firms are from Datastream. Finally, we obtain quarterly data on institutional stock holdings from 1980 to 2010 from the Thomson Reuters (formerly CDA/Spectrum) database.

Table 1 presents summary statistics for the variables we use in our analysis. Panel A reports means and standard deviations; panel B reports pairwise correlations. TK is the prospect theory variable defined in (8) whose predictive power is the focus of the paper. *Beta* is a stock's beta computed using monthly returns over the previous five years, as in Fama and French (1992); Equation (16) indicates that beta should be included in our tests.

The next few variables are known predictors of stock returns in the cross-section; we use them as controls in some of our tests. Their time t values are defined as follows. *Size* is the log of the market value of the firm's outstanding equity at the end of month $t-1$.[7] *Bm* is the log of the firm's book value of equity divided by market value of equity, where the book-to-market ratio is computed following Fama and French (2008); firms with negative book values are excluded from the analysis, and when the book value of equity is missing from Compustat, we use data from Davis, Fama, and French (2002). *Mom* is the stock's cumulative return from the start of month $t-12$ to the end of month $t-2$, a control for the momentum effect. *Rev* is the stock's return in month $t-1$, a control for the short-term reversal effect. *Illiq* is Amihud's (2002) measure of illiquidity, computed using daily data from month $t-1$. *Lt rev* is the stock's cumulative return from the start of month $t-60$ to the end of month $t-13$, a control for the long-term reversal effect. *Ivol* is the volatility of the stock's daily idiosyncratic returns over month $t-1$, as in Ang et al. (2006).

Later in the paper, we propose that some of the predictive power of the TK variable may be related to the fact that the past returns of high-TK stocks are more positively skewed than those of the typical stock, a characteristic that may be appealing to investors when they observe it in a chart of historical price fluctuations. Some skewness-related variables have already been studied in the context of the cross-section of stock returns. To understand the relationship of TK to these other variables, we include them in some of our tests. They are: *Max,* a stock's maximum one-day return in month $t-1$, as in Bali, Cakici, and Whitelaw (2011); *Min*, (the negative of) a stock's minimum one-day return in month $t-1$, as in Bali, Cakici, and Whitelaw (2011); *Skew,* the skewness of a stock's monthly returns over the previous five years; *Eiskew,* a stock's expected idiosyncratic return skewness, as in Boyer, Mitton, and Vorkink (2010); and *Coskew,* a stock's coskewness, computed using monthly returns over the previous five years in the way described by Harvey and Siddique (2000), namely, as

$$E(\varepsilon_{M,t}^2 \varepsilon_{i,t})/(E(\varepsilon_{M,t}^2)\sqrt{E(\varepsilon_{i,t}^2)}),$$

where $\varepsilon_{i,t}=r_{i,t}-\alpha_i-\beta_i r_{M,t}$ are the residuals in a regression of excess stock returns $r_{i,t}$ on excess market returns $r_{M,t}$ and where $\varepsilon_{M,t}=r_{M,t}-\mu_M$ are the residuals after de-meaning the market returns.

To be clear, *Max, Min, Eiskew,* and *Coskew* have predictive power for subsequent returns; see, for example, the papers referenced in the definitions of these variables. *Skew,* however, does not predict returns in a statistically significant way.

We compute the summary statistics in Table 1 using the full data sample, starting in July 1931 and ending in December 2010. The only exception is for *Eiskew*; this variable is available starting only in January 1988.[8]

7 We adopt the convention that month $t-j$ spans the interval from time $t-j$ to time $t-j+1$.

8 Boyer, Mitton, and Vorkink (2010), who introduce *Eiskew* to the literature, construct this variable starting in 1988 because detailed data on the trading volume of NASDAQ stocks only become available in the 1980s.

To a first approximation, the prospect theory value of a gamble is increasing in the gamble's mean, decreasing in the gamble's standard deviation (due to loss aversion), and increasing in the gamble's skewness (due to probability weighting). The results in the column labeled "TK" in panel B of Table 1 are consistent with this. Across stocks, TK is positively correlated with measures of past returns (*Rev, Mom,* and *Lt rev*), negatively correlated with a measure of past volatility (*Ivol*), and positively correlated with past skewness (*Skew*). High-TK stocks also tend to have higher market capitalizations, probably because large-cap stocks are less volatile; they are also more likely to be growth stocks.

2.2 Time-series tests

Our main hypothesis is that the prospect theory value of a stock's past return distribution – the stock's TK value – will predict the stock's subsequent return in the cross-section. We now test this hypothesis using decile sorts. In Section 2.4, we test it using the Fama-MacBeth methodology.

We conduct the decile-sort test as follows. At the start of each month, beginning in July 1931 and ending in December 2010, we sort stocks into deciles based on TK. We then compute the average return of each TK-decile portfolio over the next month, both value-weighted and equal-weighted. This gives us a time series of monthly returns for each TK decile. We use these time series to compute the average return of each decile over the entire sample. More precisely, in Table 2, we report the average return of each decile in excess of the risk-free rate; the four-factor alpha for each decile (following Carhart 1997, the return adjusted by the three factors from Fama and French 1993 and by a momentum factor); the five-factor alpha for each decile (the return adjusted by the three Fama-French factors, the momentum factor, and the Pastor and Stambaugh 2003 liquidity factor); and the characteristics-adjusted return for each decile, computed in the way described by Daniel et al. (1997) and denoted DGTW. In the right-most column, we report the difference between the returns of the two extreme decile portfolios, in other words, the return of a "low minus high" zero investment portfolio that buys the stocks in the lowest TK decile and shorts the stocks in the highest TK decile. In the case of the five-factor alpha, our analysis begins in January 1968 because data on the liquidity factor become available in this month.

The most important column in Table 2 is the right-most column, which reports the average return of the low-minus-high portfolio. Our prediction is that this return will be significantly positive. We expect this prediction to hold more strongly for equal-weighted returns – in other words, for small-cap stocks, where individual investors play a more important role.

The results in the right-most column support our hypothesis. The average equal-weighted return on the low-TK portfolio is significantly higher than on the high-TK portfolio across all four types of return that we compute (excess return, four-factor alpha, five-factor alpha, and DGTW return). As predicted, the difference in average returns is larger for equal-weighted returns than for

Table 2
Decile portfolio analysis

		P1 Low TK	P2	P3	P4	P5	P6	P7	P8	P9	P10 High TK	TK Low-high portfolio
Excess return	EW	**2.144**	**1.320**	**1.155**	**1.073**	**0.996**	**0.988**	**0.925**	**0.904**	**0.889**	**0.798**	**1.346**
		(5.39)	(4.51)	(4.28)	(4.28)	(4.31)	(4.48)	(4.59)	(4.50)	(4.43)	(3.50)	(5.05)
	VW	**1.216**	**0.969**	**0.909**	**0.903**	**0.752**	**0.703**	**0.679**	**0.667**	**0.695**	**0.537**	**0.679**
		(3.57)	(3.38)	(3.68)	(3.93)	(3.52)	(3.39)	(3.74)	(3.66)	(3.99)	(2.83)	(2.62)
Four-factor alpha	EW	**1.025**	**0.343**	**0.204**	**0.167**	**0.112**	**0.112**	**0.081**	0.038	−0.018	**−0.210**	**1.236**
		(6.05)	(4.19)	(3.39)	(3.24)	(2.27)	(2.37)	(1.89)	(0.79)	(−0.34)	(−3.19)	(6.83)
	VW	**0.405**	**0.261**	**0.242**	**0.318**	**0.141**	0.098	**0.098**	0.005	0.040	**−0.218**	**0.622**
		(2.72)	(2.39)	(2.73)	(4.01)	(1.89)	(1.48)	(1.80)	(0.09)	(0.73)	(−3.66)	(3.67)
Five-factor alpha (1968 onward)	EW	**1.242**	**0.330**	**0.181**	**0.170**	**0.169**	**0.177**	**0.135**	0.057	0.066	−0.057	**1.300**
		(4.98)	(2.82)	(2.19)	(2.63)	(3.03)	(3.46)	(2.61)	(1.03)	(1.06)	(−0.73)	(5.21)
	VW	**0.551**	0.115	**0.251**	**0.320**	**0.192**	0.132	**0.155**	0.011	0.010	**−0.155**	**0.706**
		(2.40)	(0.76)	(2.14)	(3.27)	(2.07)	(1.62)	(2.12)	(0.16)	(0.13)	(−2.17)	(2.88)
DGTW	EW	**0.720**	**0.156**	0.061	0.046	0.051	0.051	0.037	−0.012	−0.046	**−0.110**	**0.830**
		(6.56)	(2.70)	(1.42)	(1.23)	(1.51)	(1.50)	(1.08)	(−0.32)	(−1.19)	(−2.22)	(6.61)
	VW	**0.255**	0.077	**0.096**	**0.113**	0.009	0.066	0.038	−0.044	−0.025	**−0.096**	**0.351**
		(2.16)	(1.02)	(1.68)	(2.20)	(0.18)	(1.62)	(1.14)	(−1.31)	(−0.76)	(−2.46)	(2.65)

The table reports average monthly excess returns and monthly alphas, on both an equal-weighted (EW) and value-weighted (VW) basis, of portfolios of stocks sorted on TK, the prospect theory value of a stock's historical return distribution (see Section 1.2). Each month, all stocks are sorted into deciles based on TK. For each of the decile portfolios, P1 (low TK) through P10 (high TK), we report the average excess return, four-factor alpha (following Carhart 1997), five-factor alpha (Carhart four-factor model augmented by Pastor and Stambaugh's (2003) liquidity factor), and characteristics-adjusted return calculated as in Daniel et al. (1997) and denoted DGTW. The sample runs from July 1931 to December 2010, except in the case of the five-factor alpha, where it starts in January 1968 due to constraints on the availability of the liquidity factor. t-statistics are in parentheses, and bold typeface indicates a coefficient significant at the 10% level.

value-weighted returns. Nonetheless, we find a significant effect even for value-weighted returns. Moreover, the economic magnitudes of the excess returns and alphas in the right-most column are sizable.[9]

Figure 2 presents a graphical view of the results in Table 2. It plots the equal-weighted (top panel) and value-weighted (bottom panel) four-factor alphas on the ten TK-decile portfolios. The figure makes clear another aspect of the results in Table 2, namely, that the alphas on the ten portfolios decline in a near-monotonic fashion as we move from the lowest TK portfolio to the highest TK portfolio.

