From $\ln S_t = \beta_{10} + \beta_{11}\ln P_{Dt} + \beta_{12}\ln P_{x2t} + \beta_{13}\ln P_{x3t} + \beta_{14}\ln P_{x4t} + \varepsilon_{1t}$ — ①

$\ln D_t = \beta_{20} + \beta_{21}\ln P_{Dt} + \beta_{22}\ln GDP_t + \varepsilon_{2t}$ — ②

To make the Reduce Form Model, We need 3 equation for 3 unknown endogenous Variable.

Thus we assume that the Market always clears.

$\ln D_t = \ln S_t$ — ③

From ③ : $\beta_{10} + \beta_{11}\ln P_{Dt} + \beta_{12}\ln P_{x2t} + \beta_{13}\ln P_{x3t} + \beta_{14}\ln P_{x4t} + \varepsilon_{1t} = \beta_{20} + \beta_{21}\ln P_{Dt} + \beta_{22}\ln GDP_t + \varepsilon_{2t}$.

$\beta_{11}\ln P_{Dt} - \beta_{21}\ln P_{Dt} = \beta_{20} - \beta_{10} + \beta_{22}\ln GDP_t - \beta_{12}\ln P_{x2t} - \beta_{13}\ln P_{x3t} - \beta_{14}\ln P_{x4t} + \varepsilon_{2t} - \varepsilon_{1t}$

$(\beta_{11} - \beta_{21})\ln P_{Dt} = \beta_{20} - \beta_{10} + \beta_{22}\ln GDP_t - \beta_{12}\ln P_{x2t} - \beta_{13}\ln P_{x3t} - \beta_{14}\ln P_{x4t} + \varepsilon_{2t} - \varepsilon_{1t}$.

$\ln P_{Dt} = \frac{\beta_{20}-\beta_{10}}{\beta_{11}-\beta_{21}} + \frac{\beta_{22}}{\beta_{11}-\beta_{21}}\ln GDP_t - \frac{\beta_{12}}{\beta_{11}-\beta_{21}}\ln P_{x2t} - \frac{\beta_{13}}{\beta_{11}-\beta_{21}}\ln P_{x3t} - \frac{\beta_{14}}{\beta_{11}-\beta_{21}}\ln P_{x4t} + \frac{\varepsilon_{2t}-\varepsilon_{1t}}{\beta_{11}-\beta_{21}}$ — ④

④ in ② $\ln D_t = \beta_{20} + \frac{\beta_{21}(\beta_{20}-\beta_{10})}{\beta_{11}-\beta_{21}} + \frac{\beta_{22}+\beta_{21}\beta_{22}}{\beta_{11}-\beta_{21}}\ln GDP_t - \frac{\beta_{21}\beta_{12}}{\beta_{11}-\beta_{21}}\ln P_{x2t} - \frac{\beta_{13}\beta_{21}}{\beta_{11}-\beta_{21}}\ln P_{x3t} - \frac{\beta_{14}\beta_{21}}{\beta_{11}-\beta_{21}}\ln P_{x4t} + \frac{\beta_{21}(\varepsilon_{2t}-\varepsilon_{1t})}{\beta_{11}-\beta_{21}} + \varepsilon_{2t}$ — ⑤

A. let $\pi_{20} = \beta_{20} + \frac{\beta_{21}(\beta_{20}-\beta_{10})}{\beta_{11}-\beta_{21}}$ $\pi_{21} = \frac{\beta_{22}+\beta_{21}\beta_{22}}{\beta_{11}-\beta_{21}}$ $\pi_{22} = -\frac{\beta_{21}\beta_{12}}{\beta_{11}-\beta_{21}}$ $\pi_{23} = -\frac{\beta_{13}\beta_{21}}{\beta_{11}-\beta_{21}}$ $\pi_{24} = -\frac{\beta_{14}\beta_{21}}{\beta_{11}-\beta_{21}}$ $V_{2t} = \frac{\beta_{21}(\varepsilon_{2t}-\varepsilon_{1t})}{\beta_{11}-\beta_{21}} + \varepsilon_{2t}$

