

CH8. A Closed-Economy One-Period Macroeconomic Model (cont): Equilibrium

EE312 Topic 5, 6 (for Section 046402)

Read: Williamson Ch.4 - 5

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1 Introduction :

- Course outline

1. Macroeconomics Measurement, Business Cycles VS. Trend

Part I Business cycles and Economics fluctuations: Short-run analysis

2. IS-LM model and labour market
3. Open economy macroeconomics

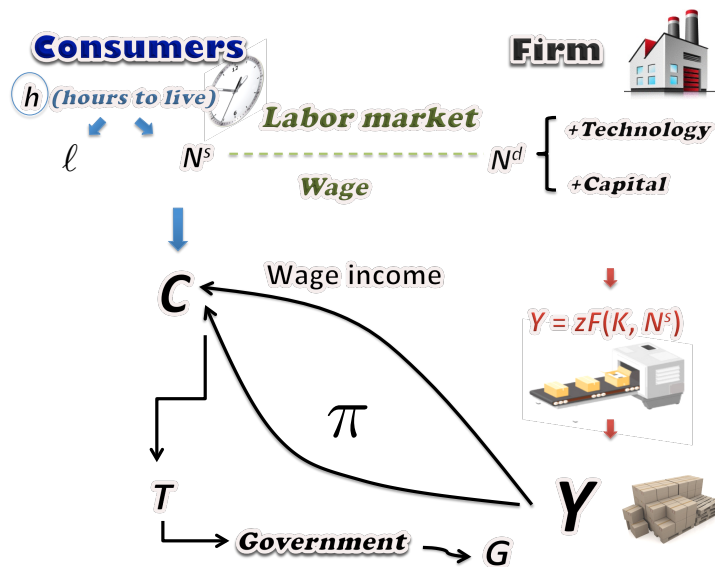
Part II Business cycles and Economics fluctuations: Medium run analysis

4. Keynes and Monetarist
5. New Classicals
6. New Keynesian

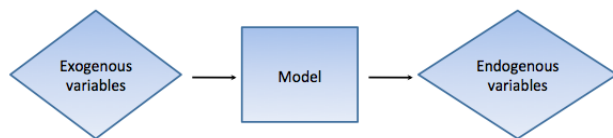
Part III Business cycles and Economics fluctuations: Micro-founded macroeconomic approach

7. A Closed-Economy One-Period Macroeconomic Model: Optimizing-agent decision
8. A Closed-Economy One-Period Macroeconomic Model: Equilibrium
9. Two-Period Model: the Consumption-Savings Decision
10. A Real Intertemporal Model with Investment
11. Long-term Economic Growth

2 Overview



One-Period macroeconomic model



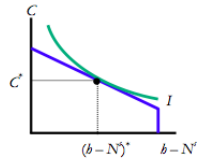
- Exogenous variables: z , G and K .
- Endogenous variables: C , Y , N^d , N^S , w , T .

3 Competitive equilibrium and Pareto Optimality

3.1 Competitive Equilibrium

- The values of endogenous variables (C , Y , N^d , N^S , w , T). at which, given z , K and G :
 - The representative consumer chooses C and N^S so that utility is maximized, given w , T and π .
 - The representative firm chooses Y and N^d so that profit is maximized, given w , z and K .
 - Competitive refers to the fact that all consumers and firms are price-takers.
 - Equilibrium refers to the state when the actions of all consumers and firms are consistent.
 - The labor market clears: $N^d = N^S$.
 - The government budget constraint: $G = T$.

