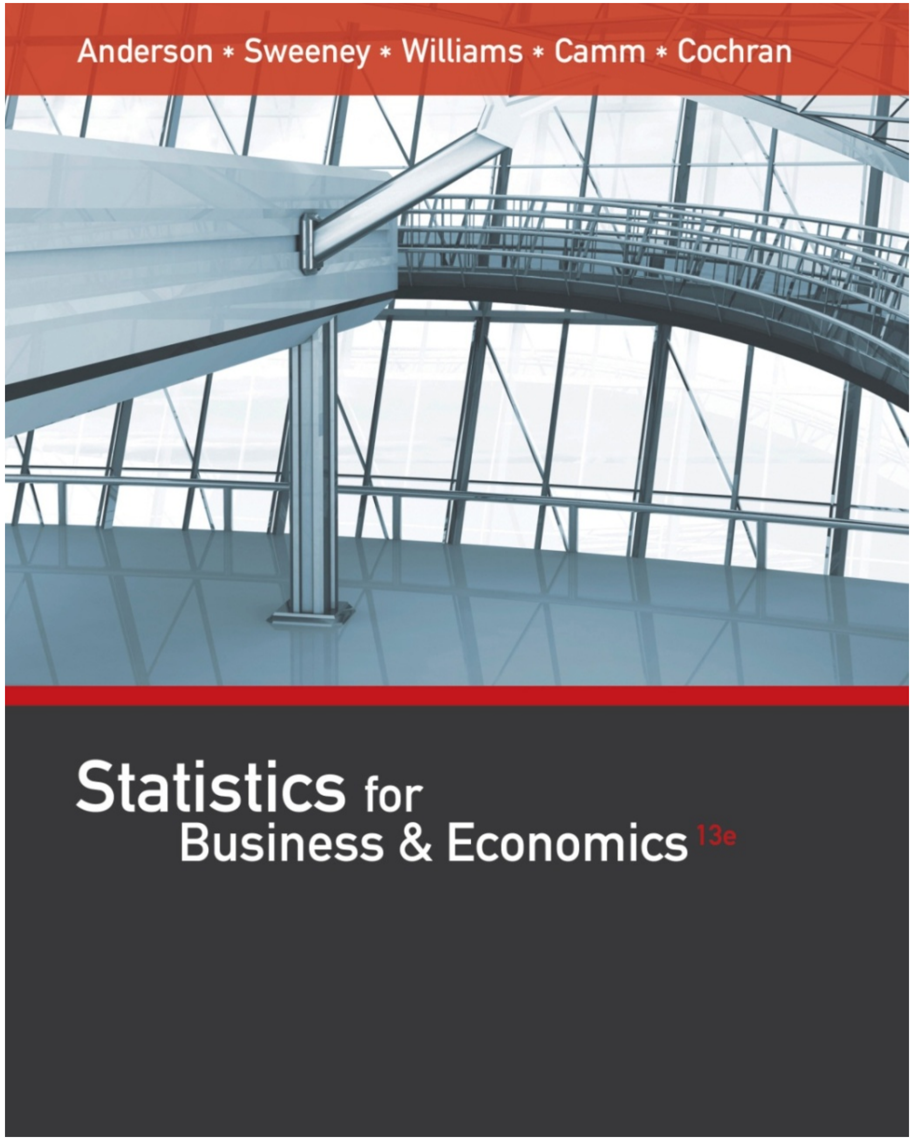


Statistics for Business and Economics (13e)

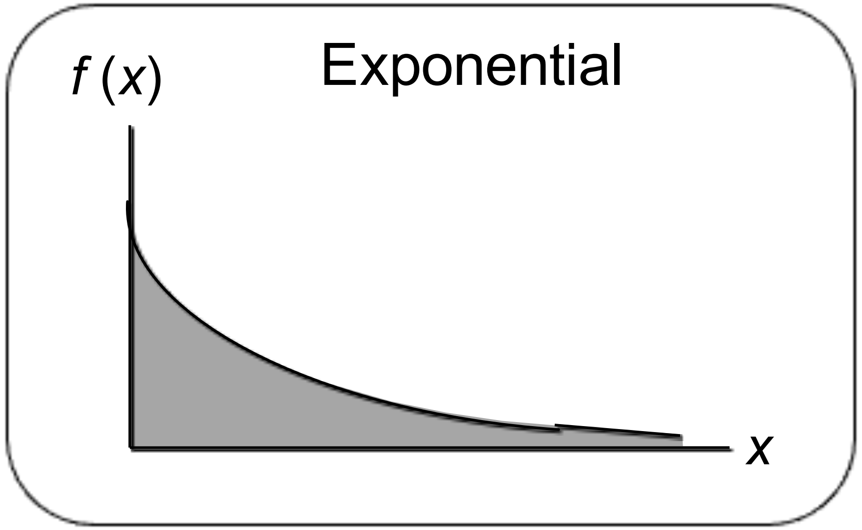
Anderson, Sweeney, Williams, Camm, Cochran

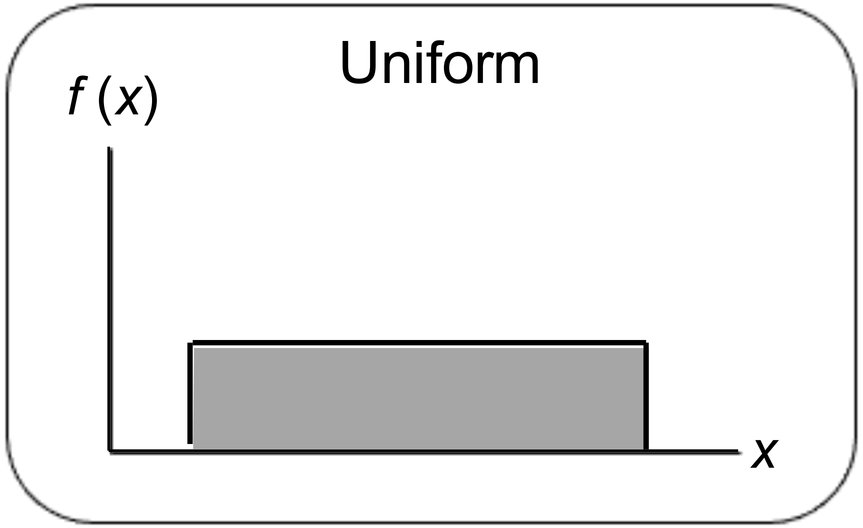
Slides by John Loucks

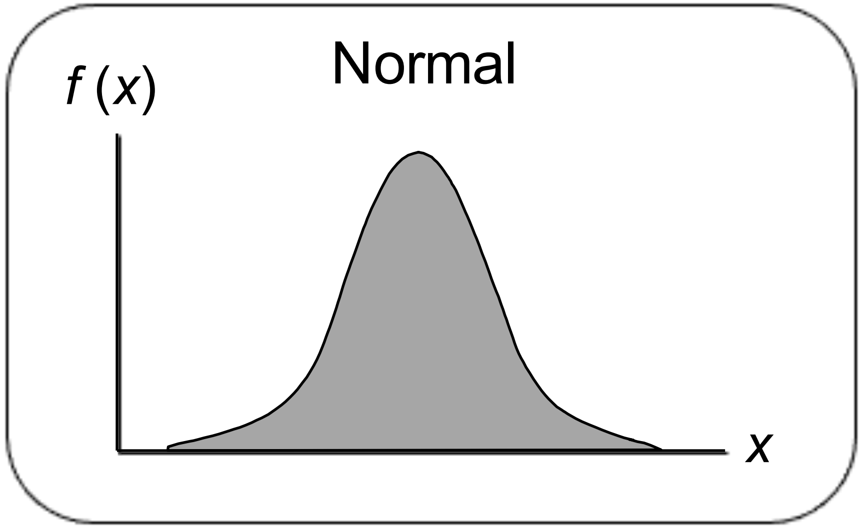
St. Edwards University

Chapter 6
Continuous Probability Distributions

- Uniform Probability Distribution
- Normal Probability Distribution
- Exponential Probability Distribution





