BACHELOR OF ECONOMICS
THAMMASAT UNIVERSITY

EE325 Introductory Econometrics, Semester 1/2019 (Section 046402)

Due Date: Thursday 27th February 2020 by 09.30 via Assignment Submission in Moodle.

Instruction: Do all questions with your own handwriting and your own attempt.

Use 4 decimal places for numerical answers

1. In Table 1. $X_i$is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

Table 1

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3.0	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i,\, u_i \sim NIID(0, \sigma^2)$
Use OLS to find the estimator of $\beta_1 and \beta_2$. Interpret the regression.

1.2 Find $\hat{Y}_i \quad and\ \hat{u}_i$ and show that $\sum_{i=1}^{n} \hat{u}_i \approx 0$

1.3 Find $var(\hat{u}_i), var(\hat{\beta}_1), and\ var(\hat{\beta}_2)$

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

2.4 If $X_i = 18$, what is the predicted Y?

2.5 Find $var(\hat{u}_i)$, $var(\hat{\beta}_1), var(\hat{\beta}_2)$

3. Consider the below regression function: consider the two-variable model

$\boldsymbol{Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)}$

Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

"Practice makes Perfect."

1.1

x_i	y_i	$x_i-\bar{x}$	$(x_i-\bar{x})^2$	$y_i-\bar{y}$	$(x_i-\bar{x})(y_i-\bar{y})$
63	2.8	-14.625	213.890625	-0.4125	6.0328125
72	3.4	-5.625	31.640625	0.1875	-1.0546875
78	3	0.375	0.140625	-0.2125	-0.0796875
81	3.5	3.375	11.390625	0.2875	0.9703125
87	3.6	9.375	87.890625	0.3875	3.6328125
75	3	-2.625	6.890625	-0.2125	0.5578125
75	2.7	-2.625	6.890625	-0.5125	1.3453125
90	3.7	12.375	153.140625	0.4875	6.0328125
Σ 621	25.7	0	511.875	0	17.4375

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n} = \frac{621}{8} = 77.625$$

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n} = \frac{25.7}{8} = 3.2125$$

$$\hat{\beta}_2 = \frac{\Sigma(x_i-\bar{x})(y_i-\bar{y})}{\Sigma(x_i-\bar{x})^2} = \frac{17.4375}{511.875} = 0.0341$$

$$\hat{\beta}_1 = \bar{y} - \hat{\beta}_2\bar{x} = 3.2125 - 0.0341(77.625) = 0.5655$$

then $\hat{Y}_i = 0.5655 + 0.0341 X_i$

$\therefore$ $\hat{\beta}_2 = 0.0341$ means that if total microeconomics exam point change by 1 point, on average, student's GPA will change by 0.0341.

$\hat{\beta}_1 = 0.5655$ means that if total microeconomics exam point is equal to zero, student's GPA is 0.5655

1.2)

x_i	y_i	$\hat{y}_i$	$\hat{u}_i = y_i - \hat{y}_i$	$\hat{u}_i^2$	x_i^2
63	2.8	2.7138	0.0862	0.00743044	3969
72	3.4	3.0207	0.3793	0.14386849	5189
78	3	3.2253	-0.2253	0.05076009	6084
81	3.5	3.3276	0.1724	0.02972176	6561
87	3.6	3.5322	0.0678	0.00459684	7569
75	3	3.123	-0.123	0.015129	5625
75	2.7	3.123	-0.423	0.178929	5625
90	3.7	3.6345	0.0655	0.0042902	8100
			$\sum_{i=1}^{n} \hat{u}_i = 0.0001 \approx 0$	$\sum_{i=1}^{n} \hat{u}_i^2 = 0.43472587$	$\sum_{i=1}^{n} x_i^2 = 48,717$

1.3 $$\text{var}(\hat{u}_i) = \hat{\sigma}^2 = \frac{\sum_{i=1}^{n} \hat{u}_i^2}{n-2} = \frac{0.43472587}{8-2} = 0.0725$$

$$\text{var}(\hat{\beta}_1) = \frac{\hat{\sigma}^2 \sum x_i^2}{n \sum (x_i - \bar{x})^2} = \frac{0.0725(48,717)}{511.875} = 0.000142$$

$$\text{var}(\hat{\beta}_2) = \frac{\hat{\sigma}^2}{\sum (x_i - \bar{x})^2} = \frac{0.0725}{511.875} = 0.000142$$