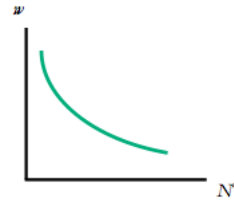
1. C and N^S solves the consumer's problem. $\max U(C, \ell)$ subject to $C = N^S w + \pi - T$.



$$MRS_{\ell, C} = w, N^S = h - \ell$$

2. N^d solves the firm's problem given z , K and w .

$$\max \pi = zF(K, N^d) - wN^d$$



$$MP_N = w \Rightarrow N^d = MP_N = w$$

3. Labor Market Clears: $N = N^S = N^d$

4. Good Market Clears: $Y = C + G$

5. Government balance budget constraint: $G = T$

Income-expenditure identity

$$Y = C + G$$

- In a competitive equilibrium, the goods market clears:

Y = total output or income; C = consumption expenditure; G = government expenditure.

The consumer's budget constraint

$$C = N^S w + \pi - T$$

$$\text{as } \pi = Y - wN^d$$

$$G = T$$

$$C = N^S w + \pi - G$$

– In equilibrium, $N^S = N^d$ and the equation is reduced to $Y = C + G$.

- Step 1:** Derive the production possibilities frontier (PPF), which describes the technological possibilities for the entire economy, in terms of the production of C and l .

Production Function : $Y = zF(K, N^d)$

Economy produces 2 goods : Consumption Goods (Y) and Leisure (ℓ)

$$\ell \downarrow \Rightarrow N \uparrow \Rightarrow zF(K, N) \uparrow \rightarrow Y \uparrow.$$

There is a trade-off between leisure and consumption goods. **PPF** (Y, ℓ) slopes downward

$MP_N \downarrow$ as $N \uparrow$. PPF is concave to the origin.

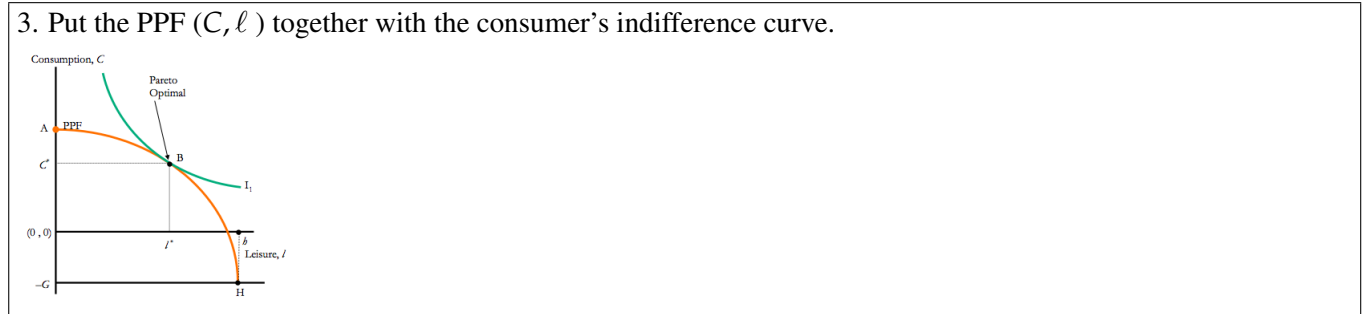
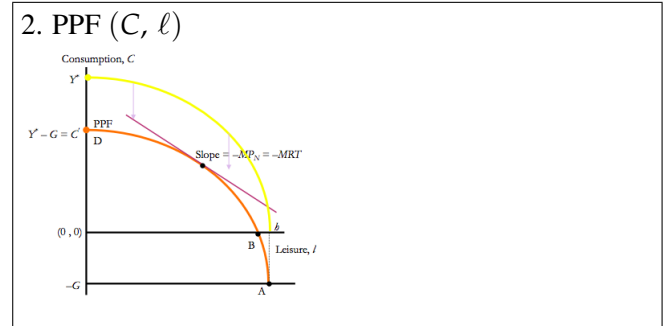
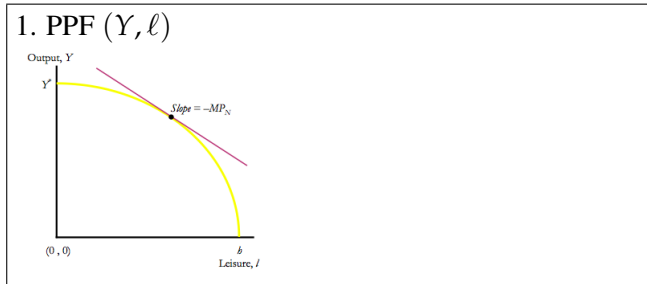
$$Y = C + G; G = \text{a constant, exogenously given}$$