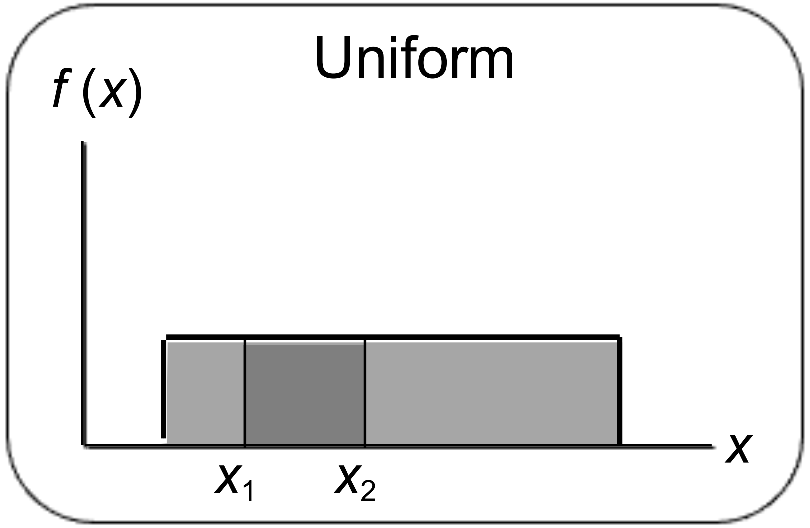


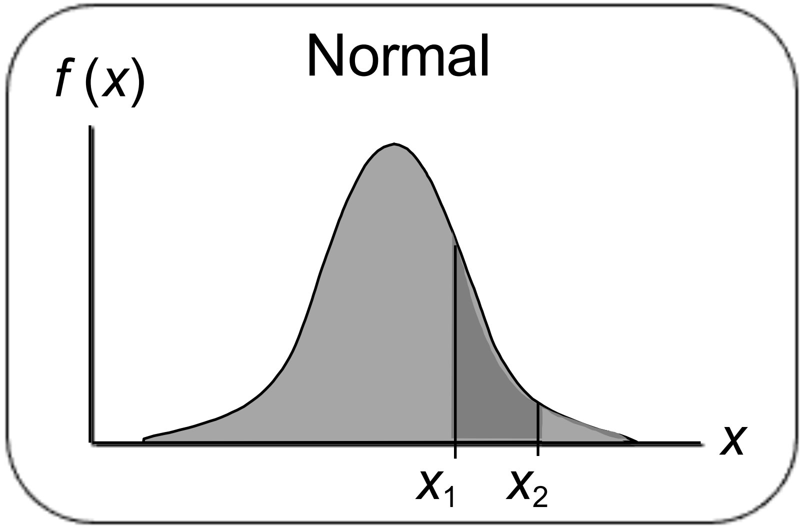
Continuous Probability Distributions

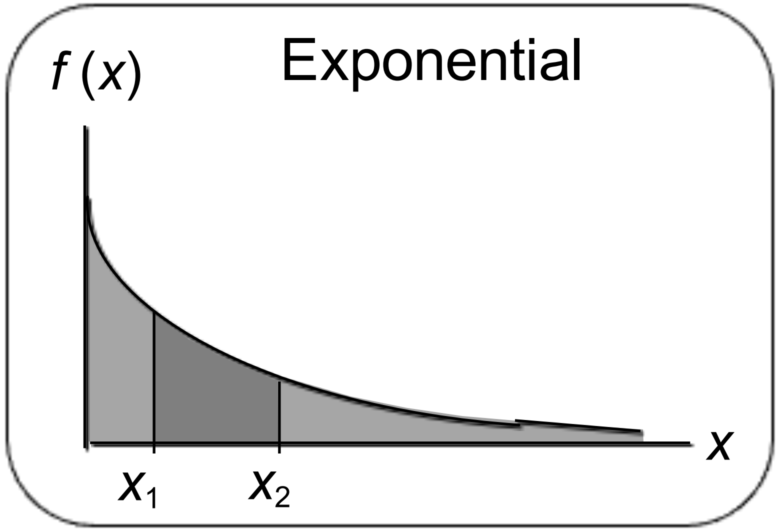
- A continuous random variable can assume any value in an interval on the real line or in a collection of intervals.
- It is not possible to talk about the probability of the random variable assuming a particular value.
- Instead, we talk about the probability of the random variable assuming a value within a given interval.

Continuous Probability Distributions

- The probability of the random variable assuming a value within some given interval from x_1 to x_2 is defined to be the area under the graph of the probability density function between x_1 and x_2.







Uniform Probability Distribution

- A random variable is uniformly distributed whenever the probability is proportional to the interval's length.
- The uniform probability density function is:

$$f(x) = 1/(b - a) \quad \text{for } a \leq x \leq b$$
$$= 0 \quad \text{elsewhere}$$

where: a = smallest value the variable can assume

b = largest value the variable can assume

Uniform Probability Distribution

- Expected Value of *x*

$$E(x) = (a + b)/2$$

- Variance of *x*

$$Var(x) = (b - a)^2/12$$

Uniform Probability Distribution

- Example: Slater's Buffet

Slater's customers are charged for the amount of salad they take. Sampling suggests that the amount of salad taken is uniformly distributed between 5 ounces and 15 ounces.

Uniform Probability Distribution

- Uniform Probability Density Function

$$f(x) = 1/10 \quad \text{for } 5 \leq x \leq 15$$
$$= 0 \quad \text{elsewhere}$$

where:

x = salad plate filling weight

Uniform Probability Distribution

- Expected Value of x

$$E(x) = (a + b)/2$$
$$= (5 + 15)/2$$
$$= 10$$

- Variance of x

$$Var(x) = (b - a)^2/12$$
$$= (15 – 5)^2/12$$
$$= 8.33$$

Uniform Probability Distribution

- Salad Plate Filling Weight

Uniform Probability Distribution

What is the probability that a customer will take between 12 and 15 ounces of salad?

Area as a Measure of Probability

- The area under the graph of $f(x)$ and probability are identical.
- This is valid for all continuous random variables.
- The probability that x takes on a value between some lower value x_1 and some higher value x_2 can be found by computing the area under the graph of $f(x)$ over the interval from x_1 to x_2.