B. let $\pi_{30} = \frac{\beta_{20}-\beta_{10}}{\beta_{11}-\beta_{21}}$ $\pi_{31} = \frac{\beta_{22}}{\beta_{11}-\beta_{21}}$ $\pi_{32} = -\frac{\beta_{12}}{\beta_{11}-\beta_{21}}$ $\pi_{33} = -\frac{\beta_{13}}{\beta_{11}-\beta_{21}}$ $\pi_{34} = \frac{\beta_{14}}{\beta_{11}-\beta_{21}}$ $W_{3t} = \frac{\varepsilon_{2t}-\varepsilon_{1t}}{\beta_{11}-\beta_{21}}$

④ in ① $\ln S_t = \beta_{10} + \frac{\beta_{11}(\beta_{20}-\beta_{10})}{\beta_{11}-\beta_{21}} + \frac{\beta_{11}\beta_{22}}{\beta_{11}-\beta_{21}}\ln GDP_t + \left(\beta_{12} - \frac{\beta_{11}\beta_{12}}{\beta_{11}-\beta_{21}}\right)\ln P_{x2t} + \left(\beta_{13} - \frac{\beta_{11}\beta_{13}}{\beta_{11}-\beta_{21}}\right)\ln P_{x3t} + \left(\beta_{14} - \frac{\beta_{11}\beta_{14}}{\beta_{11}-\beta_{21}}\right)\ln P_{x4t}$

$+ \varepsilon_{1t} + \frac{\beta_{11}(\varepsilon_{2t}-\varepsilon_{1t})}{\beta_{11}-\beta_{21}}$ — ⑥

C. let $\pi_{10} = \beta_{10} + \frac{\beta_{11}(\beta_{20}-\beta_{10})}{\beta_{11}-\beta_{21}}$ $\pi_{11} = \frac{\beta_{11}\beta_{22}}{\beta_{11}-\beta_{21}}$ $\pi_{12} = \beta_{12} - \frac{\beta_{11}\beta_{12}}{\beta_{11}-\beta_{21}}$ $\pi_{13} = \beta_{13} - \frac{\beta_{11}\beta_{13}}{\beta_{11}-\beta_{21}}$ $\pi_{14} = \beta_{14} - \frac{\beta_{11}\beta_{14}}{\beta_{11}-\beta_{21}}$ $W_{1t} = \frac{\varepsilon_{1t} + \beta_{11}(\varepsilon_{2t}-\varepsilon_{1t})}{\beta_{11}-\beta_{21}}$

$$\ln D_t = \Pi_{20} + \Pi_{21} \ln GDP_t + \Pi_{22} \ln P_{x2t} + \Pi_{23} \ln P_{x3t} + \Pi_{24} \ln P_{x4t} + W_{2t} \quad (7)$$

$$\ln S_t = \Pi_{10} + \Pi_{11} \ln GDP_t + \Pi_{12} \ln P_{x2t} + \Pi_{13} \ln P_{x3t} + \Pi_{14} \ln P_{x4t} + W_{1t} \quad (8)$$

$$\ln P_{Dt} = \Pi_{30} + \Pi_{31} \ln GDP_t + \Pi_{32} \ln P_{x2t} + \Pi_{33} \ln P_{x3t} + \Pi_{34} \ln P_{x4t} + W_{3t} \quad (9)$$