$$C = Y - G. \Rightarrow \text{PPF } (C, \ell)$$

- **Step 2:** Put the PPF (C, ℓ) together with the consumer's indifference curves, so that we can analyze a competitive equilibrium in a single diagram.

Consumer optimization : $U(C, \ell)$, $MRS_{\ell, C} = w$

$$MRS_{\ell, C} = w = MP_N$$

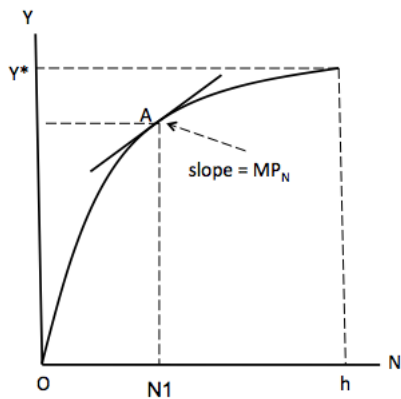


3.1.1 Step 1 : Derive the production possibilities frontier (PPF), which describes the technological possibilities for the entire economy

The production function

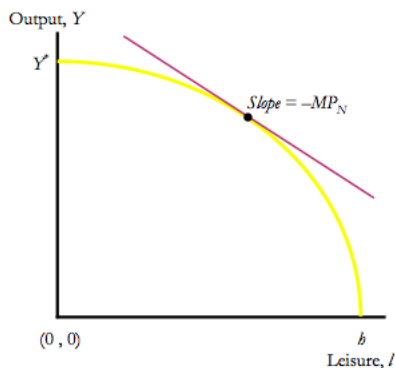
$$Y = zF(K, N)$$

- In equilibrium, $N = h - \ell$, so: $Y = zF(K, h - \ell)$



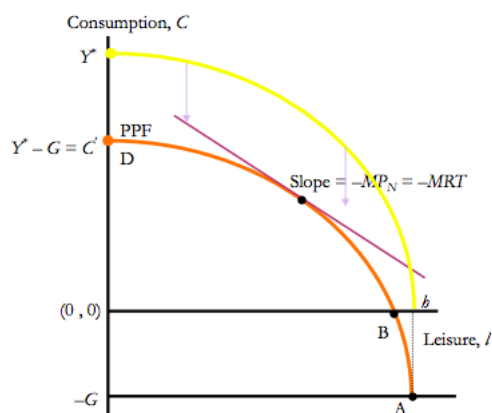
- $Y = zF(K, N)$.
- h = maximum labor supply available.
- $ON1$ = labor input.
- $N1h$ = leisure.

Output as a function of leisure



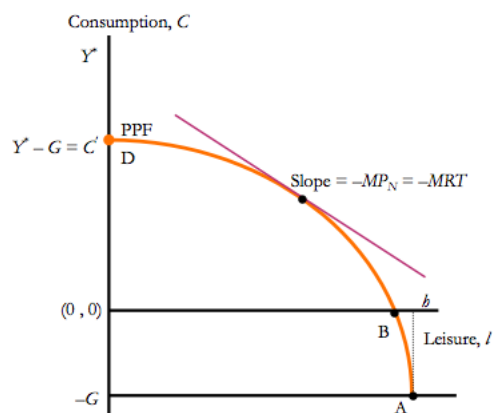
- In equilibrium, we have $N^d = N^s = N = h - \ell$.
- $Y = zF(K, h - \ell)$, which is a relationship between output and leisure.
- Y^* is the level when $l = 0$, and it is the maximum output level.
- $Y^* = zF(K, h)$
- The relation between Y and L is a mirror image of the production function with slope $= -MP_N$.