Table 2 and Figure 2 look at whether TK calculated using a stock's returns from month $t-60$ to $t-1$ can predict the stock's return in month t. We now examine whether TK can predict returns beyond the first month after portfolio construction. To do this, we again sort stocks into decile portfolios at time t using TK calculated from month $t-60$ to $t-1$, but now look at the returns of these portfolios not only in month t, but also in months $t+1$, $t+2$, and so

9 The t-statistics in Table 2 imply that the returns of the long-short low-TK minus high-TK portfolio are fairly volatile – similarly volatile to those of a long-short value minus growth portfolio. This suggests comovement in the returns of stocks with similar TK values. This is indeed the case. A stock in a given TK decile comoves more with stocks in the same TK decile than with stocks in other TK deciles, even after controlling for the four Carhart (1997) factors. This comovement may be due to investors having a similarly positive or negative attitude to stocks in the same TK decile, leading them to trade these stocks in a correlated way.

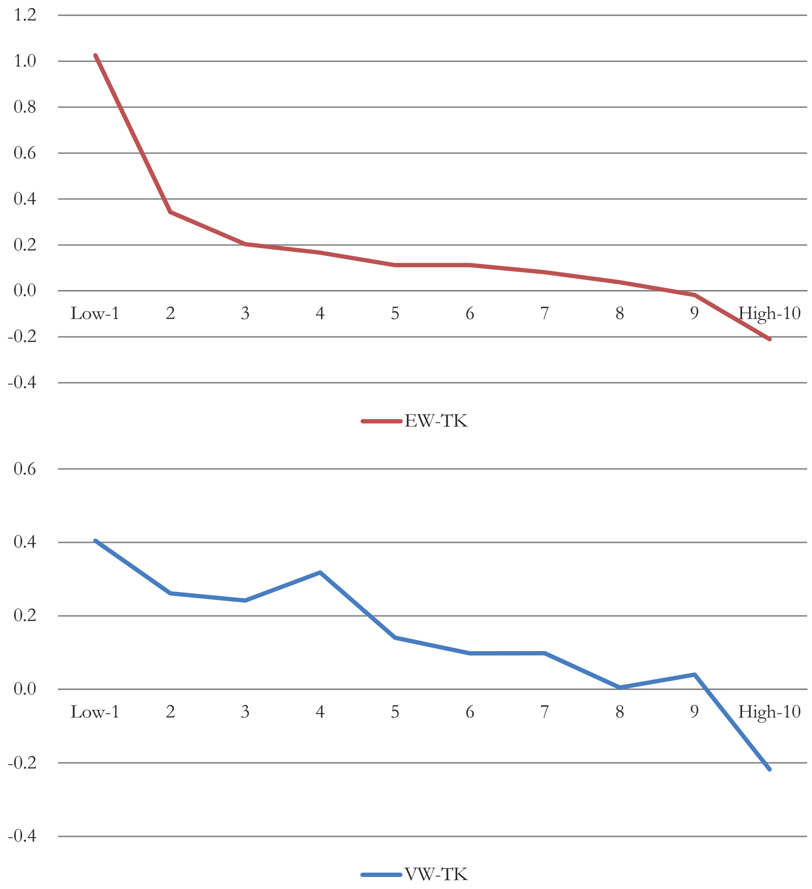

Figure 2
Performance of TK deciles
Each month, we sort all stocks into deciles by TK, the prospect theory value of a stock's historical return distribution, and record the average return of each decile over the next month on both an equal-weighted (EW) and value-weighted (VW) basis. Using the time series of average returns, we compute four-factor alphas for the deciles and plot them in the figure. The top panel is for equal-weighted returns; the bottom panel is for value-weighted returns. The vertical axis is the monthly alpha, in percent; the horizontal axis marks the decile portfolio, from decile 1 (low TK) on the left to decile 10 (high TK) on the right.

on. Figure 3 presents the results. The top chart corresponds to equal-weighted returns; the bottom chart corresponds to value-weighted returns. The charts plot four-factor alphas. Specifically, the alpha that corresponds to the $t+k$ label on the horizontal axis is the four-factor alpha of a long-short portfolio that, each month, buys stocks that were in the lowest TK decile k months previously and shorts stocks that were in the highest TK decile k months previously.

The figure shows that TK has predictive power for returns several months after portfolio construction. It also shows that TK's predictive power is particularly strong in the first month after portfolio construction. This is

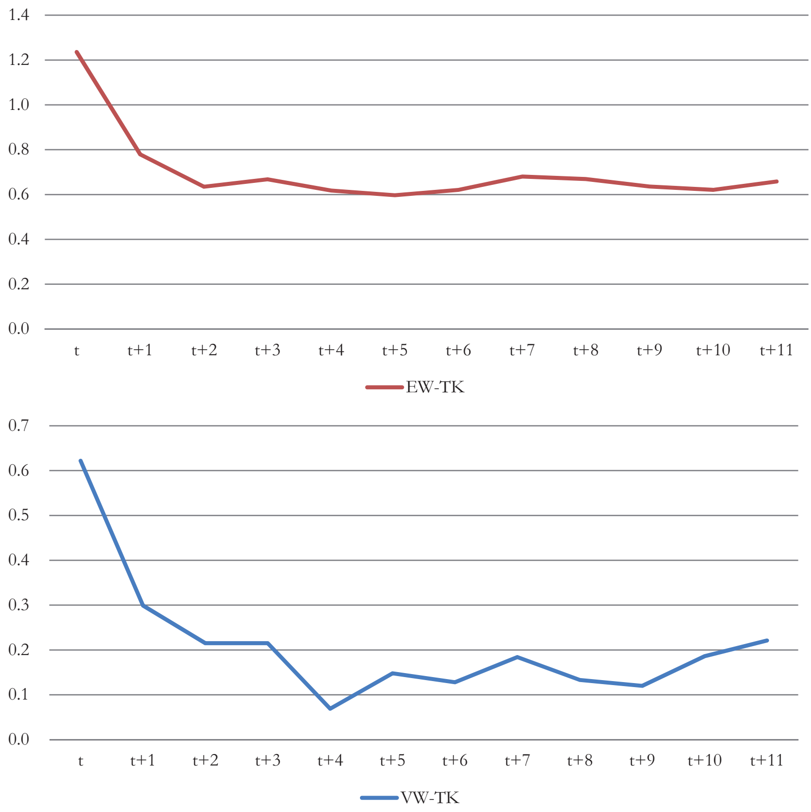

Figure 3
How do the long-short portfolio returns decline over time?
The figure plots the four-factor alpha, on both an equal-weighted (EW) and value-weighted (VW) basis, of a long-short portfolio that buys (shorts) stocks that were in the lowest (highest) TK decile at some point in the past. The vertical axis is the monthly alpha, in percent; the horizontal axis indicates the time lag, in months, between the moment of portfolio construction and the moment at which we start measuring returns: the alpha that corresponds to the label $t+k$ is the alpha of the long-short portfolio that buys (shorts) stocks that were in the lowest (highest) TK decile k months previously.

primarily a feature of U.S. data; in our analysis of international data in Section 2.6, we find that the drop-off after the first month is smaller. The pattern in U.S. data may be related to the short-term reversal phenomenon. However, we will show that TK retains its predictive power for returns even after we control for short-term reversals using both regressions and double sorts. Moreover, we will show, using both U.S. and international data, that TK retains its predictive power even when we skip a month between the moment at which TK is computed and the moment at which we start measuring returns.

We report both value-weighted *and* equal-weighted returns in Table 2 because equal-weighted returns put more weight on small-cap stocks – stocks

Table 3
Factor loadings

Model/measure	MktRf	SMB	HML	UMD	PS_Liq
Four-factor model	−0.116	1.172	0.613	−0.679	
EW	(−3.20)	(21.02)	(11.60)	(−16.51)	
Four-factor model	0.027	1.044	0.469	−0.752	
VW	(0.78)	(19.98)	(9.47)	(−19.48)	
Five-factor model	0.100	0.967	0.671	−0.710	−0.057
EW	(1.77)	(12.33)	(7.85)	(−12.99)	(−0.86)
Five-factor model	0.226	1.132	0.673	−0.858	−0.158
VW	(4.10)	(14.70)	(8.02)	(−15.99)	(−2.40)

The table reports the factor loadings of a long-short portfolio that, each month, buys stocks whose TK values at the start of the month are in the bottom decile and shorts stocks whose TK values are in the top decile. We report results for two factor models – the Carhart (1997) four-factor model and a five-factor model which augments the four-factor model by the Pastor and Stambaugh (2003) liquidity factor – and for both equal-weighted (EW) and value-weighted (VW) returns. The sample runs from July 1931 to December 2010, except in the case of the five-factor alpha, where it starts in January 1968 due to constraints on the availability of the liquidity factor. t-statistics are in parentheses.

for which we expect our prediction to hold more strongly. However, equal-weighted returns can be biased in some circumstances. We do two things to verify that any such bias is small in our case. First, we compute "return-weighted" portfolio returns in which the return of each stock is weighted by one plus its lagged monthly return; Asparouhova, Bessembinder, and Kalcheva (2010, 2013) note that this approach helps to correct for biases in equal-weighted returns. We find that the return-weighted four-factor alpha on a long-short portfolio that buys low-TK stocks and shorts high-TK stocks is 0.978%. This is slightly lower than the equal-weighted four-factor alpha of 1.236%, but is of a similar order of magnitude and remains highly significant. Second, we compute the *value*-weighted four-factor alpha using all stocks except for those in the four highest market-capitalization deciles. This measure puts more weight on small-cap stocks but is not subject to the biases that can affect equal-weighted returns. The value-weighted four-factor alpha on the long-short portfolio for the restricted sample is 0.799%, which is still very statistically significant. These calculations suggest that any bias in our equal-weighted return measures is small.