(7), (8) and (9) are reduced form of these system model

2. Estimate reduce form model using OLS and prediction of the endogenous variables.

```
. g lndt=ln(dt)

. g lnst=ln(st)

. g lnpdt =ln(pdt)
pdt not found
r(111);

. g pdt = pm+t

. g lnpdt=ln(pdt)

. g lngdp=ln(gdp)

. g lnpx2=ln(px2)

. g lnpx3=ln(px3)

. g lnpx4=ln(px4)

. reg lndt lngdp lnpx2 lnpx3 lnpx4

      Source |       SS           df       MS      Number of obs   =        22
-------------+----------------------------------   F(4, 17)        =     26.43
       Model |   3.4026552         4  .850663799   Prob > F        =    0.0000
    Residual |   .54721789        17  .032189288   R-squared       =    0.8615
-------------+----------------------------------   Adj R-squared   =    0.8289
       Total |  3.94987309        21  .188089195   Root MSE        =    .17941

------------------------------------------------------------------------------
        lndt |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       lngdp |   .1265855    .194594     0.65   0.524    -.2839719    .5371429
       lnpx2 |  -.4887365   .1541691    -3.17   0.006    -.8140049   -.1634682
       lnpx3 |  -.7243134   .2830597    -2.56   0.020    -1.321517   -.1271097
       lnpx4 |   -.577921   .4293997    -1.35   0.196    -1.483875    .3280333
       _cons |   27.18614   5.399879     5.03   0.000     15.79339    38.57889
------------------------------------------------------------------------------

. predict lndthat, xb
```

```
. reg lnst lngdp lnpx2 lnpx3 lnpx4

      Source |       SS           df       MS      Number of obs   =        22
-------------+----------------------------------   F(4, 17)        =     37.32
       Model |  4.64569724         4  1.16142431   Prob > F        =    0.0000
    Residual |  .529104674        17  .031123804   R-squared       =    0.8978
-------------+----------------------------------   Adj R-squared   =    0.8737
       Total |  5.17480192        21  .246419139   Root MSE        =    .17642

------------------------------------------------------------------------------
        lnst |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       lngdp |   .3438812   .1913463     1.80   0.090    -.0598242    .7475865
       lnpx2 |  -.4503744   .1515961    -2.97   0.009    -.7702142   -.1305347
       lnpx3 |  -.9242052   .2783356    -3.32   0.004    -1.511442   -.3369685
       lnpx4 |  -.3883793   .4222332    -0.92   0.371    -1.279214    .5024549
       _cons |   24.65741   5.309757     4.64   0.000      13.4548    35.86002
------------------------------------------------------------------------------

. predict lnsthat, xb

. reg lnpdt lngdp lnpx2 lnpx3 lnpx4

      Source |       SS           df       MS      Number of obs   =        22
-------------+----------------------------------   F(4, 17)        =      6.76
       Model |   .17707359         4  .044268398   Prob > F        =    0.0019
    Residual |  .111247189        17  .006543952   R-squared       =    0.6142
-------------+----------------------------------   Adj R-squared   =    0.5234
       Total |  .288320779        21  .013729561   Root MSE        =    .08089

------------------------------------------------------------------------------
       lnpdt |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       lngdp |   .1632779   .0877392     1.86   0.080    -.0218357    .3483914
       lnpx2 |   .1318015   .0695123     1.90   0.075    -.0148567    .2784596
       lnpx3 |   .0939842    .127627     0.74   0.472    -.1752851    .3632535
       lnpx4 |   .4939641   .1936093     2.55   0.021     .0854842    .9024439
       _cons |    2.87652   2.434717     1.18   0.254    -2.260283    8.013322
------------------------------------------------------------------------------

.
. predict lnpdthat, xb
```

3. Estimate structural form using predicted endogenous variables as independent variables in the structural form model

```
. reg lndt lnpdthat lngdp

      Source |       SS           df       MS      Number of obs   =        22
-------------+----------------------------------   F(2, 19)        =     44.99
       Model |  3.26129847         2  1.63064924   Prob > F        =    0.0000
    Residual |  .688574614        19  .036240769   R-squared       =    0.8257
-------------+----------------------------------   Adj R-squared   =    0.8073
       Total |  3.94987309        21  .188089195   Root MSE        =    .19037

------------------------------------------------------------------------------
        lndt |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    lnpdthat |  -2.574157   .5697943    -4.52   0.000     -3.76675   -1.381563
       lngdp |   .5212927   .1344816     3.88   0.001     .2398194     .802766
       _cons |   35.93498   7.189835     5.00   0.000     20.88648    50.98347
------------------------------------------------------------------------------
```

```
. reg lnst lnpdthat lnpx2 lnpx3 lnpx4

      Source |       SS           df       MS      Number of obs   =        22
-------------+----------------------------------   F(4, 17)        =     37.32
       Model |  4.64569773         4  1.16142443   Prob > F        =    0.0000
    Residual |  .529104183        17  .031123775   R-squared       =    0.8978
-------------+----------------------------------   Adj R-squared   =    0.8737
       Total |  5.17480192        21  .246419139   Root MSE        =    .17642

------------------------------------------------------------------------------
        lnst |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    lnpdthat |   2.106112   1.171903     1.80   0.090    -.3663879    4.578612
       lnpx2 |   -.727963   .1840856    -3.95   0.001     -1.11635   -.3395762
       lnpx3 |  -1.122146   .2824139    -3.97   0.001    -1.717988   -.5263052
       lnpx4 |  -1.428722   .4751381    -3.01   0.008    -2.431176   -.4262679
       _cons |   18.59912   8.546622     2.18   0.044     .5673274    36.63092
------------------------------------------------------------------------------
```