Normal Probability Distribution

- The normal probability distribution is the most important distribution for describing a continuous random variable.
- It is widely used in statistical inference.
- It has been used in a wide variety of applications including:
  - Heights of people
  - Test scores
  - Amounts of rainfall
  - Scientific measurements
- Abraham de Moivre, a French mathematician, published *The Doctrine of Chances* in 1733.
- He derived the normal distribution.

Normal Probability Distribution

- Normal Probability Density Function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2((x-\mu)/\sigma)^2}$$

where:

μ = mean

σ = standard deviation

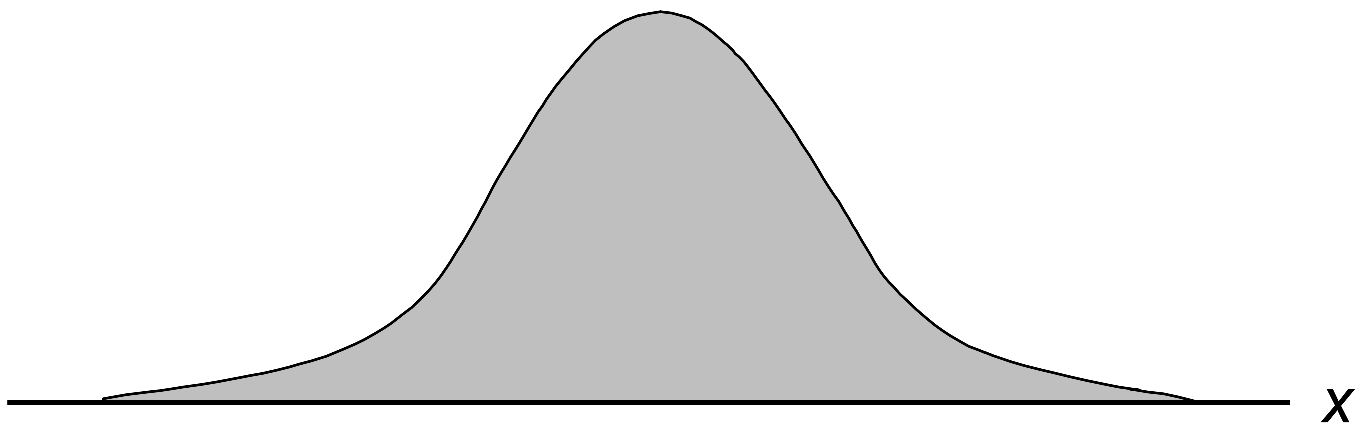
π = 3.14159

e = 2.71828

Normal Probability Distribution

- Characteristics

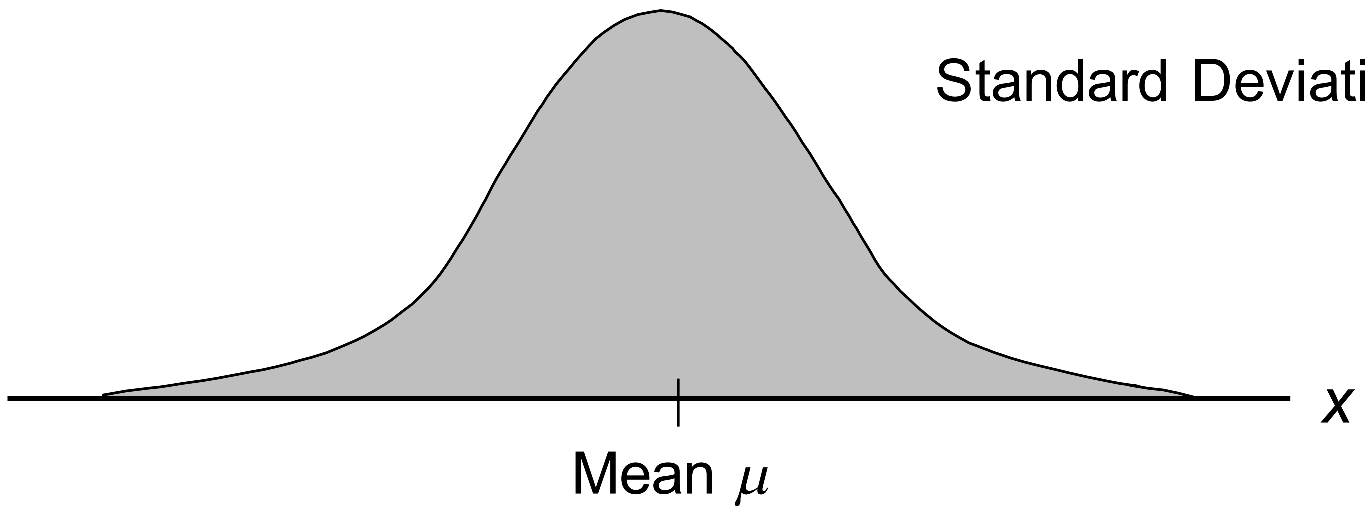
The distribution is symmetric; its skewness measure is zero.



Normal Probability Distribution

- Characteristics

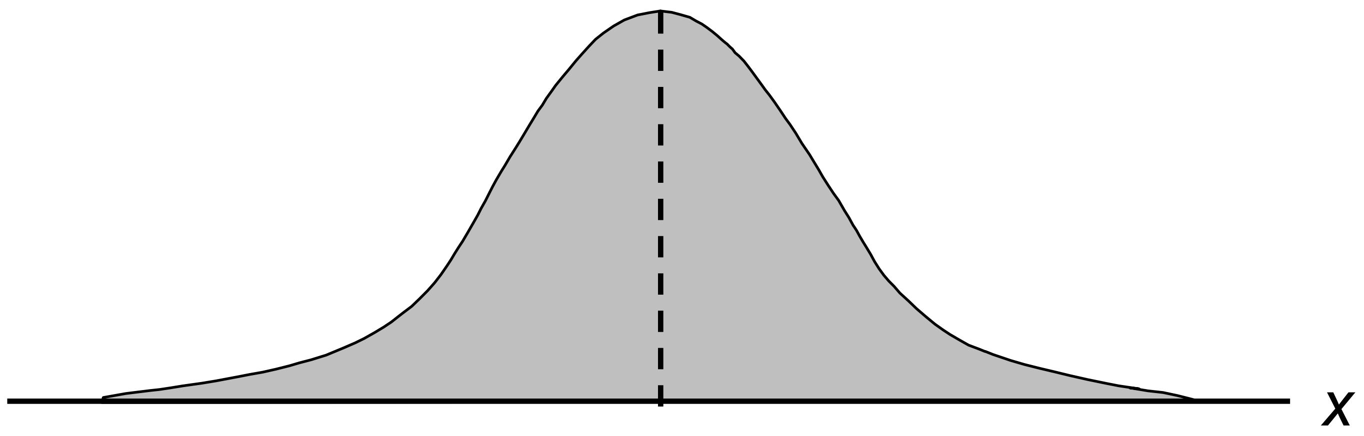
  The entire family of normal probability distributions is defined by its mean μ and its standard deviation σ.