Table 3 reports the factor loadings for the low-TK minus high-TK portfolio for both a four-factor and a five-factor model, and for both equal-weighted and value-weighted returns. The results are consistent with those in Table 1: the low-TK portfolio comoves with small stocks, value stocks, and low momentum stocks.

2.3 Robustness of time-series results

Before turning to the Fama-MacBeth analysis, we examine the robustness of the decile-sort results in Table 2. The five panels in Table 4 correspond to five different robustness checks. The two right-most columns report four-factor alphas for the low-TK minus high-TK portfolio, based on either equal-weighted or value-weighted returns.

Table 4
Robustness

		TK	
		EW	VW
Subperiods	Jul 1931 – Jun 1963	**1.252**	**0.459**
		(4.35)	(1.89)
	Jul 1963 – Dec 2010	**1.211**	**0.634**
		(5.34)	(2.81)
Window for constructing TK	Past three years	**1.283**	**0.674**
		(6.72)	(3.77)
	Past four years	**1.244**	**0.557**
		(6.79)	(3.24)
	Past six years	**1.193**	**0.643**
		(6.56)	(3.71)
Other return measures	Raw returns	**1.204**	**0.464**
		(5.48)	(2.17)
	Returns in excess of the risk-free rate	**1.049**	**0.282**
		(6.31)	(1.69)
	Returns in excess of the sample mean	**0.797**	**0.543**
		(4.07)	(2.94)
Exclude low-priced stocks	price>=$5	**0.373**	**0.365**
		(3.71)	(2.85)
Skip a month		**0.779**	**0.299**
		(4.58)	(1.86)

The table presents the results of five robustness checks. The right column reports the equal-weighted (EW) and value-weighted (VW) four-factor alphas of a long-short portfolio that, each month, buys (shorts) stocks with TK values in the lowest (highest) decile. The first panel presents results for two subperiods. In the second panel, we use three, four, or six years of monthly returns to compute TK. In the third panel, we compute TK using raw returns, returns in excess of the risk-free rate, and returns in excess of the stock's sample mean. In the fourth panel, we exclude stocks whose price falls below $5 in the month before portfolio construction. In the fifth panel, we skip a month between the moment at which TK is constructed and the moment at which we start measuring returns. The sample period runs from July 1931 to December 2010. t-statistics are in parentheses, and bold typeface indicates a coefficient significant at the 10% level.

First, we check whether our results hold not only in the full sample, but also in each of two subperiods: one that starts in July 1931 and ends in June 1963, and another that starts in July 1963 and ends in December 2010. We choose July 1963 as the breakpoint to make our results easier to compare to those of the many empirical papers that, due to data availability, begin their analyses in that month. The first panel of Table 4 confirms that our main prediction holds in both subperiods: the long-short portfolio has a significantly positive alpha, particularly in the case of equal-weighted returns.

When constructing the past return distribution for a stock, we use monthly returns over the previous five years. The second panel of Table 4 shows that, if we instead use monthly returns over the previous three, four, or six years, we obtain similar results. Also, when we construct a stock's past return distribution, we use returns in excess of the market return. The third panel of the table shows that we obtain similar results if we instead use raw returns, returns in excess of the risk-free rate, or returns in excess of the stock's own sample mean.

Empirical studies of stock returns sometimes exclude low-priced stocks. However, there is a reason to keep them as part of our analysis. We will later suggest that TK predicts returns in part because it captures a skewness-related dimension of a stock's historical return distribution: a stock with a high TK value is viewed as lottery-like, which is appealing to some investors. Earlier studies have shown that investors are more likely to think of a stock as lottery-like if it has a low price (Kumar 2009; Birru and Wang 2016). We therefore expect TK's predictive power to be at least as strong for low-priced stocks as for other stocks, making it useful to include low-priced stocks in our sample. Nonetheless, the fourth panel of Table 4 shows that, even when we exclude stocks whose prices fall below $5 in the month before portfolio construction, the equal-weighted and even the value-weighted four-factor alphas remain significant, although their magnitudes are reduced.

The fifth panel shows that, consistent with Figure 3, the long-short alphas decline somewhat in magnitude if we skip a month between the moment at which we sort stocks and the moment at which we start measuring the returns of the decile portfolios. However, as noted earlier, this decline is primarily a U.S. phenomenon: we observe it less in our international results in Section 2.6. Moreover, even after skipping a month, our prediction is confirmed for equal-weighted returns, which put more weight on the small-cap stocks for which we expect the prediction to hold more strongly.

2.4 Fama-MacBeth tests

We now test our main hypothesis using the Fama-MacBeth methodology. One advantage of this methodology is that it allows us to examine the predictive power of TK while controlling for known predictors of returns.

We implement the Fama-MacBeth technique in the usual way. Each month, starting in July 1931 and ending in December 2010, we run a cross-sectional regression of stock returns in that month on TK measured at the start of the month and on variables already known to predict returns as controls. Panel A of Table 5 reports the time-series averages of the coefficients on the independent variables. The nine numbered columns in the table correspond to nine different regression specifications which differ in how many control variables they include.

The results in the table provide support for our hypothesis. The TK variable retains significant predictive power even after we include the major known predictors of returns. In Columns (2) through (5), for example, we include controls such as market capitalization (*Size*), book-to-market (*Bm*), various measures of past returns (*Rev, Mom,* and *Lt rev*), an illiquidity measure (*Illiq*), and idiosyncratic volatility (*Ivol*). The table shows that controlling for the past month's return (*Rev*) takes a substantial bite out of the economic magnitude of the coefficient on TK; however, inclusion of the other controls does not affect the coefficient on TK very much.

In Section 2.7, we will suggest that the low returns to high-TK stocks may be due, in part, to the fact that the past returns of high-TK stocks are positively skewed. Since skewness-related variables have been studied before in connection with the cross-section of stock returns, Columns (6) through (9) include these variables as additional controls. Even after their inclusion,

Table 5
Fama-MacBeth regression analysis

A. TK is constructed using returns from month t-60 to month t-1

	Controls					Skewness controls			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
TK	**−0.107**	**−0.108**	**−0.059**	**−0.050**	**−0.043**	**−0.043**	**−0.046**	**−0.067**	**−0.043**
	(−3.77)	(−4.94)	(−3.07)	(−2.60)	(−2.16)	(−2.18)	(−1.92)	(−2.28)	(−2.16)
Beta		0.132	0.166	0.192	**0.240**	**0.243**	**0.249**	**0.480**	**0.243**
		(1.14)	(1.22)	(1.40)	(1.99)	(2.07)	(2.11)	(2.59)	(2.00)
Size		**−0.132**	**−0.125**	**−0.078**	**−0.097**	**−0.089**	**−0.092**	**−0.066**	**−0.092**
		(−4.07)	(−3.65)	(−2.44)	(−3.38)	(−3.24)	(−3.55)	(−1.94)	(−3.27)
Bm		**0.151**	**0.203**	**0.177**	**0.126**	**0.127**	**0.112**	0.121	**0.124**
		(2.67)	(3.44)	(3.06)	(2.26)	(2.29)	(2.03)	(1.31)	(2.27)
Mom		**0.01**	**0.009**	**0.009**	**0.008**	**0.009**	**0.008**	**0.005**	**0.009**
		(7.99)	(6.47)	(6.73)	(6.39)	(6.47)	(6.30)	(3.44)	(6.55)
Rev			**−0.079**	**−0.079**	**−0.078**	**−0.081**	**−0.082**	**−0.053**	**−0.092**
			(−16.39)	(−16.28)	(−15.20)	(−16.47)	(−16.02)	(−9.64)	(−15.85)
Illiq				**0.286**	**0.597**	**0.622**	**0.631**	**1.299**	**0.620**
				(2.37)	(4.90)	(5.08)	(5.20)	(6.88)	(5.11)
Lt rev					−0.041	−0.039	**−0.035**	−0.000	−0.033
					(−1.40)	(−1.31)	(−1.70)	(−0.04)	(−1.13)
Ivol					**−0.138**	0.068	0.067	0.073	0.068
					(−4.27)	(1.43)	(1.43)	(1.03)	(1.44)
Max						**−0.036**	**−0.036**	−0.022	**−0.036**
						(−3.45)	(−3.42)	(−1.36)	(−3.45)
Min						**−0.059**	**− 0.060**	**−0.093**	**−0.059**
						(−4.50)	(−4.61)	(−6.72)	(−4.56)
Skew							0.013		
							(0.03)		
Eiskew								−0.194	
								(−1.61)	
Coskew									−0.039
									(−0.41)
N	954	954	954	954	954	954	954	276	954

(*continued*)

the coefficient on TK remains largely unaffected in magnitude and statistical significance.