- $Y = C + G$.
- So, In equilibrium, $C = Y - G = zF(K, h - \ell) - G$.
- The equation shows a relationship between C and ℓ , given the exogenous variables z, K and G .
- Total output is deducted by G to give the net amount available for consumption — the PPF.
- This is the PPF which captures the trade-off between leisure and consumption given the production technology.
- Graphical Illustration, next page.



Production Possibility Frontier

- PPF gives the trade-off between consumption and leisure, given technology.
- DB is feasible ($C \geq 0$); AB is not feasible (C is negative).

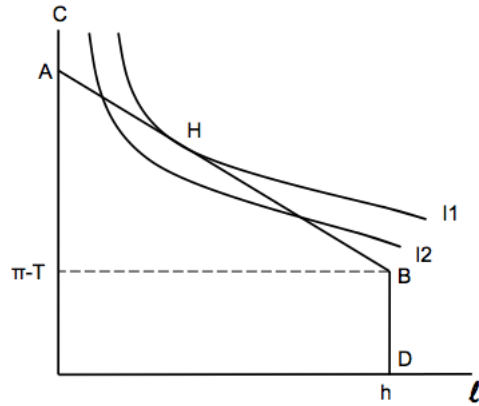


- The slope of PPF is **the marginal rate of transformation (MRT)** of ℓ to C , the rate at which leisure is converted to consumption through work, given technology.

$$MRT_{\ell, C} = MP_N = -\text{slope of PPF}$$

- This helps us determine the equilibrium values C^* and ℓ^* .

3.1.2 Step 2. Put the PPF (C, ℓ) together with the consumer's indifference curves



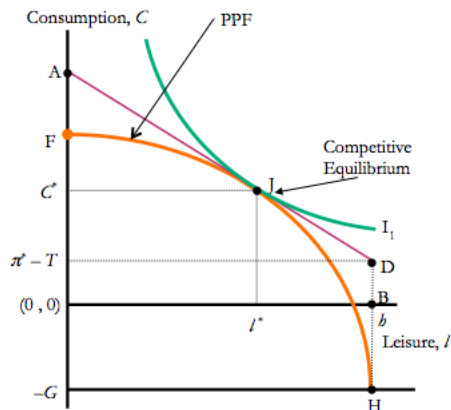
The consumer's max. utility

- The consumer trades off between C and ℓ , given w .

PPF and the Consumer

- The firm chooses the point on PPF which maximizes profits. $MRT_{\ell,C} = MPN = w$.
- The consumer's budget constraint has a slope $MRS_{\ell,C} = -w$.
- That point is on the firm's PPF and on the consumer's budget constraint — tangent point.
- In equilibrium, C^* and ℓ^* are chosen by the representative consumer.

Competitive equilibrium



The consumer's max. utility

- Note that ADB is the budget constraint.
- J is the equilibrium consumption bundle (C^*, ℓ^*) where

$$MRS_{\ell,C} = w$$

- In equilibrium,

$$\begin{aligned} MP_N &= w = MRT_{\ell,C} \\ &= MRS_{\ell,C} \end{aligned}$$

- The firm and the consumer both optimize at J.

Properties of competitive equilibrium

- The values of C , Y , N^d , N^s , w and T at which, given z , K and G :
- The representative consumer chooses C and N^s so that utility is maximized, given w , T and π .
- The representative firm chooses Y and N^d so that profit is maximized, given w , z and K .
- The labor market clears: $N^d = N^s$. The government budget constraint: $G = T$.

The firm's optimization

- The firm maximizes profits at J, given technology: $MP_N = w = MRT_{\ell,C} = \text{slope of the budget line AD}$.

- The firm pays the real wage = w = the real wage received by the consumer.
- The firm demands labor equal to $h - \ell^*$ and produces $Y^* = zF(K, h - \ell)$.
- Max. profit: $\pi^* = zF(K, h - \ell) - w(h - \ell^*) = DH$.
- $DB = \pi^* - G = \pi^* - T$.