Normal Probability Distribution

- Characteristics

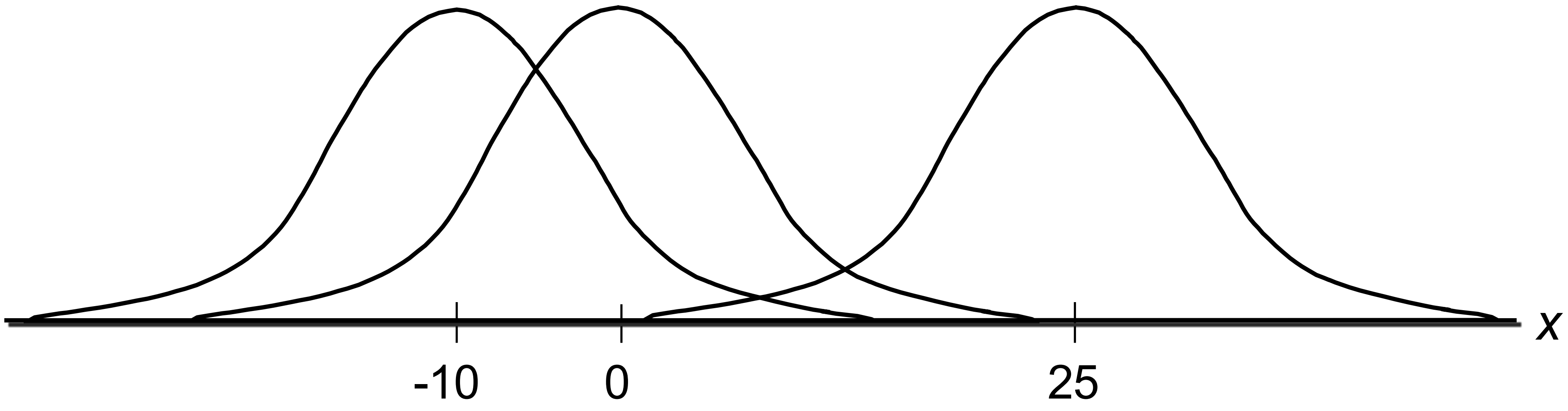
  The highest point on the normal curve is at the mean, which is also the median and mode.



Normal Probability Distribution

- Characteristics

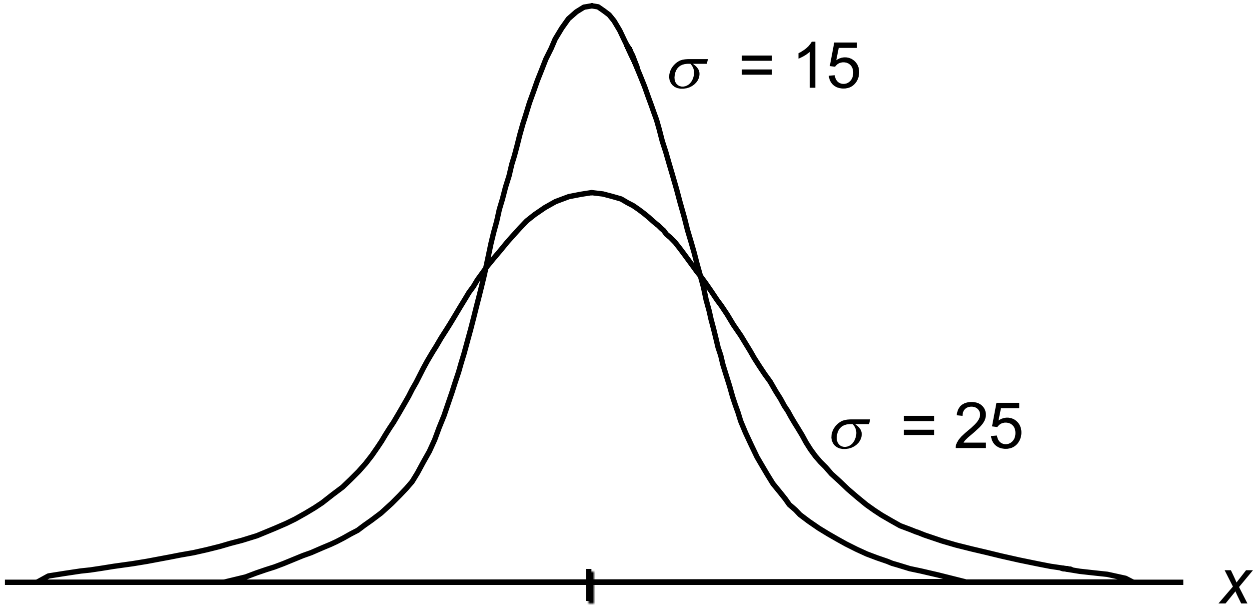
  The mean can be any numerical value: negative, zero, or positive.



Normal Probability Distribution

- Characteristics

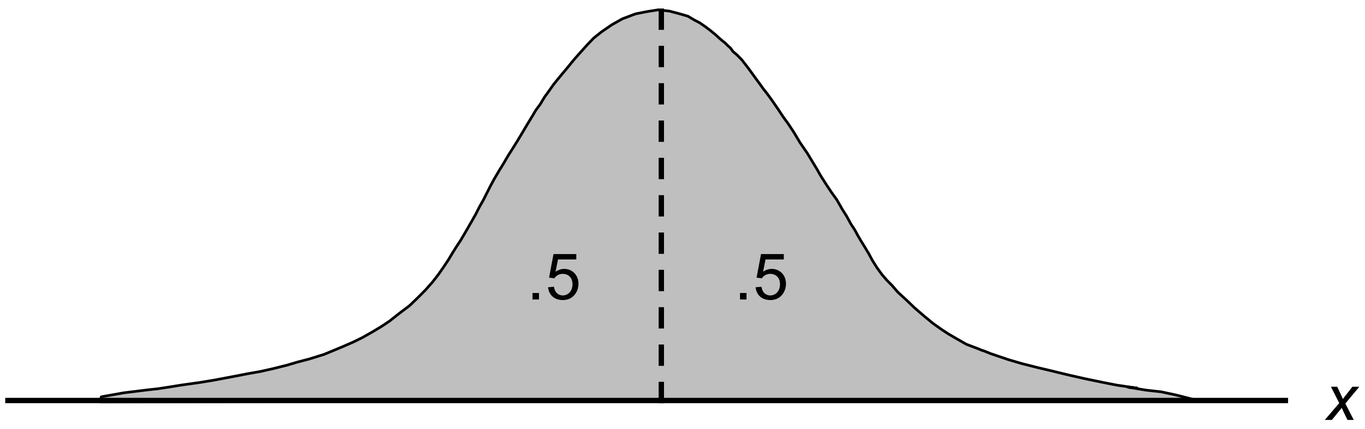
  The standard deviation determines the width of the curve: larger values result in wider, flatter curves.



Normal Probability Distribution

- Characteristics

Probabilities for the normal random variable are given by areas under the curve. The total area under the curve is 1 (.5 to the left of the mean and .5 to the right).