Panel A of Table 5 shows that, while TK predicts returns at the conventional 5% level of statistical significance, its predictive power is not as strong as that of, say, the past year's return or idiosyncratic volatility. We emphasize, however, that the construction of TK is constrained by prior theory and evidence in a way that the construction of many other predictor variables is not. For example, we use the exact functional forms suggested by Tversky and Kahneman (1992), as well as the exact parameter values that they estimate. Moreover, given the typical format of price charts, we restrict ourselves to using five years of monthly data when computing TK. Given that we have tied our hands on these dimensions, the statistical significance with which TK predicts returns is, if anything, strikingly high rather than low. The only specification in panel A of Table 5 for which the statistical significance of the coefficient on TK falls below the 5% level is the one in Column (7) that includes past return skewness as a control; here, the coefficient is significant at the 5.5% level. However, if, as we later suggest, past skewness is an integral part of the prospect theory

Table 5
Continued

B. TK is constructed using returns from month t-61 to month t-2

	Controls					Skewness controls			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
TK	**−0.070**	**−0.040**	**−0.056**	**−0.049**	**−0.040**	**−0.040**	**−0.040**	**−0.060**	**−0.039**
	(−2.76)	(−2.51)	(−3.01)	(−2.57)	(−2.06)	(−2.05)	(−1.76)	(−2.16)	(−2.02)
Beta		0.072	0.168	0.194	**0.240**	**0.244**	**0.252**	**0.486**	**0.242**
		(0.68)	(1.22)	(1.40)	(1.96)	(2.05)	(2.11)	(2.61)	(1.97)
Size		**−0.164**	**−0.127**	**−0.079**	**−0.098**	**−0.090**	**−0.094**	**−0.068**	**−0.093**
		(−4.63)	(−3.67)	(−2.46)	(−3.40)	(−3.27)	(−3.65)	(−2.00)	(−3.30)
Bm		**0.209**	**0.206**	**0.179**	**0.128**	**0.128**	**0.115**	0.123	**0.125**
		(3.73)	(3.49)	(3.09)	(2.28)	(2.31)	(2.10)	(1.33)	(2.29)
Mom		**0.008**	**0.008**	**0.009**	**0.008**	**0.008**	**0.008**	**0.005**	**0.009**
		(6.46)	(6.39)	(6.67)	(6.20)	(6.31)	(6.13)	(3.35)	(6.38)
Rev			**−0.081**	**−0.081**	**−0.080**	**−0.083**	**−0.083**	**−0.056**	**−0.083**
			(−16.12)	(−15.88)	(−14.89)	(−16.23)	(−15.66)	(−9.87)	(−15.62)
Illiq				**0.284**	**0.597**	**0.623**	**0.631**	**1.297**	**0.621**
				(2.36)	(4.93)	(5.11)	(5.22)	(6.89)	(5.14)
Lt rev					**−0.050**	−0.047	−0.047	−0.008	−0.041
					(−1.75)	(−1.67)	(−1.60)	(−0.32)	(−1.47)
Ivol					**−0.137**	0.069	0.069	0.076	0.069
					(−4.21)	(1.43)	(1.46)	(1.06)	(1.44)
Max						**−0.036**	**−0.036**	−0.022	**−0.036**
						(−3.45)	(−3.43)	(−1.37)	(−3.45)
Min						**−0.058**	**−0.059**	**−0.092**	**−0.059**
						(−4.46)	(−4.57)	(−6.64)	(−4.52)
Skew							0.001		
							(0.03)		
Eiskew								**−0.201**	
								(−1.70)	
Coskew									−0.040
									(−0.42)
N	954	954	954	954	954	954	954	276	954

The table reports the results of Fama-MacBeth regressions. The dependent variable is percentage return. TK is the prospect theory value of a stock's historical return distribution (see Section 1.2). In panel A, TK is constructed using returns from month $t-60$ to month $t-1$; in panel B, it is constructed using returns from month t-61 to month t-2. *Beta* is calculated from monthly returns over the previous five years, following Fama and French (1992). *Size* is the log market capitalization at the end of the previous month. *Bm* is the log book-to-market ratio. When the book value of equity is missing from Compustat, we use data from Davis, Fama, and French (2002); observations with negative book value are removed. *Mom* is the cumulative return from the start of month t-12 to the end of month t-2. *Rev* is the return in month t-1. *Illiq* is Amihud's (2002) measure of illiquidity, scaled by 10^5. *Lt rev* is the cumulative return from the start of month t-60 to the end of month t-13. *Ivol* is idiosyncratic return volatility, as in Ang et al. (2006). *Max* and *Min* are the maximum and the negative of the minimum daily returns in month t-1, as in Bali, Cakici, and Whitelaw (2011). *Skew* is the skewness of monthly returns over the previous five years. *Eiskew* is expected idiosyncratic skewness, as in Boyer, Mitton, and Vorkink (2010). *Coskew* is coskewness, computed as in Harvey and Siddique (2000) using five years of monthly returns. TK, *Mom*, *Rev*, *Ivol*, *Max*, and *Min* are scaled up by 100. The sample period runs from July 1931 to December 2010, except in the case of the *Eiskew* regression, where it starts in January 1988 due to data availability. t-statistics, in parentheses, are Newey-West adjusted with twelve lags, and bold typeface indicates a coefficient significant at the 10% level.

interpretation we are proposing, it is not clear that it should even be included as a control. We also note that the t-statistics in Table 5 are computed using a conservative specification of standard errors, namely, Newey-West adjusted with twelve lags.[10]

10 Harvey, Liu, and Zhu (2016) argue that, due to the prevalence of data mining, empirical results should be held to a higher level of statistical significance than is currently the case. However, they also note that an exception

In Section 1.2, we introduced a modified TK measure, TK(ρ), which downweights, by a multiplicative factor ρ, the components of TK associated with more distant past returns. We rerun the Fama-MacBeth regression in Column (6) of Table 5, panel A, but now use TK(ρ) in place of TK. To more easily compare the coefficients on TK(ρ) for different values of ρ, we normalize all of the independent variables, including TK(ρ), to have a mean of 0 and a standard deviation of 1. We find that, when $\rho < 1$, the statistical significance of the prospect theory variable goes up. For example, when $\rho = 1$, the coefficient on TK(ρ) is -0.162 with a t-statistic of 2.18; when $\rho = 0.95$, it is -0.151 with a t-statistic of 2.64; when $\rho = 0.85$, it is -0.167 with a t-statistic of 2.89; and when $\rho = 0.8$, it is -0.184 with a t-statistic of 3.07. These results are consistent with investors' putting less weight on more distant past returns.

One of the assumptions of our framework is that, after investors observe a stock's historical return distribution, they evaluate it according to prospect theory. We now consider the possibility that they instead evaluate it according to expected (power) utility. The prediction in this case is that a stock whose historical return distribution has a high (low) *expected utility* value will have low (high) subsequent returns. To test this prediction, we replace TK in the regression in Column (6) of Table 5, panel A, with the variable EU, defined as

$$\mathrm{EU} \equiv \frac{1}{60} \sum_{i=-m}^{n} \frac{(1+r_i)^{1-\theta}}{1-\theta}, \tag{17}$$

where, using the notation of Section 1.2, $\{r_{-m}, \ldots, r_n\}$ are the stock's monthly returns over the previous five years. We consider several values of θ, ranging from 0.5 to 10, but find no evidence that, after controlling for the important known predictors in Column (6), the EU variable has any predictive power for subsequent returns. This suggests that the results in Table 5 are driven by something specific to the way prospect theory weights stocks' past returns.

To ensure that the results in panel A are not driven by microstructure effects associated with small-cap stocks, we rerun the regression in Column (6) of the panel, while excluding all stocks in the lowest market-capitalization decile. We find that the coefficient on TK is again -0.043 and that it remains significant, with a t-statistic of 2.29.

Given the strength of the short-term reversal phenomenon, we also rerun the Fama-MacBeth regressions in panel A using a TK variable lagged by a month, in other words, one constructed using returns from month $t-61$ to $t-2$. Panel B of Table 5 presents the results. The TK variable continues to be

can be made for tests that, like ours, are heavily motivated and constrained by prior theory and evidence. In Section 2.6, we address the data-mining concern directly by conducting an out-of-sample test of our hypothesis in international data.

significant at the 5% level, even after including the many control variables. The only exception is when we include *Skew,* the measure of past return skewness. However, we see the lower significance of TK in this case as consistent with the skewness-related interpretation of TK's predictive power that we discuss later in the paper.

From Tables 4 and 5, we see that, while TK remains a statistically significant predictor of returns even after we control for the past month's return or skip a month between portfolio construction and performance measurement, both of these operations reduce TK's predictive power. This is surprising – TK is built using information on stock returns going back sixty months – and suggests that an outsize portion of TK's predictive power comes from the component of TK associated with the previous month's return, a component we label TK_{-1}. Indeed, if we replace the TK variable in regression (6) of Table 5, panel A, with two variables – TK_{-1} and the residual, TK-TK_{-1} – we find that, while both variables are significant predictors of subsequent returns, TK_{-1} is disproportionately so.

Why would the previous month's return play a sizable role in TK's predictive power? We noted earlier that some investors may receive their investment charts with a delay of approximately one month, say, and may therefore obtain a stock's most recent monthly return from another source. This separate sourcing of the previous month's return may cause it to receive greater weight, implying that investors' impression of the stock will be given not by TK in (8), but by a modified TK measure in which the component associated with the previous month's return is weighted more heavily. Interestingly, when we include TK_{-1} in the regression, as described above, the magnitude of the coefficient on *Rev* falls by about 30%. Our framework may therefore help explain part of the short-term reversal effect: if a stock had a particularly good return in the previous month, investors are substantially more likely to form a positive impression of the stock, causing it to become overvalued and to earn a lower subsequent return.