4.Estimate the structural models of these system equations using OLS, 2SLS, 3SLS, and I3SLS. Concerning of the asymptotic property, which model is the most appropriated model? Why?

```
. reg3 (lnst lnpdt lnpx2 lnpx3 lnpx4) (lndt lnpdt lngdp), ols
```

Multivariate regression

| Equation | Obs | Parms | RMSE | "R-sq" | F-Stat | P |
|---|---|---|---|---|---|---|
| lnst | 22 | 4 | .1652258 | 0.9103 | 43.14 | 0.0000 |
| lndt | 22 | 2 | .1391259 | 0.9069 | 92.53 | 0.0000 |

| | | Coef. | Std. Err. | t | P>\|t\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|---|
| lnst | | | | | | | |
| | lnpdt | -1.111835 | .4515147 | -2.46 | 0.019 | -2.027549 | -.1961207 |
| | lnpx2 | -.4189546 | .1431634 | -2.93 | 0.006 | -.7093034 | -.1286059 |
| | lnpx3 | -.9424196 | .2585266 | -3.65 | 0.001 | -1.466736 | -.4181034 |
| | lnpx4 | -.521346 | .3441643 | -1.51 | 0.139 | -1.219344 | .1766516 |
| | _cons | 41.4946 | 3.661911 | 11.33 | 0.000 | 34.0679 | 48.9213 |
| lndt | | | | | | | |
| | lnpdt | -2.181329 | .2946999 | -7.40 | 0.000 | -2.779008 | -1.58365 |
| | lngdp | .5776586 | .0887536 | 6.51 | 0.000 | .397658 | .7576593 |
| | _cons | 31.03578 | 3.761201 | 8.25 | 0.000 | 23.40771 | 38.66385 |

```
. reg3 (lnst lnpdt lnpx2 lnpx3 lnpx4) (lndt lnpdt lngdp), 2sls nodfk inst(lnpx2 lnpx3 lnpx4 lngdp)
```

Two-stage least-squares regression

| Equation | Obs | Parms | RMSE | "R-sq" | F-Stat | P |
|---|---|---|---|---|---|---|
| lnst | 22 | 4 | .329951 | 0.6424 | 13.81 | 0.0000 |
| lndt | 22 | 2 | .1454858 | 0.8982 | 89.20 | 0.0000 |

| | | Coef. | Std. Err. | t | P>\|t\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|---|
| lnst | | | | | | | |
| | lnpdt | 2.10611 | 1.926677 | 1.09 | 0.282 | -1.801371 | 6.013591 |
| | lnpx2 | -.7279628 | .3026471 | -2.41 | 0.021 | -1.34176 | -.114166 |
| | lnpx3 | -1.122146 | .464304 | -2.42 | 0.021 | -2.063798 | -.180494 |
| | lnpx4 | -1.428722 | .7811544 | -1.83 | 0.076 | -3.012977 | .1555325 |
| | _cons | 18.59914 | 14.05113 | 1.32 | 0.194 | -9.897873 | 47.09616 |
| lndt | | | | | | | |
| | lnpdt | -2.574157 | .4046743 | -6.36 | 0.000 | -3.394875 | -1.75344 |
| | lngdp | .5212921 | .0955104 | 5.46 | 0.000 | .327588 | .7149961 |
| | _cons | 35.93499 | 5.106302 | 7.04 | 0.000 | 25.57893 | 46.29105 |

Endogenous variables: lnst lnpdt lndt
Exogenous variables: lnpx2 lnpx3 lnpx4 lngdp

```
. reg3 (lnst lnpdt lnpx2 lnpx3 lnpx4) (lndt lnpdt lngdp), 3sls inst(lnpx2 lnpx3 lnpx4 lngdp)
```