Normal Probability Distribution

- Characteristics (basis for the empirical rule)

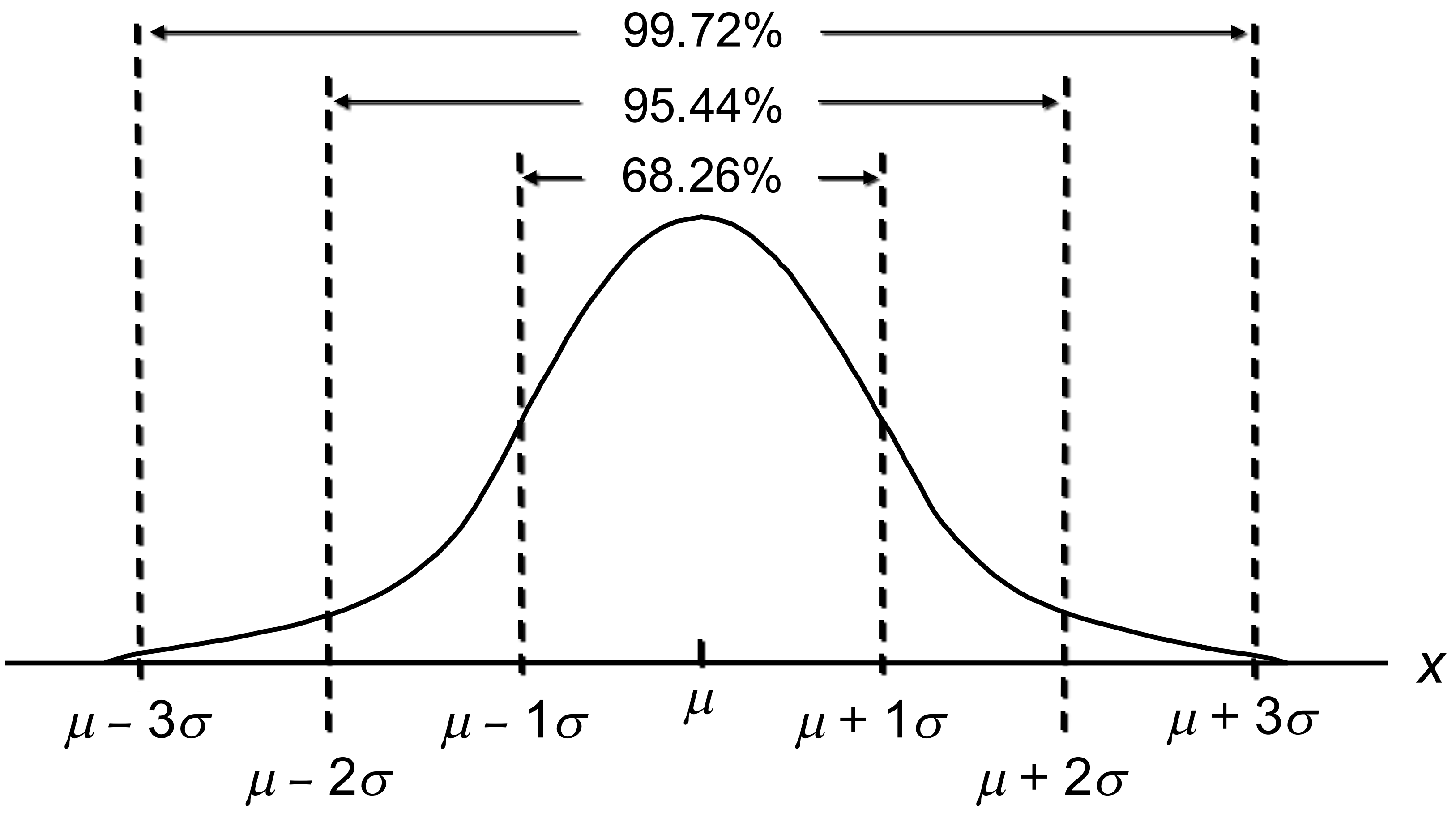
68.26% of values of a normal random variable are within +/- 1 standard deviation of its mean.

95.44% of values of a normal random variable are within +/- 2 standard deviations of its mean.

99.72% of values of a normal random variable are within +/- 3 standard deviations of its mean.

Normal Probability Distribution

- Characteristics (basis for the empirical rule)



Standard Normal Probability Distribution

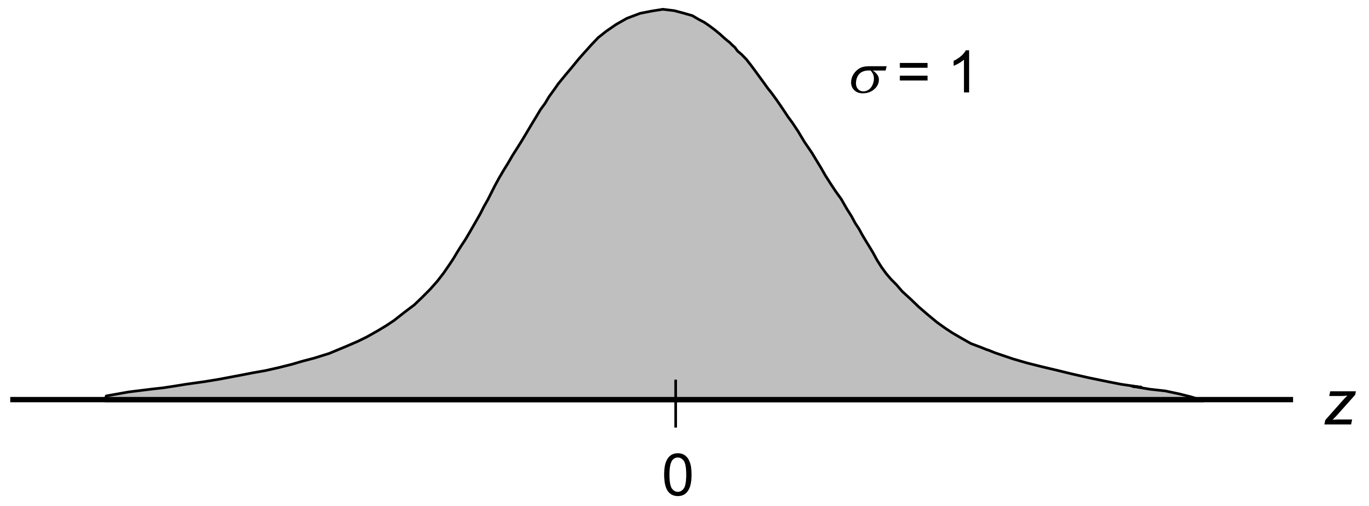
- Characteristics

  A random variable having a normal distribution with a mean of 0 and a standard deviation of 1 is said to have a standard normal probability distribution.

Standard Normal Probability Distribution

- Characteristics

  The letter *z* is used to designate the standard normal random variable.



Standard Normal Probability Distribution

- Converting to the Standard Normal Distribution

$$z = \frac{x-\mu}{\sigma}$$

We can think of z as a measure of the number of standard deviations x is from μ.

Standard Normal Probability Distribution

- Example: Pep Zone

Pep Zone sells auto parts and supplies including a popular multi-grade motor oil. When the stock of this oil drops to 20 gallons, a replenishment order is placed.

The store manager is concerned that sales are being lost due to stockouts while waiting for a replenishment order.

Standard Normal Probability Distribution

- Example: Pep Zone

It has been determined that demand during replenishment lead-time is normally distributed with a mean of 15 gallons and a standard deviation of 6 gallons.

The manager would like to know the probability of a stockout during replenishment lead-time. In other words, what is the probability that demand during lead-time will exceed 20 gallons?

$$P(x > 20) = ?$$

Standard Normal Probability Distribution

- Solving for the Stockout Probability

Step 1: Convert *x* to the standard normal distribution.

$$z = (x - \mu)/\sigma$$
$$= (20 - 15)/6$$
$$= .83$$

Step 2: Find the area under the standard normal curve to the left of z = .83.