Fama-MacBeth regressions allow us to examine the predictive power of TK while controlling for known predictors, but they have a limitation: they assume that the relationship between stock returns and the various predictors is linear. We therefore also use double sorts to study TK's predictive power. Specifically, we use the following procedure. Suppose that we want to know whether the predictive power of TK is subsumed by control variable X. At the beginning of each month, we sort stocks into quintiles based on X. Within each quintile, we again sort stocks into quintiles, this time based on TK. The returns, over the next month, of the five TK-quintile portfolios are then averaged across different quintiles of the control variable X. More precisely, if $r_{i,j}$ is the return, over the next month, of the portfolio of stocks in the i'th quintile of X and j'th quintile of TK, we compute, for $j=1,\ldots,5$,

$$\overline{r}_j = \frac{r_{1,j}+\ldots+r_{5,j}}{5}. \tag{18}$$

We then compute

$$\overline{r}_1-\overline{r}_5=\frac{(r_{1,1}-r_{1,5})+\ldots+(r_{5,1}-r_{5,5})}{5} \tag{19}$$

as a measure of the return of the low-TK minus high-TK portfolio, controlling for variable X.

We report the results of this exercise in Table 6. Each column corresponds to a specific control variable. Within each column, we report the four-factor alphas of the five TK-quintile portfolios on both an equal-weighted and value-weighted basis – in other words, $\overline{r}_1$, $\overline{r}_2$, $\overline{r}_3$, $\overline{r}_4$, and $\overline{r}_5$, defined above, adjusted for the four Carhart (1997) factors – and, in the bottom row of each column, the four-factor alpha of the low-TK minus high-TK portfolio, in other words, $\overline{r}_1-\overline{r}_5$ adjusted for the four factors. The control variables we consider are the past month's return (*Rev*), Amihud's (2002) illiquidity measure (*Illiq*), the long-term past return (*Lt rev*), idiosyncratic volatility (*Ivol*), the maximum one-day return over the past month (*Max*), skewness of past returns (*Skew*), expected idiosyncratic skewness (*Eiskew*), and coskewness of past returns (*Coskew*).

The bottom row in Table 6 is the most important one. It shows that, consistent with the Fama-MacBeth results in Table 5, the TK variable retains significant predictive power for returns even after controlling for known predictors of returns.

2.5 The role of limits to arbitrage

We expect the predictive power of TK to be stronger for stocks less subject to the forces of arbitrage – for example, stocks with low market capitalizations, illiquid stocks, stocks with high idiosyncratic volatility, and stocks with low institutional ownership. We now test this hypothesis.

In Table 7, we present the results of four Fama-MacBeth regressions that are the same as the regression in Column (6) of Table 5, panel A, except that they include four new independent variables: TK interacted with *Size,* TK interacted with *Illiq,* TK interacted with *Ivol,* and TK interacted with *Institutional holding. Size* is the log market capitalization at the end of the previous month, *Illiq* is Amihud's (2002) measure of illiquidity, *Ivol* is idiosyncratic volatility, computed as in Ang et al. (2006), and *Institutional holding* is the log of one plus the fraction of a stock's outstanding shares held by institutional investors. Our analysis is based on the full sample from July 1931 to December 2010, except in the case of the *Institutional holding* regression, where, due to data availability, the sample begins in 1980.[11]

[11] For a given stock in a given quarter, we obtain the stock's institutional holdings by aggregating the positions of its institutional investors. Different institutional funds may have different report dates; we assume that fund managers do not trade between the report date and the quarter end. If the Thomson Reuters database does not have data on a particular stock, we set the stock's institutional holdings to zero. We also winsorize the institutional ownership variable at 1% in both tails and lag it by one quarter to match returns.

Table 6
Double sorts

TK	Rev		Illiq		Lt rev		Ivol		Max		Skew		Eiskew		Coskew	
	EW	VW	EW	VW	EW	VW	EW	VW	EW	VW	EW	VW	EW	VW	EW	VW
Low 1	0.493	0.267	0.626	0.285	0.572	0.295	0.679	0.231	0.764	0.281	0.684	0.234	0.690	0.267	0.693	0.259
	(4.77)	(2.93)	(7.59)	(3.57)	(6.56)	(3.61)	(7.72)	(2.68)	(7.87)	(3.13)	(5.93)	(2.36)	(6.23)	(2.92)	(6.13)	(2.68)
2	0.186	0.261	0.262	0.061	0.249	0.169	0.273	0.047	0.201	0.048	0.167	0.119	0.202	0.229	0.163	0.199
	(3.87)	(3.77)	(5.37)	(1.27)	(5.09)	(3.08)	(5.60)	(0.76)	(4.00)	(0.78)	(3.36)	(1.77)	(4.25)	(3.61)	(3.45)	(2.93)
3	0.154	0.095	0.120	−0.051	0.147	0.074	0.089	−0.091	0.086	−0.054	0.100	0.072	0.114	0.114	0.121	0.093
	(3.75)	(1.92)	(2.99)	(−1.22)	(3.84)	(1.52)	(2.25)	(−1.73)	(2.23)	(−1.06)	(2.60)	(1.45)	(2.77)	(2.23)	(3.10)	(1.85)
4	0.105	0.091	0.049	−0.111	0.060	−0.008	0.025	−0.200	0.002	−0.136	0.035	−0.022	0.043	0.058	0.034	0.059
	(2.60)	(2.33)	(1.14)	(−2.77)	(1.60)	(−0.18)	(0.64)	(−4.29)	(0.05)	(−2.91)	(0.87)	(−0.52)	(1.12)	(1.47)	(0.83)	(1.53)
High 5	−0.007	−0.061	−0.127	−0.345	−0.100	−0.120	−0.135	−0.318	−0.118	−0.141	−0.056	−0.170	−0.100	−0.059	−0.079	−0.078
	(−0.14)	(−1.35)	(−2.59)	(−6.44)	(−2.05)	(−2.53)	(−2.91)	(−5.48)	(−2.55)	(−2.83)	(−1.30)	(−4.24)	(−2.10)	(−1.44)	(−1.64)	(−1.93)
1-5	**0.500**	**0.328**	**0.753**	**0.630**	**0.673**	**0.416**	**0.814**	**0.549**	**0.882**	**0.421**	**0.740**	**0.403**	**0.790**	**0.326**	**0.772**	**0.337**
	(4.34)	(2.94)	(7.94)	(6.23)	(7.02)	(4.11)	(8.34)	(4.83)	(8.15)	(3.79)	(5.77)	(3.47)	(6.38)	(2.95)	(6.18)	(2.93)

Each month, stocks are sorted into quintiles based on a control variable (one of *Rev*, *Illiq*, *Lt rev*, *Ivol*, *Max*, *Skew*, *Eiskew*, or *Coskew*). Then, within each quintile, stocks are further sorted into quintiles based on TK. The returns of the five TK portfolios over the next month are averaged across the five control variable quintiles. We report the four-factor alphas, on both an equal-weighted (EW) and value-weighted (VW) basis, of the five TK portfolios and of the low-TK minus high-TK long-short portfolio. TK is the prospect theory value of a stock's historical return distribution. *Rev* is the return in month t-1. *Illiq* is Amihud's (2002) measure of illiquidity. *Lt rev* is the cumulative return from the start of month t-60 to the end of month t-13. *Ivol* is idiosyncratic return volatility, as in Ang et al. (2006). *Max* is the maximum daily return in month t-1, as in Bali, Cakici, and Whitelaw (2011). *Skew* is the skewness of monthly returns over the previous five years. *Eiskew* is expected idiosyncratic skewness, as in Boyer, Mitton, and Vorkink (2010). *Coskew* is coskewness, computed as in Harvey and Siddique (2000) using five years of monthly returns. The sample period runs from July 1931 to December 2010, except in the case of *Eiskew*, where it starts in January 1988 due to data availability. t-statistics are in parentheses.

Table 7
Fama-MacBeth analysis of limits to arbitrage

Interactions of limits-to-arbitrage measures with TK	Size (1)	Illiq (2)	Ivol (3)	Institutional holding (4)
TK	**−0.314**	−0.025	0.002	**−0.055**
	(−4.79)	(−1.31)	(0.10)	(−1.88)
TK*Size	**0.029**			
	(4.90)			
TK*Illiq		**−0.060**		
		(−2.07)		
TK*Ivol			**−0.014**	
			(−2.41)	
TK*Institutional holding				**0.014**
				(2.84)
Beta	0.254	0.124	0.258	0.240
	(2.15)	(2.24)	(2.17)	(1.48)
Size	0.042	−0.109	−0.106	−0.094
	(1.34)	(−3.92)	(−3.82)	(−2.74)
Bm	0.128	0.124	0.124	0.233
	(2.32)	(2.24)	(2.23)	(2.99)
Mom	0.008	0.008	0.008	0.006
	(6.26)	(6.34)	(6.07)	(4.17)
Rev	−0.081	−0.080	−0.080	−0.061
	(−16.42)	(−6.34)	(−16.22)	(−12.45)
Illiq	0.501	0.084	0.568	0.405
	(3.80)	(0.36)	(4.46)	(3.69)
Lt rev	−0.087	−0.063	−0.065	−0.011
	(−2.80)	(−2.17)	(−2.13)	(−0.50)
Ivol	0.060	0.072	−0.032	0.024
	(1.26)	(1.51)	(−0.57)	(0.41)
Max	−0.034	−0.035	−0.039	0.012
	(−3.18)	(−3.28)	(−3.77)	(0.82)
Min	−0.058	−0.058	−0.055	−0.065
	(−4.51)	(−4.43)	(−4.14)	(−5.04)
Institutional holding				0.196
				(7.08)
N	954	954	954	366

The table reports the results of Fama-MacBeth regressions of stock returns (in percentage terms) on TK – the prospect theory value of a stock's historical return distribution – and on TK interacted with four variables that proxy for limits to arbitrage: *Size*, the log market capitalization; *Illiq*, Amihud's (2002) measure of illiquidity (scaled by 10^5); *Ivol*, idiosyncratic return volatility, as in Ang et al. (2006); and *Institutional holding*, the log of one plus the fraction of outstanding shares held by institutional investors. The other variables are controls, defined in Table 1. The sample runs from July 1931 to December 2010, except in the case of the *Institutional holding* regression, where it starts in 1980. TK, *Mom*, *Rev*, *Ivol*, *Max*, and *Min* are scaled up by 100. t-statistics, in parentheses, are Newey-West adjusted with twelve lags.

