Topic 4 : Capital Asset Pricing Model and Abitrage Pricing Theory (Part 2)

EE431/438

Copeland, Thomas E. and J. Fred Weston, Financial Theory and Corporate Policy (4th ed), Addison-Wesley, 2005: Ch6 (pp 147 -157)
Federic Mishkin, The Economics of Money, Banking and Financial Markets (Appendix to Chapter 5, available in the internet):

October 2013

- Portfolio Diversification and Individual Asset Risk [Part 1]
- The Capital Asset Pricing Model (CAPM) [Part 1]
  - Assumptions
  - Efficiency of the Market Portfolio
  - Derivation of the CAPM
  - Properties of the CAPM
  - Risk premium and diversification
- Arbitrage Pricing Theory (APT) [Part 2]

- Arbitrage : an abitrage opportunity arises when an investor can construst a zero investment portfolio that will yield a sure profit (risk-free)
- Example :
-

	S_1	S_2	*Price*
A_1	1	0	0.2
A_2	0	1	0.1

- If there exists a security C pay 2 $ when S_1 occurs and it is priced at $ 0.5, we can contruct an abitrage portfolio; selling at a high price and buying at a low price.
- In equilibrium, no abitrage portfolio exists.
- Law of one price

- Arbitrage : an abitrage opportunity arises when an investor can construct a zero investment portfolio that will yield a sure profit (risk-free)
- The CAPM : $R_i = R_f + \beta_i(R_m - R_f) + \epsilon_i$: only one source of systematic risk, unsystematic risk can be eliminated through diversification
- The APT: several sources of risk that cannot be eliminated through diversification
- The sources of risk : inflation, aggregate output, .. etc.
- $R_i = \beta_0^i + \beta_1^i(\text{Factor 1}) + \beta_2^i(\text{Factor 2}) + ... + \beta_k^i(\text{Factor k}) + \epsilon^i$
- If there is only one factor and that factor is R_m, the APT is the same as the CAPM.
- the APT is more general than the CAPM

- CAPM: $E(R_i) = a_i + \beta_i(E(R_m) - R_f))$;
  $E(R_i) = \text{constant}_1 + \beta_i(\text{constant}_2) = a_i + \beta_i E(R_m)$
- APT :
  $E(R_i) = \beta_0^i + \beta_1^i(\text{Factor 1}) + \beta_2^i(\text{Factor 2}) + ... + \beta_k^i(\text{Factor k})$
- APT : $R_i = \beta_0^i + \beta_1^i F_1 + \beta_2^i F_2 + ... + \beta_k^i F_K$
- Assets with the same values of β_j for all factors j must have the same rate of returns.

- Example : $E(R_a) = 0.08 + 0.6F_1$, $E(R_b) = 0.02 - 0.2F_1$.
  - Determine the portfolio weights you need to place on a and b in order to construct a portfolio which has no risk ($\beta_1 = ...$).
  - If the risk free rate is equal to 0.01 (you can lend or borrow at 1% interest rate), can you make an arbitrage profit?

- At equilibrium, there must be no abitrage opportunity.
- From the last example, $E(R_a) = 0.08 + 0.6F_1$, $E(R_b) = 0.02 - 0.2F_1$.
    - Suppose there is asset C, $E(R_C) = 0.10 + 0.6F_1$.
    - Can you make an arbitrage profit?
    - Buy low and sell high.
    - Buy .... and short-sell ....
    - get a profit of ....=0.02 = 2%, regardless of the value of F_1.

- Short-sell : The sale of shares not owned by the investor but borrowed through a broker and later repurchased to preplace the loan. Profit is earned if the initial sale is at higher price than the repurchase price.
- $E(R_i) = \beta_0^i + \beta_1^i F_1 + \beta_2^i F_2 + ... + \beta_k^i F_K$
- Assets with the same values of β_j for all factors j must have the same rate of returns.
- Example : $E(R_a) = 0.08 + 0.6F_1$, $E(R_b) = 0.02 - 0.2F_1$.
    - R_f = the rate of return of the portfolio which has $\beta_1 = 0$.
    - Determine the portfolio weights you need to place on a and b in order to construct a portfolio which has $\beta_1 = 1$.
    - $R_m = 0.11 + F_1$
    - Determine the rate of returns of portiolio i which has 0, 0.6, 0.2, 1, 0.5, 2.

- $E(R_i) = R_f + (E(R_m) - R_f)\beta_1^i$
- $E(R_i) =$ ....+ ....

- $\beta = 0.6 \Rightarrow$
  $E(R_i) = 0.035 + 0.6(0.11 + F_1 - 0.035) = 0.08 + 0.6F_1$
- $\beta = -0.2 \Rightarrow$
  $E(R_i) = 0.035 - 0.2(0.11 + F_1 - 0.035) = 0.02 - 0.2F_1$
- CAPM : $R_i = R_f + \beta(\text{Market Risk Premium})$
- APT :
  $E(R_i) = \beta_0^i + \beta_1^i(\text{Factor 1}) + \beta_2^i(\text{Factor 2}) + ... + \beta_k^i(\text{Factor k})$
- $E(R_i) = R_f + (\lambda_1 - R_f)\beta_1^i + (\lambda_2 - R_f)\beta_2^i + ... + (\lambda_k - R_f)\beta_k^i$;
- λ_j = the expected rate of return on portfolio with unit sensitivity to the *j* the factor and zero sensitivity to all other factors.
- At equilibrium, there is no arbitrage opportunity.

- Examples: 2 Factors
  $E(R_a) = 0.10F_1 - 0.5F_2$
  $E(R_b) = 0.08 + 2F_1 + F_2$
  $E(R_c) = 0.05 + 0.5F_1 + 0.5F_2$
  $E(R_p) = w_a E(R_a) + w_b E(R_b) + w_c E(R_c)$
- Determine the portfolio weights you need to place on a and b in order to construct a portfolio which has no risk
  $0.10w_a + 2w_b + 0.5w_c = 0$
  $-0.5w_a + 1w_b + 0.5w_c = 0$
  $w_a = \frac{5}{13},\ w_b = -\frac{3}{13},\ w_c = \frac{11}{13}$
- Determine the rate of return on portfolio which has $\beta_1 = 1$ and $\beta_2 = 0$.
- Determine the rate of return on portfolio which has $\beta_1 = 0$ and $\beta_2 = 1$
- $E(R_i) = R_f + (\lambda_1 - R_f)\beta_1^i + (\lambda_2 - R_f)\beta_2^i$; λ_j = the expected rate of return on portfolio which has $\beta_j = 1$ and other betas equal to zero.