Assignment 2: Due date: February 17, 2022 before 2.00 pm

Question 1 (30 Points) Score.....................

At this moment, all of your assets are invested in asset A with the following return and risk characteristics:

$$E(r_A) = 10\%$$

$$\sigma_A = 10\%$$

Another asset (call it"B") becomes avaiable; the characteristics of B are as follows;

$$E(r_B) = 20\%$$

$$\sigma_B = 25\%$$

. Furthermore, the correlation of A's and B's return patterns is -1.

Questions:

(1) Using MATLAB to write down the syntax (.m file) for determining the optimal weight (w) of asset A and B in order to achieve the lowest variance, or, in other words, determining the optimal weight for the minimum-variance portfolio.

(2) Find out the Expected return and its variance of the min-variance portfolio using the MATLAB.

(3) By reallocating your portfolio to include some of asset B, how much additional return could you expect to receive if you wanted to maintain your portfolio's risk at $\sigma_p = 10\%$. (Hint: Solve for W_B, not for the W_A).

Note: You must submit both the.m file and your answer in the next page's supplied space.

Given b = weight of asset B

$b^* = \arg\max_b (1-b)(1.1) + b(1.2) \longrightarrow 1.1 + 0.1b$

from $(0.1)^2 = (1-b)^2(0.1)^2 + b^2(0.25)^2 + 2(-1)(1-b)\,b\,(0.1)(0.25) \rightarrow$ Constraint.

$$L(b,\lambda) = (1-b)(1.1) + b(1.2) + \lambda\left[0.01 - (1-b)^2(0.01) - b^2(0.0625) + 2(1-b)b(0.025)\right] = 0 \quad - 1)$$

FOC $\frac{\partial L}{\partial b} = 0 \Rightarrow -1.1 + 1.2 + \lambda^*\left[0.02(1-b^*) - 0.125b^* + 0.05(1-2b^*)\right] = 0 \quad - 2)$

$\frac{\partial L}{\partial \lambda} = 0 \Rightarrow (1-b^*)^2(0.01) + (b^*)^2(0.0625) - 2(1-b^*)b^*(0.1)(0.25) = 0.01 \quad - 3)$

rearange from 3.) $|0.1(1-b^*) - 0.25b^*| = 0.1$

So $b^* = 0$, $b^* = \underline{0.5714}$

× $b > 0$

$R_p^* = (1-b^*)(1.1) + b^*(1.2) = 1.157$

So, The additional return $= R_p^* - 1.1 = 1.157 - 1.1 = 0.057$

$= 5.7\%$