The coefficients on the interaction terms in the four regressions in Table 7 confirm our hypothesis that the predictive power of TK is greater among stocks with low market capitalizations, illiquid stocks, stocks with high idiosyncratic volatility, and stocks with low institutional ownership – in other words, among stocks for which the forces of arbitrage are likely to be weaker.[12]

12 We also conduct a double-sort analysis to test the hypothesis that the predictive power of TK is stronger among stocks for which limits to arbitrage are greater. The results of this analysis are again consistent with the hypothesis.

2.6 International evidence

Our main hypothesis is that a stock's prospect theory value can predict the stock's subsequent return in the cross-section. We have provided some evidence for this using data on U.S. stocks. We now implement an important out-of-sample test: we test our hypothesis using data from Datastream on forty-six international stock markets.

For each stock market in turn, we conduct a test similar to the decile-sort tests in Table 2. Each month, we sort stocks into quintiles based on TK and record the return of each quintile over the next month. This gives us a time series of returns for each TK quintile. We use these time series to compute the average return of each quintile over the entire sample, and hence also the average return of a long-short portfolio that goes long the stocks in the low-TK quintile and short the stocks in the high-TK quintile. Specifically, we compute, on both an equal-weighted and a value-weighted basis, the raw average return of the long-short portfolio, but also the average return of the long-short portfolio adjusted for "global factors," for "international factors," and for "local factors." For a given market, the local factors are the four Carhart (1997) factors constructed from the universe of stocks traded in that market; the global factors are the four Carhart factors constructed from the universe of *all* stocks across all forty-six markets plus the U.S. market; and the international factors are a set of eight factors: the four local factors supplemented by the four global factors (Hou, Karolyi, and Kho 2011). As before, we expect our prediction to hold more strongly for equal-weighted returns.

In panel A of Table 8, we report the average value, across the forty-six markets, of the eight types of long-short portfolio returns we compute in each market ("Average alpha"), the number of markets for which the portfolio return is positive/negative and significant/insignificant at the 10% level, and the percentage of markets in which the portfolio return is positive or positive and significant. In panel B, we report analogous results for the case in which we skip a month between the moment of portfolio construction and the moment at which we start measuring returns.[13]

The international evidence is consistent with our prediction. Panel A of the table shows that, across all specifications, the vast majority of the markets we consider – as many as 80% to 90% of them – generate an average long-short portfolio return with the predicted positive sign. And for equal-weighted returns, the average return is positive in a statistically significant way in a strong majority of the forty-six markets.

In Section 2.2, we noted that, in U.S. data, TK's predictive power is particularly strong in the first month after portfolio construction. Panel B of Table 8 shows that, by contrast, in the international data, skipping a month between the moment when TK is computed and the moment at which we start

[13] The table caption contains additional information about our methodology. Results on a market-by-market basis are available from the authors on request.

Table 8
Quintile portfolio analysis in forty-six international stock markets

	Long-short		Global factors		International factors		Local factors	
	EW	VW	EW	VW	EW	VW	EW	VW
A. Do not skip a month								
Average alpha (%)	1.017	0.494	1.237	0.692	1.471	0.878	1.397	0.892
Positive and significant	26	8	29	16	38	18	35	16
Positive and insignificant	12	24	13	19	5	23	8	23
Negative and significant	3	1	0	0	0	0	0	0
Negative and insignificant	5	13	4	11	3	5	3	7
Percent positive	83%	70%	91%	76%	93%	89%	93%	85%
Percent positive and significant	57%	17%	63%	35%	83%	39%	76%	35%
B. Skip a month								
Average alpha (%)	0.745	0.264	0.946	0.460	1.208	0.695	1.132	0.690
Positive and significant	23	4	24	11	31	16	32	13
Positive and insignificant	13	22	18	21	12	21	11	26
Negative and significant	3	1	1	0	0	0	0	0
Negative and insignificant	7	19	3	14	3	9	3	7
Percent positive	78%	57%	91%	70%	93%	80%	93%	85%
Percent positive and significant	50%	9%	52%	24%	67%	35%	70%	28%

For each of forty-six international stock markets, we sort stocks each month into quintiles based on TK and then compute, on both an equal-weighted (EW) and value-weighted (VW) basis, the average raw return and alphas from models with "global factors," "local factors," and "international factors" of the portfolio that, each month, buys (shorts) stocks in the lowest (highest) TK quintile. All returns are in U.S. dollars. The risk-free rate is the one-month U.S. Treasury rate. For a given market, the local factors are the four Carhart (1997) factors constructed from the universe of stocks traded in that market; the global factors are the four Carhart factors constructed from the universe of all stocks across all forty-six markets and the United States; and the international factors are the union of the four local factors and four global factors (Hou, Karolyi, and Kho 2011). In panel A, we report the average value, across the forty-six markets, of the eight types of long-short portfolio returns we compute in each market ("Average alpha"), the number of markets for which the portfolio return is positive/negative and significant/insignificant at the 10% level, and the percentage of markets in which the portfolio return is positive or positive and significant. In panel B, we report analogous results for the case in which we skip a month between the moment of TK construction and the moment at which we start measuring returns.

For each market, we include all common stocks listed on the major exchange(s) in that market. Following Griffin, Kelly, and Nardari (2010), we eliminate noncommon stocks, such as preferred stocks, warrants, unit or investment trusts, funds, REITs, ADRs, and duplicates. A cross-listed stock is included only in its home-market sample. If a stock has multiple share classes, only the primary class is included; for example, we include only A-shares in the Chinese stock market and only bearer-shares in the Swiss stock market. To alleviate survivorship bias, both active and delisted stocks are included. We filter out suspicious stock returns by setting returns above 100% to 100% and returns below −95% to −95% (Ang et al. 2009; Chui, Titman, and Wei 2010). To be included in the sample, we require that a stock has no missing monthly return in the past five years. As in Table 2, we construct TK using returns in excess of the value-weighted market return; the results are similar if we instead use raw returns or returns in excess of the risk-free rate. We also require that, for a given month, the number of stocks with a valid TK measure is at least fifty. Following Hou, Karolyi, and Kho (2011), for all three factor models, the value factor is constructed using the price-to-cash flow ratio rather than the price-to-book ratio.

measuring returns affects our results to a lesser extent: even after skipping a month, the vast majority of markets generate a long-short equal-weighted alpha with the predicted positive sign, and, in more than half of the markets, this alpha is both positive and statistically significant. Even in the value-weighted case, there is evidence that TK has predictive power for subsequent returns.

2.7 Mechanism

In previous sections, we have presented evidence that a stock's prospect theory value predicts the stock's subsequent return in the cross-section. Our interpretation of this is that, when thinking about a stock, some investors mentally represent it by its historical return distribution and then evaluate this distribution according to prospect theory. They tilt their portfolios toward stocks whose past return distributions are appealing under prospect theory, thereby causing these stocks to become overvalued and to earn low subsequent returns. Similarly, they tilt away from stocks whose past return distributions are unappealing under prospect theory, causing these stocks to become undervalued and to earn high subsequent returns.

What is it exactly about the past returns of stocks with high prospect theory values that makes these stocks appealing to investors? Conversely, what is it about the past returns of stocks with low prospect theory values that makes these stocks unappealing? To answer this, we start by looking more closely at the characteristics of low-TK and high-TK stocks. In general, gambles with high prospect theory values have a high mean payoff, a low standard deviation (loss aversion lowers the prospect theory value of a gamble with a high standard deviation), and high skewness (probability weighting raises the prospect theory value of a positively skewed gamble). Given this, we expect stocks with high TK values to be stocks with high past returns, low past volatility, and high past skewness.

To examine this conjecture, we sort stocks each month into deciles based on TK, and then, for each decile, and for a variety of characteristics, we record the average value of each characteristic across all stocks in that decile at that time. We repeat this exercise in every month from July 1931 to December 2010. For each decile in turn, we then compute the time-series averages of the decile-level characteristics across all months in the sample. In effect, this exercise allows us to dig deeper into the correlations in panel B of Table 1.

The results, reported in Table 9, confirm our conjecture to some extent, but also indicate some subtleties. As expected, measures of past returns (*Rev*, *Mom*, and *Lt rev*) increase monotonically from TK decile 1 to TK decile 10. Past skewness (*Skew*) also generally increases from TK-decile 1 to TK-decile 10; however, the increase in skewness occurs largely as we move from decile 8 to decile 9, and from decile 9 to decile 10. Interestingly, while measures of past volatility – *Beta*, *Ivol*, and *Sd*, the standard deviation of monthly returns over the past five years – decrease monotonically from TK-decile 1 to TK-decile 8, they increase slightly in decile 9 and then rise significantly in decile 10; indeed, far from being the least volatile stocks, the stocks in decile 10 are *more* volatile than average. The reason for this is that volatility and skewness are correlated in the cross-section; since stocks in TK-decile 10 are much more skewed than the average stock, they are also more volatile than the average stock. Put differently, the stocks in decile 10 are those that trade off past return, volatility, and skewness in a way that is maximally attractive to a prospect theory individual. Table 9 shows that these are stocks with high past returns and high past skewness, but, if anything, higher past volatility than the average stock.