Standard Normal Probability Distribution

- Cumulative Probability Table for the Standard Normal Distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.	.	.	.	.	.	.	.	.	.	.
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
.	.	.	.	.	.	.	.	.	.	.

$P(z \leq .83) = .7967$

Standard Normal Probability Distribution

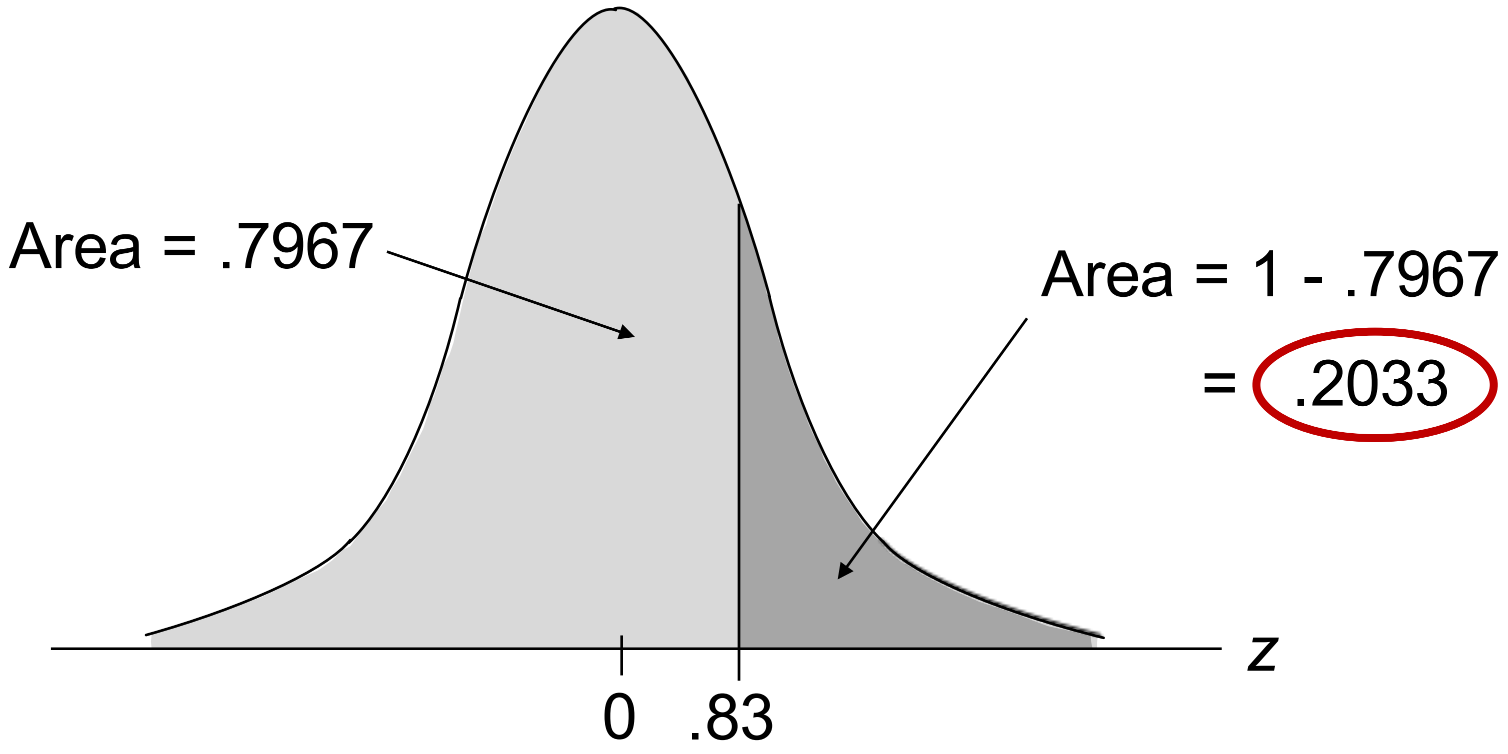
- Solving for the Stockout Probability

  Step 3: Compute the area under the standard normal curve to the right of z = .83.

$$P(z > .83) = 1 - P(z \leq .83)$$
$$= 1 - .7967$$
$$= .2033$$

Standard Normal Probability Distribution

- Solving for the Stockout Probability



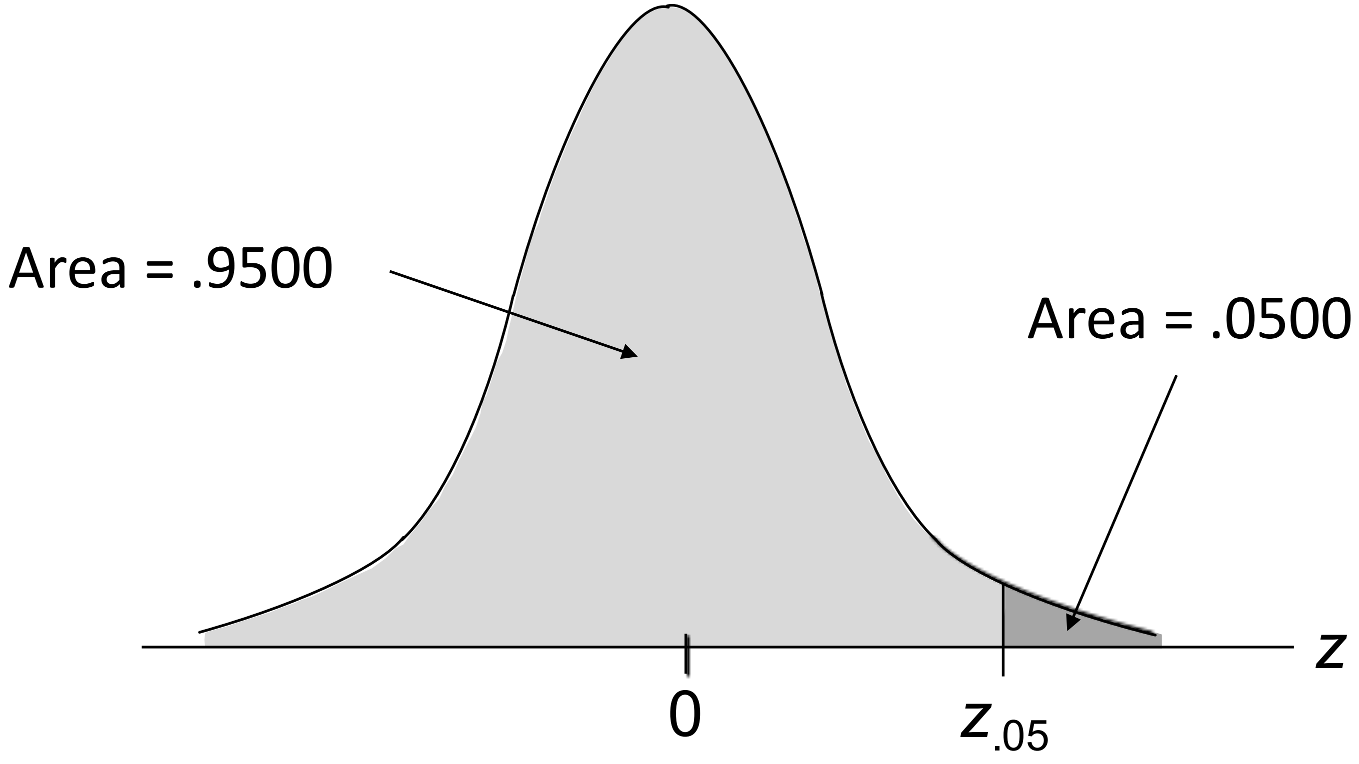
Standard Normal Probability Distribution

If the manager of Pep Zone wants the probability of a stockout during replenishment lead-time to be no more than .05, what should the reorder point be?