Three-stage least-squares regression

| Equation | Obs | Parms | RMSE | "R-sq" | chi2 | P |
|---|---|---|---|---|---|---|
| lnst | 22 | 4 | .2963642 | 0.6266 | 57.47 | 0.0000 |
| lndt | 22 | 2 | .135203 | 0.8982 | 178.41 | 0.0000 |

| | Coef. | Std. Err. | z | P>\|z\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| lnst | | | | | | |
| lnpdt | 2.171576 | 1.926095 | 1.13 | 0.260 | -1.603501 | 5.946652 |
| lnpx2 | -.7990055 | .2985983 | -2.68 | 0.007 | -1.384247 | -.2137635 |
| lnpx3 | -1.329743 | .4560002 | -2.92 | 0.004 | -2.223487 | -.4359989 |
| lnpx4 | -1.171403 | .775654 | -1.51 | 0.131 | -2.691657 | .348851 |
| _cons | 17.84948 | 14.04122 | 1.27 | 0.204 | -9.670808 | 45.36976 |
| lndt | | | | | | |
| lnpdt | -2.574157 | .4046743 | -6.36 | 0.000 | -3.367304 | -1.78101 |
| lngdp | .5212921 | .0955104 | 5.46 | 0.000 | .3340951 | .708489 |
| _cons | 35.93499 | 5.106302 | 7.04 | 0.000 | 25.92682 | 45.94316 |

Endogenous variables: lnst lnpdt lndt
Exogenous variables: lnpx2 lnpx3 lnpx4 lngdp

```
. reg3 (lnst lnpdt lnpx2 lnpx3 lnpx4) (lndt lnpdt lngdp), 3sls ireg3 inst(lnpx2 lnpx3 lnpx4 lngdp)

Iteration 1:   tolerance =  .1059484
Iteration 2:   tolerance = .04569793
Iteration 3:   tolerance = .01846611
Iteration 4:   tolerance = .00725496
Iteration 5:   tolerance = .00281814
Iteration 6:   tolerance = .00108981
Iteration 7:   tolerance = .00042072
Iteration 8:   tolerance = .00016231
Iteration 9:   tolerance =  .0000626
Iteration 10:   tolerance =  .00002414
Iteration 11:   tolerance =  9.310e-06
Iteration 12:   tolerance =  3.590e-06
Iteration 13:   tolerance =  1.384e-06
Iteration 14:   tolerance =  5.339e-07

Three-stage least-squares regression, iterated
```

| Equation | Obs | Parms | RMSE | "R-sq" | chi2 | P |
|---|---|---|---|---|---|---|
| lnst | 22 | 4 | .3022006 | 0.6117 | 54.83 | 0.0000 |
| lndt | 22 | 2 | .135203 | 0.8982 | 178.41 | 0.0000 |

| | Coef. | Std. Err. | z | P>\|z\| | [95% Conf. | Interval] |
|---|---|---|---|---|---|---|
| lnst | | | | | | |
| lnpdt | 2.212666 | 2.005956 | 1.10 | 0.270 | -1.718936 | 6.144268 |
| lnpx2 | -.8435967 | .3049354 | -2.77 | 0.006 | -1.441259 | -.2459342 |
| lnpx3 | -1.460044 | .4623671 | -3.16 | 0.002 | -2.366267 | -.5538216 |
| lnpx4 | -1.009892 | .7998393 | -1.26 | 0.207 | -2.577548 | .557764 |
| _cons | 17.37893 | 14.61488 | 1.19 | 0.234 | -11.26571 | 46.02357 |
| lndt | | | | | | |
| lnpdt | -2.574157 | .4046743 | -6.36 | 0.000 | -3.367304 | -1.78101 |
| lngdp | .5212921 | .0955104 | 5.46 | 0.000 | .3340951 | .708489 |
| _cons | 35.93499 | 5.106302 | 7.04 | 0.000 | 25.92682 | 45.94316 |

```
Endogenous variables:  lnst lnpdt lndt
Exogenous variables:   lnpx2 lnpx3 lnpx4 lngdp
```

Concerning on the asymptotic property,2SLS is the most appropriated. 2SLS model ensures consistency since we estimated reduced form model using OLS to predict endogenous variable and use predict endogenous variable as instrumental variables (IV) for lagged endogenous variable instead of actual endogeous variable to predict betas. In 2SLS, We assume that endogenity bias is solved therefore, The consistency will be the most strong. In OLS, there exists endogenity problem so, coefficient of independent vatiables won't be consistent bias and ineffiecient. In 3SLS and I3SLS, there are a risk of specification effor in one equation or any equations which might speard through all the equations in system and lead to inconsistency.

5. What do β21 and β22 mean?

β21 means the price elasticity of demand since it indicate how many percentage change in quantity of domestic demand will change from 1% change in price.

β22 means income elasticity of demand since it indicate how many percentage change in quantity of domestic demand will change from 1% change in GDP which represent income.