Table 9
Characteristics of TK portfolios

Portfolios	TK	Beta	Size	Bm	Mom	Rev	Illiq	Lt rev	Ivol	Max	Min	Skew	Eiskew	Coskew	Sd
Low TK	−0.110	1.312	9.357	0.433	−0.073	−0.010	3.417	−0.331	0.042	0.112	0.089	0.571	0.809	−0.017	0.161
2	−0.083	1.241	10.129	0.170	0.042	0.005	1.172	−0.018	0.030	0.081	0.065	0.579	0.625	−0.021	0.140
3	−0.071	1.187	10.544	0.028	0.086	0.009	0.677	0.183	0.026	0.070	0.057	0.580	0.545	−0.017	0.130
4	−0.062	1.139	10.859	−0.072	0.110	0.012	0.443	0.369	0.023	0.063	0.051	0.568	0.477	−0.013	0.122
5	−0.055	1.099	11.110	−0.157	0.134	0.014	0.314	0.544	0.021	0.057	0.047	0.569	0.433	−0.011	0.115
6	−0.048	1.076	11.320	−0.216	0.160	0.016	0.220	0.727	0.019	0.053	0.044	0.568	0.399	−0.007	0.110
7	−0.042	1.059	11.512	−0.301	0.186	0.018	0.197	0.928	0.018	0.051	0.042	0.581	0.369	0.002	0.106
8	−0.035	1.068	11.677	−0.377	0.219	0.020	0.174	1.170	0.017	0.049	0.040	0.632	0.353	0.012	0.106
9	−0.027	1.098	11.798	−0.472	0.265	0.023	0.176	1.547	0.017	0.049	0.040	0.730	0.356	0.020	0.109
High TK	−0.007	1.319	11.658	−0.669	0.432	0.037	0.572	2.919	0.021	0.063	0.048	1.314	0.483	0.033	0.156

Each month, we sort stocks into deciles based on TK and then, for each decile, compute the mean values of the characteristics listed in the top row of the table across all stocks in the decile. The table reports, for each TK decile, the time-series averages of these mean characteristic values. All but one of the variables is defined in Table 1. The remaining variable, *Sd*, is the standard deviation of a stock's monthly returns over the past five years. The sample runs from July 1931 to December 2010, except in the case of *Eiskew*, where it starts in January 1988 due to data availability.

Table 10
Fama-MacBeth regressions using different components of prospect theory

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	LA	PW	CC	LACC	LAPW	CCPW	TK
TK components	**−0.219**	**−0.136**	**−0.149**	**−0.160**	**−0.152**	**−0.134**	**−0.162**
	(−1.90)	**(−2.29)**	**(−1.62)**	**(−1.46)**	**(−2.18)**	**(−2.37)**	**(−2.18)**
Beta	0.126	0.183	0.187	0.119	0.164	0.188	0.165
	(1.80)	(2.32)	(2.24)	(1.80)	(2.07)	(2.34)	(2.07)
Size	−0.172	−0.253	−0.233	−0.181	−0.198	−0.253	−0.195
	(−3.06)	(−3.88)	(−3.51)	(−3.23)	(−3.27)	(−3.86)	(−3.24)
Bm	0.107	0.120	0.097	0.111	0.125	0.117	0.120
	(2.05)	(2.28)	(1.85)	(2.15)	(2.38)	(2.24)	(2.29)
Mom	0.537	0.544	0.576	0.511	0.549	0.557	0.556
	(6.05)	(5.35)	(6.78)	(5.62)	(6.22)	(5.71)	(6.47)
Rev	−1.269	−1.272	−1.266	−1.277	−1.269	−1.268	−1.268
	(−16.55)	(−16.08)	(−16.71)	(−16.56)	(−16.42)	(−16.15)	(−16.47)
Illiq	3.095	3.059	3.123	3.104	3.046	3.065	3.058
	(5.21)	(4.99)	(5.08)	(5.23)	(5.05)	(5.00)	(5.08)
Lt rev	−0.023	−0.190	−0.038	−0.099	−0.111	−0.163	−0.083
	(−0.32)	(−2.16)	(−0.44)	(−1.44)	(−1.66)	(−1.99)	(−1.31)
Ivol	0.133	0.213	0.159	0.146	0.175	0.208	0.169
	(1.23)	(1.73)	(1.33)	(1.36)	(1.46)	(1.68)	(1.43)
Max	−0.283	−0.311	−0.303	−0.281	−0.306	−0.310	−0.303
	(−3.29)	(−3.57)	(−3.46)	(−3.28)	(−3.48)	(−3.55)	(−3.45)
Min	−0.298	−0.284	−0.281	−0.297	−0.293	−0.284	−0.293
	(−4.64)	(−4.35)	(−4.30)	(−4.62)	(−4.50)	(−4.34)	(−4.50)
N	954	954	954	954	954	954	954

We report the results of seven Fama-MacBeth regressions. The dependent variable is percentage return. The seven specifications vary by which components of prospect theory are incorporated into the prospect theory variable TK. "LA," "PW," and "CC" indicate that only loss aversion, probability weighting, and concavity/convexity are incorporated into TK, respectively. "LA,CC" indicates that only loss aversion and concavity/convexity are incorporated; "LA,PW" and "CC,PW" are defined similarly. "TK" in Column (7) incorporates all elements of prospect theory and is identical to the TK variable in previous tables. To make it easier to compare across different specifications, we standardize the independent variables to have mean 0 and standard deviation 1. The sample runs from July 1931 to December 2010. t-statistics, in parentheses, are Newey-West adjusted with twelve lags.

Given that high-TK stocks have much higher past returns than low-TK stocks, one might conjecture that the negative relation between TK and subsequent returns in the cross-section has something to do with the negative relation between *Rev* / *Lt rev* and subsequent returns in the cross-section. However, in our earlier tests, using two different methodologies, we saw that controlling for *Rev* and *Lt rev* did not take away TK's predictive power. Nor is it easy to argue that TK's predictive power is related to the predictive power of volatility for subsequent returns: the volatility of stocks in TK-decile 1 is not very different from the volatility of stocks in TK-decile 10. A more promising avenue is to focus on skewness: high-TK stocks are much more highly skewed than low-TK stocks.

Some evidence that suggests a role for skewness comes from examining which component of prospect theory is most responsible for TK's ability to predict returns. Table 10 presents some results on this. The seven numbered columns in the table correspond to seven different Fama-MacBeth regressions. The regressors in each case are a prospect theory variable (first row) and ten well-known predictors of returns, such as market capitalization and book-to-market, as controls. The control variables are the same across the seven regression specifications; only the prospect theory variable changes.

The right-most column uses TK as the prospect theory variable; this column therefore corresponds to the Fama-MacBeth regression we previously ran in Column (6) of Table 5, panel A.[14] In the other six columns, we "turn off" one or more features of prospect theory. For example, the label "LA" in the first column stands for "loss aversion." The prospect theory variable in this regression features loss aversion, but turns off probability weighting and the concavity/convexity feature of the value function; in other words, it is the quantity on the right-hand side of Equation (8), computed using $\lambda = 2.25$ as before, but also using $(\alpha, \gamma, \delta) = (1, 1, 1)$ in place of $(0.88, 0.61, 0.69)$. Similarly, in the column labeled "PW" ("probability weighting"), the prospect theory variable uses $(\gamma, \delta) = (0.61, 0.69)$, as in our main analysis, but also $(\alpha, \lambda) = 1$, thereby retaining probability weighting but turning off loss aversion and concavity/convexity; in the column labeled "CC" ("concavity/convexity"), we use $(\alpha, \gamma, \delta, \lambda) = (0.88, 1, 1, 1)$. The column labeled "LA,CC" retains loss aversion and concavity/convexity but turns off probability weighting; in other words, it corresponds to $(\alpha, \gamma, \delta, \lambda) = (0.88, 1, 1, 2.25)$. "LA,PW" and "CC,PW" are defined in a similar way.

The results in Table 10 suggest that the element of prospect theory primarily responsible for the predictive power of TK is probability weighting: the four most significant coefficients in the first row of the table correspond to prospect theory variables that involve probability weighting (Columns (2), (5), (6), and (7)), while for the three least significant coefficients, probability weighting has been turned off (Columns (1), (3), and (4)). Put another way, by comparing Columns (4) and (7), we see that, if we remove probability weighting from the TK variable while retaining the other features of prospect theory, the significance of the coefficient on the prospect theory variable drops markedly. We stress that this evidence on the role of probability weighting is tentative: comparing *t*-statistics across columns does not constitute a formal statistical test.

Table 11 makes the same point in a different way. The eleven numbered columns in the table correspond to eleven different Fama-MacBeth regressions. As before, the regressors in each specification are a prospect theory variable (first row) and a set of control variables, with only the prospect theory variable changing across specifications. In this table, we vary only the values of the probability weighting parameters. Specifically, each specification sets the loss aversion and concavity/convexity parameters to $\alpha = 0.88$ and $\lambda = 2.25$ as before,

[14] The regression coefficients in Column (7) of Table 10 are different from those in Column (6) of Table 5, panel A. In Table 10, we normalize each independent variable to have a mean of 0 and a standard deviation of 1. This makes it easier to compare the coefficients on the various prospect theory variables in the first row of the table. In Table 5, the independent variables are *not* normalized, thereby enabling comparison with other empirical studies that follow the same procedure.