The consumer's optimization

- The consumer maximizes utility at J subject to the budget constraint:
- ADB is the budget constraint; the slope = $-w$.
- DB = the consumer's dividend income minus taxes = $\pi^* - T = \pi^* - G$ = the firm's max. profit minus G .
- C^* = consumption goods demanded by the consumer = quantity of consumption goods produced by the firm.
- $h - \ell^*$ = quantity of labor supplied by the consumer = quantity of labor demanded by the firm;
- ℓ^* = leisure desired by the consumer.
- Point J on AD is also tangent to the consumer's highest indifference curve where $MRS_{\ell,C} = W$.

Equilibrium in production and consumption

$$MRS_{\ell,C} = w = MRT_{\ell,C} = MP_N$$

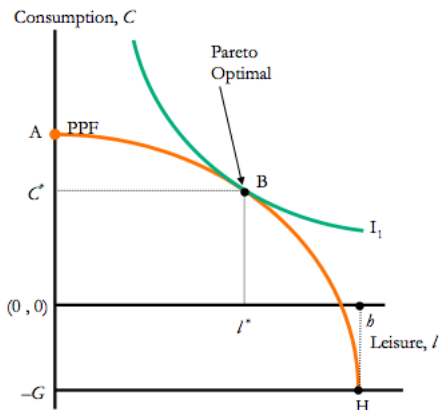
- A competitive equilibrium is achieved when both the consumer and the firm optimize, given z , G and K .
- The real wage (w) is the price signal for both parties to adjust and achieve a simultaneous equilibrium.

3.2 Pareto optimality

- Questions: *Is the competitive equilibrium efficient? Are there any other ways to obtain a better outcome?*
- A competitive equilibrium is **Pareto optimal** if there is no way to rearrange production or to reallocate goods so that someone is made better off and no one is made worse off.
- Since there is only one consumer, we can ignore how consumption goods are allocated among consumers.
- Rather, we focus on how production is arranged.
- An allocation of C , ℓ (and Y) at which an increase in the utility of one agent cannot be made without reducing the utility of another agent.
- The maximum efficiency is achieved as the competitive outcome.

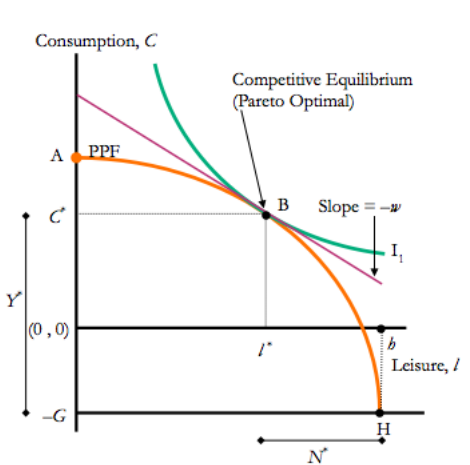
$$MRS_{\ell,C} = w = MRT_{\ell,C} = MP_N$$

3.2.1 Social Planner's Problem



- Consider a social planner who runs the representative firm and chooses the quantities C and ℓ so as to maximize consumer's utility.
- Graphically, the social planner chooses a consumption bundle that is on the PPF and is on the highest possible indifference curve for the consumer.

3.2.2 Comparison between Social Planner's solution and Competitive Equilibrium



- Comparison:
 - Representative consumer faces a linear or kinked budget constraint.
 - Social planner faces a concave PPF.
- The Pareto optimum is at B where the equality holds

$$MRS_{\ell,C} = MRT_{\ell,C} = MP_N$$

- Note that we have the same condition for a competitive equilibrium.

3.2.3 Fundamental theorems in welfare economics

- Assuming convex and monotone preferences and technologies.
 - **First welfare theorem:**
 - * Under certain conditions, a competitive equilibrium is Pareto optimal. Competition results in a socially efficient outcome.
 - * Adam Smith's 'the Wealth of Nations' (1776).

