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EE 325 Section 1 (Aj.Wanwiphang) Homework Assignment 1

Due date: 31 January 2020 before 11pm

**** Please submit this assignment on Moodle. For those who work on paper, please scan or submit the pictures of your work. ****

1. Find the answers following questions (please also show your calculation)

a. $\sum_{i=1}^5 (a + bx_i) = 5a + b \sum_{i=1}^5 x_i = 5a + b(x_1 + \dots + x_5)$

b. $\sum_{y=0}^5 f(x+y) = f(x) + f(x+1) + f(x+2) + \dots + f(x+5)$

c. $\sum_{i=1}^{10} i^2 = 1^2 + 2^2 + \dots + 10^2 = \frac{n(n+1)(2n+1)}{6} = \frac{10(11)(21)}{6} = 385$

d. $\sum_{x=1}^2 \sum_{y=2}^3 (2x+y) = (2(1)+2) + (2(1)+3) + (2(2)+2) + (2(2)+3) = 23$

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b	b	2.25b	2b	1.5b	0.5b	0.25b

** when b is constant number

a. Find the value of b

$\sum f(x) = 1 = 0.5b + b + \dots + 0.25b = 8b$ so $b = 1/8$

b. Find the answer for $P(X \leq 2)$

$P(X \leq 2) = 1 - P(X=3) - P(X=4) = 1 - \frac{0.5}{8} - \frac{0.25}{8} = \frac{8 - 0.75}{8} = \frac{7.25}{8} = 0.90625$

c. Find the answer for $P(-2 \leq X \leq 3)$

$P(-2 \leq X \leq 3) = 1 - P(X=4) = 1 - \frac{0.25}{8} = \frac{7.75}{8} = 0.96875$

d. Find the answer for $P(X \geq 1)$

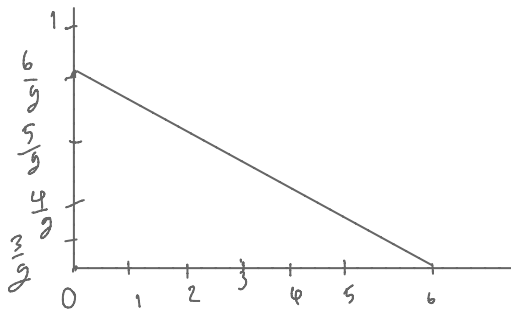
$P(X \geq 1) = 1 - P(X=-2) - P(X=-1) - P(X=0)$
 $= 1 - 3.75b = \frac{8 - 3.75}{8} = 0.53125$ 1

3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \leq x \leq 3$$

= 0 otherwise

- a. Plot graph for $f(x)$



- b. Find the answer for $P(1 \leq X \leq 3)$

$$\begin{aligned} \int_1^3 f(x) dx &= \left. -\frac{1}{9} \frac{x^2}{2} + \frac{6}{9}x \right|_1^3 \\ &= \left(-\frac{1}{2} + 2 \right) - \left(-\frac{1}{18} + \frac{12}{18} \right) = \frac{3}{4} - \frac{11}{18} = \frac{27 - 22}{36} \\ &= \frac{5}{36} \end{aligned}$$

- c. Find the answer for $P(X \geq 2)$

$$\begin{aligned} \int_2^{\infty} f(x) dx &= \int_2^3 f(x) dx + \int_3^{\infty} f(x) dx = \int_2^3 f(x) dx + \int_3^{\infty} 0 dx \\ &= \left. -\frac{1}{9} \frac{x^2}{2} + \frac{6}{9}x \right|_2^3 = -\frac{1}{2} + 2 - \left(-\frac{2}{9} + \frac{12}{9} \right) = \frac{3}{2} - \frac{10}{9} = \frac{27 - 20}{18} = \frac{7}{18} \end{aligned}$$

- d. Find the expected value of X

$$\begin{aligned} \int_{-\infty}^{\infty} x f(x) dx &= \int_0^3 x \left(-\frac{1}{9}x + \frac{6}{9} \right) dx = \left. -\frac{1}{9} \frac{x^3}{3} + \frac{6x^2}{18} \right|_0^3 = \left| -1 + 3 - 0 \right| = 2 \\ &\downarrow \\ \int_{-\infty}^c x(0) dx &= 0, \quad \int_c^{\infty} x(0) dx = 0 \end{aligned}$$

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4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0.

a. Construct the joint probability distribution function (PDF) table of X and Y

$Y \backslash X$	1	2	3	4	5	6	
1	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.5
0	0.0833	0.0833	0.0833	0.0833	0.0833	0.0833	0.5
	0.1666	0.1666	0.1666	0.1666	0.1666	0.1666	1

b. Find the marginal probability distribution function (PDF) of X

$$P(X=1, 2, 3, 4, 5, 6) = 0.1666$$

c. Find the marginal probability distribution function (PDF) of Y

$$P(Y=1) = 0.5, P(Y=0) = 0.5$$

d. Find the conditional probability distribution function (PDF) of X given Y is equal to 1

$$P(X=x_i | Y=1) = \frac{P(X=x_i, Y=1)}{P(Y=1)}$$

e. Find the expected value of X given Y is equal to 1

$$E(X=x_i | Y=1) = \sum_{i=1}^n x_i P(X=x_i | Y=1) = \sum_{i=1}^n x_i \frac{P(X=x_i, Y=1)}{P(Y=1)} = \frac{1(0.0833) + 2(0.0833) + \dots + 6(0.0833)}{0.5}$$