Table 11
Fama-MacBeth regressions that vary the degree of probability weighting

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
γ/δ	0.31/ 0.39	0.41/ 0.49	0.51/ 0.59	0.61/ 0.69	0.71/ 0.79	0.81/ 0.89	0.91/ 0.99	1.01/ 1.09	1.11/ 1.19	1.21/ 1.29	1.31/ 1.39
TK	**−0.068**	**−0.090**	**−0.121**	**−0.162**	**−0.198**	**−0.206**	**−0.186**	**−0.143**	**−0.131**	**−0.109**	**−0.093**
	(−1.75)	**(−2.02)**	**(−2.16)**	**(−2.18)**	**(−2.09)**	**(−1.91)**	**(−1.69)**	**(−1.36)**	**(−1.30)**	**(−1.15)**	**(−1.02)**
Beta	0.161	0.166	0.167	0.165	0.155	0.141	0.131	0.124	0.127	0.127	0.129
	(2.07)	(2.09)	(2.09)	(2.07)	(2.00)	(1.91)	(1.86)	(1.85)	(1.88)	(1.90)	(1.92)
Size	−0.218	−0.216	−0.209	−0.195	−0.182	−0.176	−0.180	−0.189	−0.194	−0.199	−0.204
	(−3.42)	(−3.40)	(−3.35)	(−3.24)	(−3.13)	(−3.10)	(−3.18)	(−3.34)	(−3.40)	(−3.49)	(−3.56)
Bm	0.136	0.133	0.128	0.120	0.112	0.108	0.108	0.111	0.112	0.114	0.116
	(2.59)	(2.53)	(2.44)	(2.29)	(2.14)	(2.05)	(2.06)	(2.14)	(2.16)	(2.21)	(2.25)
Mom	0.519	0.530	0.543	0.556	0.558	0.548	0.532	0.509	0.507	0.500	0.495
	(5.63)	(5.83)	(6.14)	(6.47)	(6.53)	(6.32)	(6.03)	(5.61)	(5.59)	(5.45)	(5.36)
Rev	−1.277	−1.275	−1.272	−1.268	−1.266	−1.268	−1.273	−1.279	−1.280	−1.282	−1.283
	(−16.31)	(−16.32)	(−16.38)	(−16.47)	(−16.55)	(−16.60)	(−16.62)	(−16.60)	(−16.63)	(−16.63)	(−16.63)
Illiq	3.026	3.031	3.042	3.058	3.076	3.092	3.104	3.116	3.122	3.128	3.133
	(4.96)	(4.98)	(5.02)	(5.08)	(5.14)	(5.19)	(5.21)	(5.23)	(5.22)	(5.23)	(5.22)
Lt rev	−0.224	−0.196	−0.149	−0.083	−0.029	−0.018	−0.043	−0.102	−0.109	−0.132	−0.151
	(−2.48)	(−2.38)	(−2.08)	(−1.31)	(−0.43)	(−0.24)	(−0.59)	(−1.45)	(−1.54)	(−1.89)	(−2.15)
Ivol	0.192	0.191	0.184	0.169	0.151	0.140	0.142	0.153	0.157	0.163	0.168
	(1.55)	(1.55)	(1.51)	(1.43)	(1.32)	(1.27)	(1.31)	(1.41)	(1.44)	(1.49)	(1.53)
Max	−0.307	−0.308	−0.307	−0.303	−0.295	−0.288	−0.285	−0.284	−0.286	−0.287	−0.289
	(−3.47)	(−3.49)	(−3.48)	(−3.45)	(−3.39)	(−3.33)	(−3.31)	(−3.31)	(−3.33)	(−3.34)	(−3.35)
Min	−0.287	−0.288	−0.290	−0.293	−0.295	−0.295	−0.295	−0.295	−0.293	−0.293	−0.292
	(−4.42)	(−4.43)	(−4.46)	(−4.50)	(−4.55)	(−4.58)	(−4.58)	(−4.59)	(−4.56)	(−4.55)	(−4.54)
N	954	954	954	954	954	954	954	954	954	954	954

We report the results of eleven Fama-MacBeth regressions. The dependent variable is percentage return. The eleven specifications differ in the values of the parameters γ and δ used to construct the prospect theory variable TK. These parameters control the degree of probability weighting; Tversky and Kahneman's (1992) estimates are 0.61 and 0.69, respectively. To make it easier to compare across different specifications, we standardize the independent variables to have mean 0 and standard deviation 1. The sample period runs from July 1931 to December 2010. t-statistics, in parentheses, are Newey-West adjusted with twelve lags.

but varies the probability weighting parameters γ and δ, giving them the values listed at the top of each column. Recall that the baseline parameter values are $\gamma = 0.61$ and $\delta = 0.69$, the values used in Column (4).

The table shows that the most statistically significant prospect theory variables correspond to values of γ and δ that are below 1, in other words, to values that imply *over*weighting of tails. By contrast, the predictive power of the prospect theory variable drops markedly in Columns (8)-(11); these columns correspond to values of γ and δ that are greater than 1, in other words, to values that imply underweighting of tails. This leads to the, again, tentative suggestion that the predictive power of the prospect theory variable is related to the overweighting of tails generated by probability weighting.

That probability weighting appears to play a significant role in our results suggests that the skewness of a stock's past returns may be important for understanding the predictive power of TK: under probability weighting, the appeal of a stock depends, in part, on the skewness of its returns. Consistent with this, Table 9 shows that the past returns of high-TK stocks are indeed highly skewed. In short, then, part of what may be driving the predictive power of TK is that, when investors observe the past returns of a high-TK stock – for example, by looking at a chart of its past price movements – the skewness they see leads them to mentally represent the stock as a lottery-like gamble. Since they find such gambles appealing, they tilt toward the stock, causing it to become overpriced and to earn low subsequent returns.

An additional piece of evidence that skewness is, in part, driving the predictive power of TK comes from Columns (7) in the two panels of Table 5, which show that the predictive power of TK diminishes somewhat when past skewness (*Skew*) is included as a control. However, the same regression also suggests that skewness is unlikely to be the whole story: while TK predicts subsequent returns, *Skew* itself does not. This suggests that TK may be capturing what people find appealing, or not appealing, about the extremes of a return distribution in a way that simple measures of skewness are too crude to do. We also note that probability weighting does not affect only the tails of a distribution. In other words, TK is likely capturing what people find appealing, or not appealing, about the *entire* distribution of past returns – an appeal not captured by mean, volatility, or skewness alone.

3. Conclusion

We propose that, when deciding how much money to allocate to a stock, some investors use a two-step process. First, they form a mental representation of the stock. Second, they evaluate this representation to see whether the stock is appealing. We have made assumptions about each of these steps that we think are psychologically plausible. Specifically, we suggest that some investors mentally represent the stock by the distribution of its past returns, and that they evaluate this representation according to prospect theory. This framework predicts that, in the cross-section, the prospect theory value of a stock's historical return distribution will be negatively related to the stock's subsequent return. We find support for this prediction using data from the U.S. stock market and from forty-six international stock markets.

In the coming years, researchers are likely to develop better models of both the representation and valuation stages of decision making under risk. While prospect theory makes broadly accurate predictions about how people evaluate risks they are presented with, at least in experimental settings, superior valuation models may emerge. And while it is plausible that some investors mentally represent a stock by the distribution of its past returns, other representations may be more realistic still. The empirical framework we present in this paper can be used to test these new theories of representation and valuation. The new theories will likely lead us to new predictors of returns – predictors whose forecasting ability may well exceed that of the TK variable we focused on here.

Appendix

Proof of Proposition 1.[15] Rearranging Equation (12), we obtain

$$\omega_t = \omega_m - \eta\omega_{TK}. \tag{A1}$$

Since ω_t are the asset weights in the tangency portfolio, we know that, for some γ,

$$\mu - r_f 1 = \gamma\Sigma\omega_t, \tag{A2}$$

where μ is the $J\times 1$ vector of mean asset returns, Σ is the $J\times J$ matrix of return covariances, and 1 is a $J\times 1$ vector of ones. Substituting (A1) into (A2) gives

$$\mu - r_f 1 = \gamma(\Sigma\omega_m - \eta\Sigma\omega_{TK}). \tag{A3}$$

Premultiplying both sides of (A3) by ω_m leads to

$$\mu_m - r_f = \gamma\sigma_m^2(1-\eta\beta_{TK}). \tag{A4}$$

Dividing Equation (A3) by Equation (A4) gives

$$\frac{\mu_j - r_f}{\mu_m - r_f} = \frac{\beta_j - \eta\sigma_{j,TK}/\sigma_m^2}{1-\eta\beta_{TK}} \tag{A5}$$

for all $j\in\{1,\ldots,J\}$. Given the decomposition in the statement of the proposition, we have $\sigma_{j,TK} = \beta_j\beta_{TK}\sigma_m^2 + s_{j,TK}$. Substituting this into (A5) gives

$$\frac{\mu_j - r_f}{\mu_m - r_f} = \beta_j - \frac{\eta s_{j,TK}}{\sigma_m^2(1-\eta\beta_{TK})}, \tag{A6}$$

which is Equation (13).

Under the additional assumption that $Cov(\widetilde{\varepsilon}_i,\widetilde{\varepsilon}_j)=0$, we have $s_{j,TK} = \omega_{TK}^j s_j^2$. Substituting this into (A6) and using the definition of ω_{TK}^j in Equation (11) gives Equation (16).

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[15] We are grateful to the referee for suggesting the specific method of proof used here.

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