A competitive market economy with self-interested consumers and firms could achieve the allocation of resources and goods which is socially efficient.

Competition is 'the invisible hand' which guides individuals to act in the way which benefit both themselves and society.
 - **Second welfare theorem:**
 - * Under certain conditions, a Pareto optimum is a competitive equilibrium
- Remark: Pareto optimality ignores the distribution issue among individuals and is thus a narrow concept of social optimality.

3.3 Sources of Social Inefficiencies

A competitive equilibrium may not be Pareto optimal due to:

- **Externalities**

- An externality is any activity for which an individual firm or consumer does not take account of all associated costs and benefits.
- All the benefits or costs are not captured by the price of the goods.
- Positive externalities: social benefit > private benefit (e.g., education, innovation, health care).
- Negative externalities: social cost > private cost (e.g., pollution, noise).
- The root cause of an externality is that it is too costly, if not impossible, to set up a market to trade for the benefits and costs associated with the externalities (Market Failure).

- **Distorting taxes**, e.g., proportional income tax (t) on wages:

$$W(1 - t) = MRS_{\ell,C} < MP_N = MRT_{\ell,C}$$

- **Imperfect competition**: firms which are not price-takers.

- Undersupply of the goods: $P > MR = MC$.

- But government intervention to solve market failure may make the inefficiency worse.

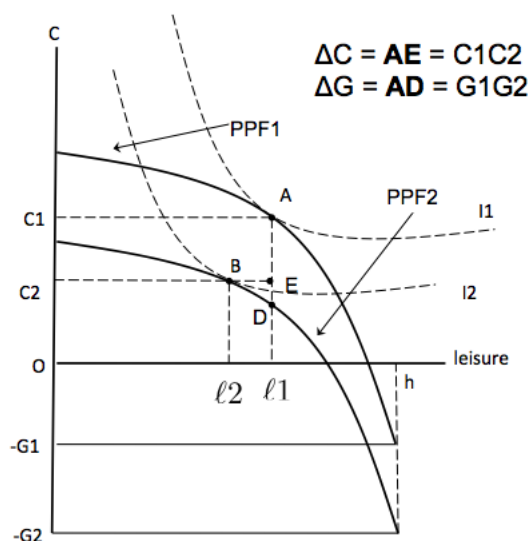
- **The competitive model** is still very powerful.

- A large number of real-world markets are close to perfect competition.
- Benchmark for analysis of inefficiency and possible private solutions.

4 Model Application

4.1 Effects of an increase in G

- Dividend income ($\pi - T$) and disposable income fall;
- C and ℓ decrease (normal goods).
- Employment ($N = h - \ell$) increases.
- Output $Y = zF(K, N)$ rises;
- C increases.
- $\Delta C = \Delta Y - \Delta G$; C does not drop as much as ΔG .
- Private consumption is partially crowded out by the increase in G .



- Consider an $\uparrow$ in G from G_1 to G_2 .
- Since $G = T$, $G \uparrow$ must be followed by $T \uparrow$ of the same amount.
- $G \uparrow$, the PPF shifts from PPF_1 to PPF_2 .
- This shift leaves the slope of the PPF constant for each ℓ .
- The new Pareto optimum is at B.
- Thus, $G \uparrow$ leads to a negative income effect on C and ℓ .

- $G \uparrow \Rightarrow C \downarrow$

- Private consumption is **crowded out** by government purchases.
- The decrease in C is smaller than the increase in G .