(Hint: Given a probability, we can use the standard normal table in an inverse fashion to find the corresponding *z* value.)

Standard Normal Probability Distribution

- Solving for the Reorder Point



Standard Normal Probability Distribution

- Solving for the Reorder Point

Step 1: Find the *z*-value that cuts off an area of .05 in the right tail of the standard normal distribution.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.	.	.	.	.	.	.	.	.	.	.
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
.	.	.	.	.	.	.	.	.	.	.

We look up the complement of the tail area (1 - .05 = .95)

Standard Normal Probability Distribution

- Solving for the Reorder Point

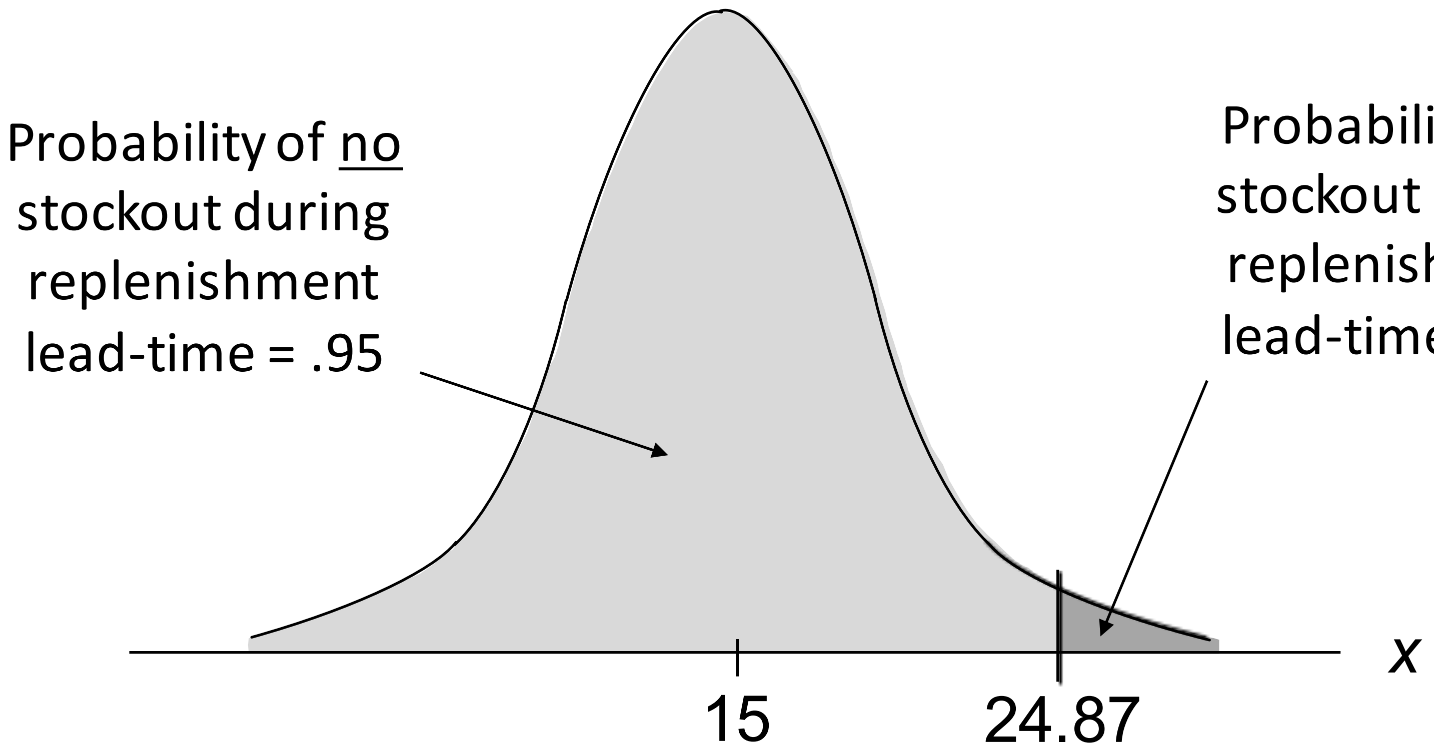
Step 2: Convert $z_{.05}$ to the corresponding value of x.

$$x = \mu + z_{.05}\sigma$$
$$= 15 + 1.645(6)$$
$$= 24.87 \text{ or } 25$$

A reorder point of 25 gallons will place the probability of a stockout during lead time at (slightly less than) .05.

Normal Probability Distribution

- Solving for the Reorder Point



Standard Normal Probability Distribution

- Solving for the Reorder Point

By raising the reorder point from 20 gallons to 25 gallons on hand, the probability of a stockout decreases from about .20 to .05.

This is a significant decrease in the chance that Pep Zone will be out of stock and unable to meet a customer’s desire to make a purchase.

Using Excel to Compute Normal Probabilities

- Excel has two functions for computing cumulative probabilities and *x* values for any normal distribution:
  - NORM.DIST is used to compute the cumulative probability given an *x* value.
  - NORM.INV is used to compute the *x* value given a cumulative probability.

Exponential Probability Distribution

- The exponential probability distribution is useful in describing the time it takes to complete a task.
- The exponential random variables can be used to describe:
  - Time between vehicle arrivals at a toll booth
  - Time required to complete a questionnaire
  - Distance between major defects in a highway
- In waiting line applications, the exponential distribution is often used for service time.

Exponential Probability Distribution

- A property of the exponential distribution is that the mean and standard deviation are equal.
- The exponential distribution is skewed to the right. Its skewness measure is 2.

