5.3 Ex.3, p. 4

(a) $y = 5 \ln x$

$y' = 5 \dfrac{d \ln x}{dx}$

$= \dfrac{5}{x}$ (use 3 in 5.3)

(b) $y = \ln(5x)$

$= \ln u$ where $u = 5x$

$= \dfrac{1}{u} \dfrac{du}{dx}$ (use 4 in 5.3)

$= \dfrac{1}{5x}(5) = \dfrac{1}{x}$

(c) $y = x \ln x$

since x is not a constant, we can view y as

$y = f(x) \cdot g(x)$ where $f(x) = x$ & $g(x) = \ln x$

$\therefore$ use product rule

$y' = f'(x) g(x) + g'(x) f(x)$

$= (1) \ln x + \dfrac{1}{x}(x)$

$= \ln x + 1$

EX. 3 (5.3) p. 5

(d) $y = \dfrac{\ln \sqrt[3]{x^2}}{x^4}$

$$= \frac{\ln x^{2/3}}{x^4} = \frac{\frac{2}{3}\ln x}{x^4} = \frac{f(x)}{g(x)}$$

log properties in 5.2.2 p. 3

$$y' = \frac{f'(x)g(x) - g'(x)f(x)}{[g(x)]^2}$$

$$= \frac{\frac{2}{3x} \cdot x^4 - x^3 \cdot \frac{2}{3}\ln x}{x^8}$$

$$= \frac{\frac{2}{3}x^3 - \frac{8}{3}x^3 \ln x}{x^8} = \frac{\frac{2}{3} - \frac{8}{3}\ln x}{x^5}$$

$$= \frac{2}{3}\left[\frac{1 - 4\ln x}{x^5}\right]$$

(e) $y = 3\ln(x^2 - 5x) = 3\ln u$

$y' = 3\dfrac{d\ln u}{dx}$ where $u = x^2 - 5x$

$$= 3 \cdot \frac{1}{u}\frac{du}{dx} = 3 \cdot \frac{1}{x^2-5x} \cdot 2x-5$$

$$= \frac{3(2x-5)}{x^2-5x}$$

(f) $y = (x + \ln x)^{3/2}$

$y = u^{3/2}$ where $u = x + \ln x$

use chain rule

$$\boxed{\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}}$$

$\therefore \frac{dy}{dx} = \frac{3}{2} u^{1/2} \cdot \frac{du}{dx}$ ——— (1)

$\because u = x + \ln x$

$\frac{du}{dx} = \frac{dx}{dx} + \frac{d \ln x}{dx} = 1 + \frac{1}{x}$ ——— (2)

substitute (2) in (1)

$$\frac{dy}{dx} = \frac{3}{2} u^{1/2} \left(1 + \frac{1}{x}\right)$$

$$= \frac{3}{2} (x + \ln x)^{1/2} \left(1 + \frac{1}{x}\right)$$

(g) $y = \log_b x = \frac{\ln x}{\ln b}$ (log properties in 5.2.2 p.3)

- We try to change $\log_b x$ to $\frac{\ln x}{\ln b}$ because we have formulae for differentiating ln functions not log functions
- b is a constant, $\therefore$ ln b is a constant not a function, we don't need quotient rule here. Simply treat ln b as a constant $\Rightarrow$

$$\therefore \frac{dy}{dx} = \frac{1}{\ln b} \frac{d \ln x}{dx} = \frac{1}{\ln b} \cdot \frac{1}{x}$$

EX. 3, 5.3 p. 5

(h) $y = \log_b u$

Here, u is not a constant but a function of x

$\therefore y = \frac{\ln u}{\ln b}$ (we change y from log form to ln form again because we can diff functions in ln form)

$\because$ b is a constant $\Rightarrow$ $\ln b$ is a constant $\therefore$ Don't need quotient rule

$$\frac{dy}{dx} = \frac{1}{\ln b} \frac{d \ln u}{dx}$$

$$= \frac{1}{\ln b} \cdot \frac{1}{u} \frac{du}{dx}$$ (use 4 in 5.3, p. 4) or this is actually chain rule as shown below

$$= \frac{1}{(\ln b) u} \frac{du}{dx}$$

Chain rule states that if $y = f(u)$ where $u = g(x)$

$$\frac{dy}{dx} = \frac{d f(u)}{du} \cdot \frac{du}{dx}$$

$$= \frac{d f(u)}{du} \cdot \frac{d g(x)}{dx}$$

How does it work in this example?

$$\frac{d \ln u}{dx} = \frac{d \ln u}{du} \cdot \frac{du}{dx}$$

$$= \frac{1}{u} \frac{du}{dx}$$

This is extra, for those who are curious!! as long as you know you can use 4 in 5.3 for example (h), you'll be fine in exam!