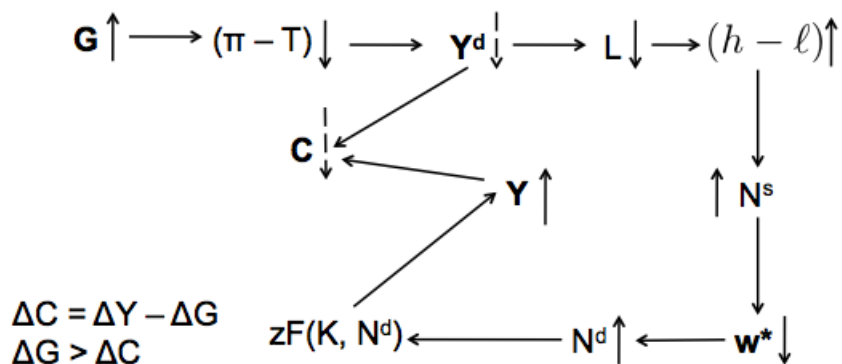
- $\ell \downarrow \Rightarrow N \uparrow \Rightarrow Y \uparrow \Rightarrow w \downarrow$

- What happens to the real wage?

- The slope of PPF_2 at B is less steep than PPF_1 at A.
- So the real wage fall. The consumer supplies more labor ($N=h-\ell$ increases).
- Given K , more labor input causes MP_N to fall.
- The firm optimizes by paying lower $w = MP_N$.
- The lower real wage (w) induces the firm to raise employment (N).

- The consumer works more, receives a lower real wage and consume less.

- Higher G crowds out C .



- Note on business cycle :

- Model prediction

$$G \uparrow \Rightarrow Y^* \dots\dots$$

$N^* \dots\dots$ procyclical
 $w^* \dots\dots$ countercyclical
 $C^* \dots\dots$ countercyclical

- facts

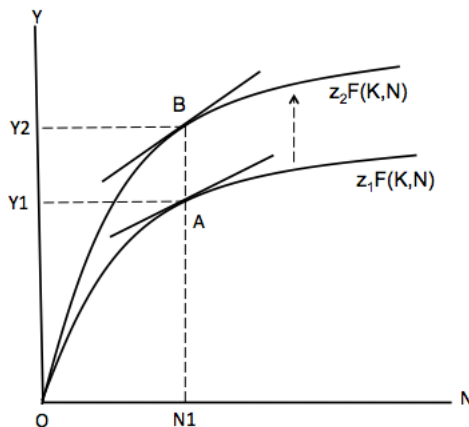
$$Y^* \uparrow$$

$N^* \uparrow$ procyclical
 $w^* \uparrow$ procyclical
 $C^* \uparrow$ procyclical

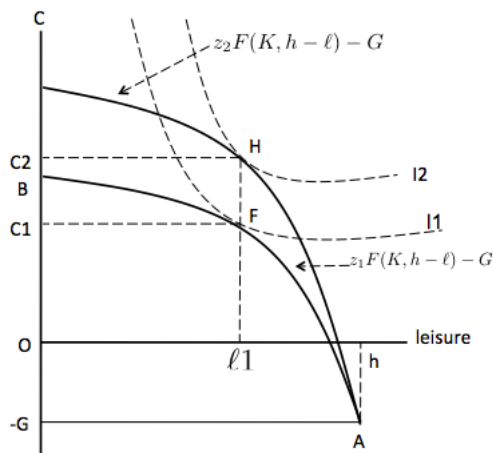
- Therefore, it is unlikely that government spending is the primary cause of business cycle fluctuation.

4.2 Effects of an increase in z (or K)

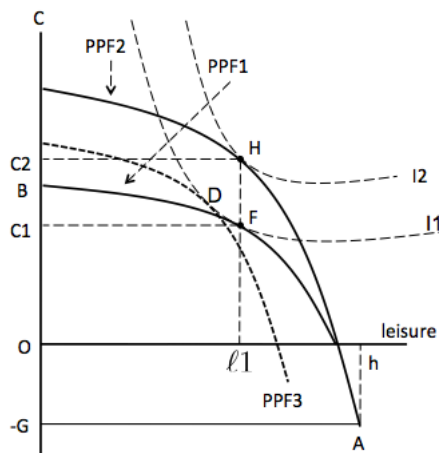
- Increases in z = better technology or organization.
 - The production function and PPF rotate upwards.
 - Higher MP_N , given N with better technology. More demand for labor by the firm. The real wage increases ($MP_N = w$). Employment and leisure ($N = h - \ell$) may rise or fall.
- Output and consumption increase, given G ($Y \uparrow = C \uparrow + G$); higher social welfare.