f. Find the variance of X given Y is equal to 1

$$= \frac{0.0833(21)}{0.5} = 3.48$$

$$\text{Var}(X) = E(X^2) - \mu^2$$

$$= \frac{\sum (x_i)^2}{n} - \mu^2 = (0.0833) \frac{(1^2 + \dots + 6^2)}{6} - (0.3481)^2 = 21(0.0833) - 0.121212$$

$$\approx 7.46$$

5. If X_1, X_2, X_3 is a random sample from a population with mean μ and variance σ^2 . X_1, X_2, X_3 are not independent

$$E(xy) = \sum_{i=1}^n x_i y_i \cdot \overbrace{P(x_i, y_i)}^{\text{joint prob.}}$$

$$\text{Cov}(X_1, X_2) = \text{Cov}(X_1, X_3) = \text{Cov}(X_2, X_3) = \frac{1}{4}\sigma^2$$

\bar{X} is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

Find $E(\bar{X})$ and $\text{var}(\bar{X})$

$$\text{Cov}(X_1, X_2) = E(X_1 X_2) - \mu_{X_1} \mu_{X_2} = E(X_1 X_2) - \mu^2$$

$$\sigma^2 = E(X_i^2) - \mu^2$$

$$\text{so } \frac{\sigma^2}{4} = \text{Cov}(X_1, X_2)$$

$$\frac{E(X_1 X_2) - \mu^2}{4} = E(X_1 X_2) - \mu^2$$

$$E(X_1 X_2) - \mu^2 = 4E(X_1 X_2) - 4\mu^2$$

$$3\mu^2 = 4E(X_1 X_2) - E(X_1^2)$$

$$\mu^2 = \frac{4E(X_1 X_2) - E(X_1^2)}{3}$$

$$\mu = \sqrt{\frac{4E(X_1 X_2) - E(X_1^2)}{3}}$$

$$E(\bar{X}) = \frac{1}{3} E(X_1 + X_2 + X_3) = \mu = \sqrt{\frac{4E(X_1 X_2) - E(X_1^2)}{3}}$$

$$\text{Var}(\bar{X}) = \frac{1}{3^2} \text{Var}(X_1 + X_2 + X_3)$$

$$= \frac{\sigma^2}{3} = \frac{E(X_i^2) - \mu^2}{3}$$

$$\text{substitute } \mu = \sqrt{\frac{4E(X_1 X_2) - E(X_1^2)}{3}}$$

$$\text{so } \text{Var}(\bar{X}) = \frac{E(X_1^2) - 4E(X_1 X_2) + E(X_1^2)}{3}$$

$$= \frac{4E(X_1^2) - 4E(X_1 X_2)}{3}$$

6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean μ and variance σ^2 . \bar{X} is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

a. Find $E(\bar{X})$ and $\text{var}(\bar{X})$ in term of μ and σ

$$E(X_1) = \mu$$

$$E(\bar{X}) = E\left(\frac{1}{4}(X_1 + \dots + X_4)\right) = \frac{E(X_1 + \dots + X_4)}{4} = \frac{4\mu}{4} = \mu$$

$$\text{Var}(\bar{X}) = \text{Var}\left(\frac{1}{4}(X_1 + \dots + X_4)\right) = \frac{1}{4^2} [\text{Var}(X_1) + \dots + \text{Var}(X_4)] = \frac{4\sigma^2}{16} = \frac{\sigma^2}{4}$$

- b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ . Show that \tilde{X} is an unbiased estimator of μ

$$\begin{aligned} E(\tilde{X}) &= E\left(\frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4\right) \\ &= \frac{1}{8}E(X_1) + \frac{1}{4}E(X_2) + \frac{1}{8}E(X_3) + \frac{1}{2}E(X_4) \\ &= \frac{\mu}{8} + \frac{\mu}{4} + \frac{\mu}{8} + \frac{\mu}{2} = \frac{\mu + 2\mu + \mu + 4\mu}{8} = \frac{8\mu}{8} = \mu \end{aligned}$$

So $E(\tilde{X}) = \mu \Rightarrow$ it's unbiased estimator of μ .

- c. Between \bar{X} and \tilde{X} , which one is the better estimator for μ ? Why?

Lower variance is said to more efficient than another estimators with higher variance.

$$\begin{aligned} \text{Var}(\bar{X}) &= \frac{\sigma^2}{4} & \text{Var}(\tilde{X}) &= \text{Var}\left(\frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4\right) \\ & & &= \text{Var}\left(\frac{X_1}{8}\right) + \text{Var}\left(\frac{X_2}{4}\right) + \text{Var}\left(\frac{X_3}{8}\right) + \text{Var}\left(\frac{X_4}{2}\right) \\ & & &= \frac{1}{64}\text{Var}(X_1) + \frac{1}{16}\text{Var}(X_2) + \frac{1}{64}\text{Var}(X_3) + \frac{1}{4}\text{Var}(X_4) \\ & & &= \frac{\sigma^2}{64} + \frac{4\sigma^2}{64} + \frac{\sigma^2}{64} + \frac{16\sigma^2}{64} \\ & & &= \frac{22\sigma^2}{64} = \frac{11\sigma^2}{32} = \frac{\sigma^2}{2.9} \end{aligned}$$

finally $\text{Var}(\bar{X}) < \text{Var}(\tilde{X})$
Therefore, \bar{X} is the better estimator for μ .