Exponential Probability Distribution

- Density Function

$$f(x) = \frac{1}{\mu} e^{-x/\mu} \quad \text{for } x \geq 0$$

where: μ = expected value or mean

e = 2.71828

Exponential Probability Distribution

- Cumulative Probabilities

$$P(x \le x_0) = 1 - e^{-x_0/\mu}$$

where:

x_0 = some specific value of x

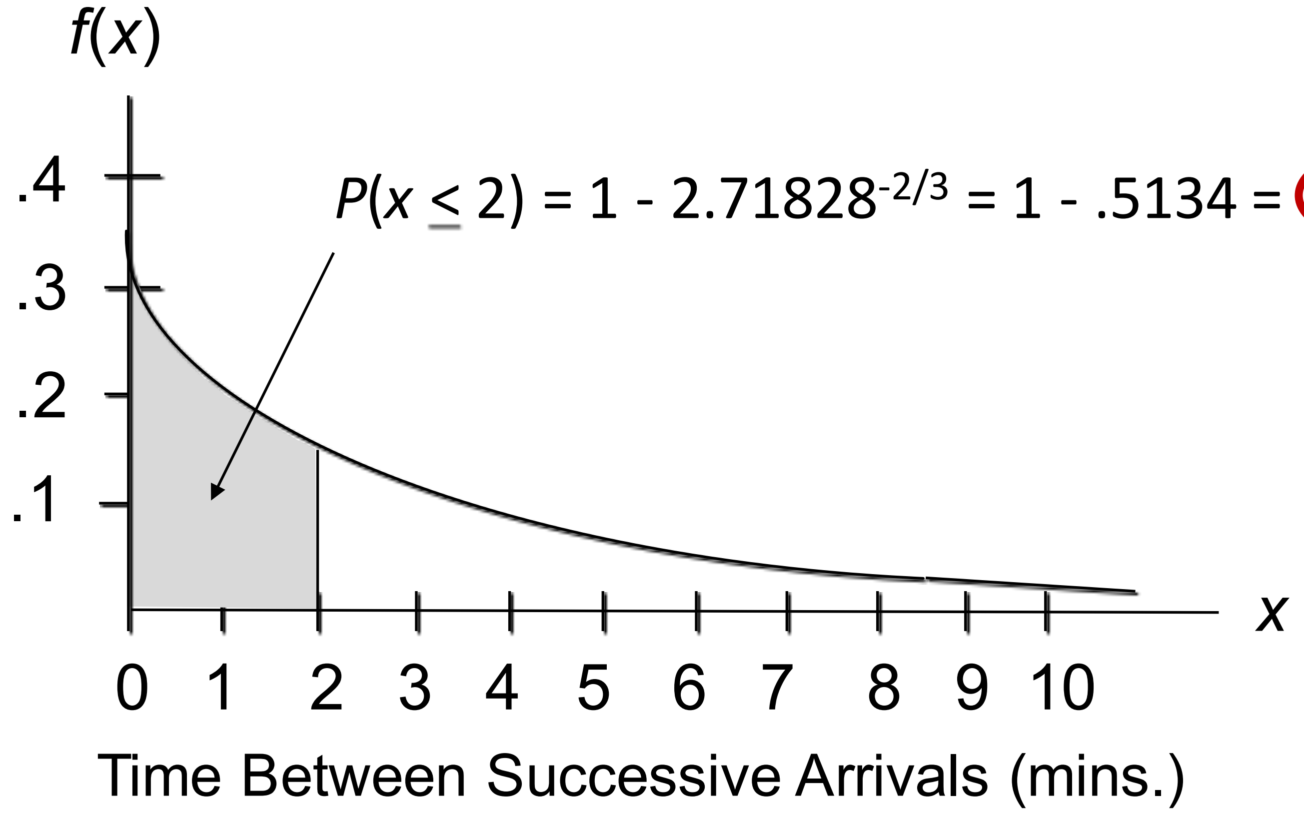
Exponential Probability Distribution

- Example: Al's Full-Service Pump

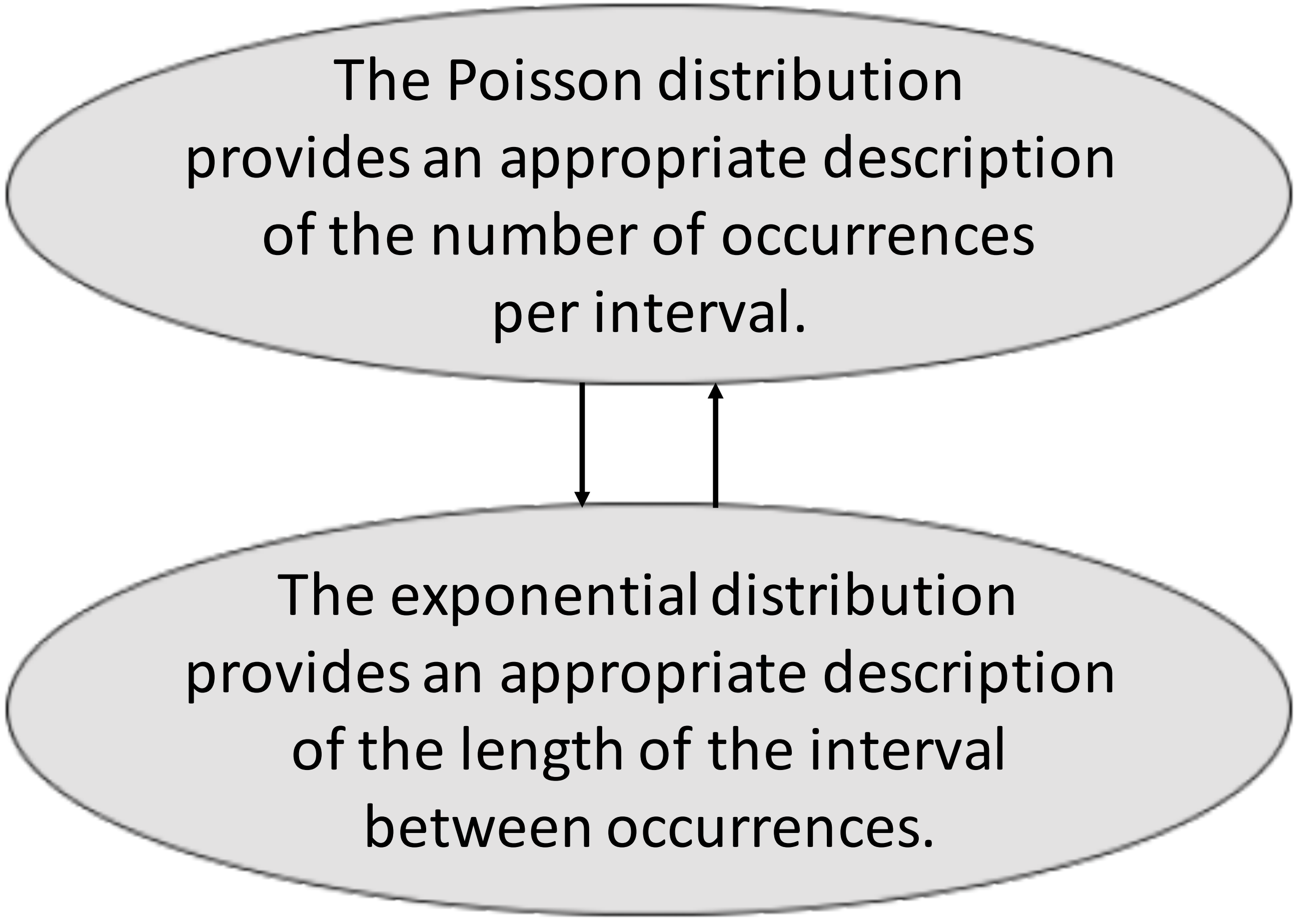
The time between arrivals of cars at Al's full-service gas pump follows an exponential probability distribution with a mean time between arrivals of 3 minutes. Al would like to know the probability that the time between two successive arrivals will be 2 minutes or less.

Exponential Probability Distribution

- Example: Al's Full-Service Pump



Relationship between the Poisson and Exponential Distributions



End of Chapter 6