- The production function rotates upwards with higher MP_N , given N .
- Not only more Y can be produced given N , but the MP_N i.e. the slope of the production function also increases for each N .
- In next page, we see that the new PPF is steeper than the original one.

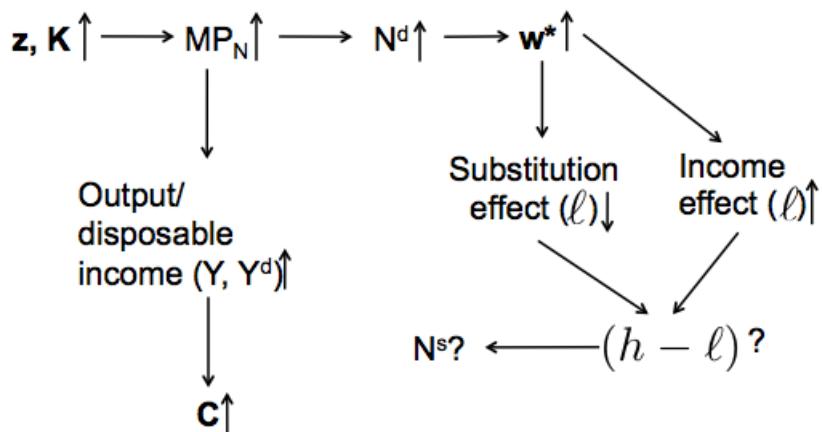


- The PPF rotates upwards.
- C , Y , MP_N and w increase. N and ℓ may rise or fall.



- FD = substitution effect (rising C and N , falling ℓ).
- DH = income effect (rising C and ℓ).
- Equal effects: no change in ℓ and N .

A higher z or K raises w , Y , C



If $SE = IE$, N^s
 If $SE > IE$, N^s
 If $SE < IE$, N^s

“tells a story about the long-term economic effects of long-run improvement in technology, such as that have occurred in the United States since WWII. ... some key observations from post-WWII US data are

that aggregate output has increased steadily, consumption has increased, the real wage has increased, and the hours worked per employed person has remained roughly constant.”

• **Note on business cycle :**

– Model prediction : assuming a stronger substitution effect or equal effect

$z \uparrow \Rightarrow Y^* \dots\dots$

$N^* \dots\dots$ procyclical(SE....IE) ,uncertain (SE....IE)
 $w^* \dots\dots$ procyclical
 $C^* \dots\dots$ procyclical

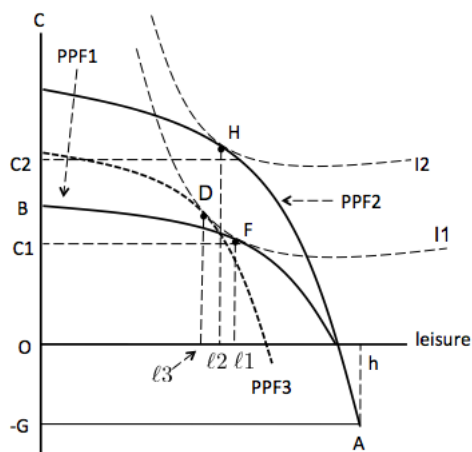
– facts

$Y^* \uparrow$

$N^* \uparrow$ procyclical
 $w^* \uparrow$ procyclical
 $C^* \uparrow$ procyclical

– Therefore, fluctuations in total factor productivity could be the primary cause of business cycle.

Stronger substitution effect



- FD = substitution effect (rising C, falling ℓ).
- DH = income effect (rising C and ℓ).
- Lower ℓ and larger N.