2.1

	X_i	Y_i	$x_i - \bar{X}$	$x_i - \bar{X}$	$Y_i - \bar{Y}$	$(x_i - \bar{X})(Y_i - \bar{Y})$
	10	0	−10	100	−9.1	91
	12	2	−8	64	−7.1	56.8
	14	5	−6	36	−4.1	24.6
	16	6	−4	16	−3.1	12.4
	18	7	−2	4	−2.1	4.2
	22	10	2	4	0.9	1.8
	24	10	4	16	0.9	3.6
	26	15	6	36	5.9	35.4
	28	16	8	64	6.9	55.2
	30	20	10	100	10.9	109
(Σ)	200	91	0	440	0	394

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} = \frac{200}{10} = 20$$

$$\bar{Y} = \frac{\sum_{i=1}^{n} Y_i}{n} = \frac{91}{10} = 9.1$$

$$\hat{\beta}_2 = \frac{\Sigma (X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma (X_i - \bar{X})^2} = \frac{394}{440} \approx 0.8955$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X} = 9.1 - (0.8955)(20) = -8.81$$

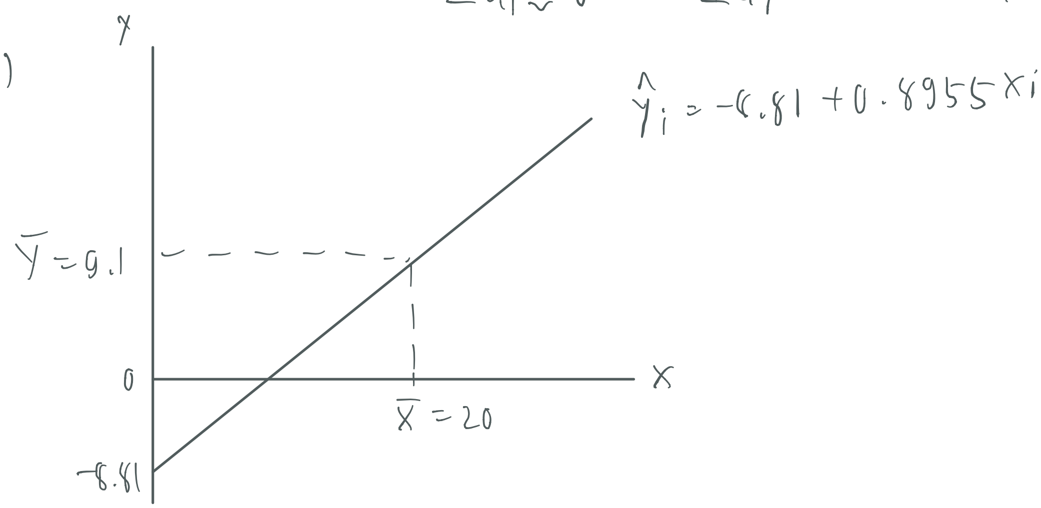
then $\hat{Y}_i = -8.81 + 0.8955 X_i$

$\therefore$ $\hat{\beta}_2 = 0.8955$ means that if X change by 1 unit, on average Y will change by 0.8955 unit.

2.2

X_i	Y_i	$\hat{Y}_i$	$\hat{u}_i = Y_i - \hat{Y}$	$\hat{u}_i^2$	X_i^2
10	0	0.145	−0.145	0.021025	100
12	2	1.936	0.064	0.004096	144
14	5	3.727	1.273	1.620529	196
16	6	5.518	0.482	0.232324	256
18	7	7.309	−0.309	0.095481	324
22	10	10.891	−0.891	0.793881	484
24	10	12.682	−2.682	7.193124	576
26	15	14.473	0.527	0.277729	676
28	16	16.264	−0.264	0.069696	784
30	20	18.055	1.945	3.783025	900
			$\sum \hat{u}_i \approx 0$	$\sum \hat{u}_i^2 = 14.09091$	$\sum X_i^2 = 4440$

2.3)



$$\hat{Y}_i = -8.81 + 0.8955X_i$$

$$\bar{Y} = -8.81 + 0.8955\bar{X}$$

$$\bar{Y} = -8.81 + 0.8955(20) = 9.1$$

∴ The regression line passes through the $\bar{X}$, $\bar{Y}$

2.4 $x_i = 18$

$\hat{Y}_i = -8.81 + 0.8955(18)$

$= 7.309$

2.5 $var(\hat{u}_i) = \hat{\sigma}^2 = \dfrac{\sum \hat{u}_i^2}{n-2} = \dfrac{14.0909}{10-2} = 1.7614$

$var(\hat{\beta}_1) = \dfrac{\hat{\sigma}^2 \sum x_i^2}{n \sum (x_i - \bar{x})^2} = \dfrac{(1.7614)(4440)}{10(440)} = 1.7774$

$var(\hat{\beta}_2) = \dfrac{\hat{\sigma}^2}{\sum (x_i - \bar{x})^2} = \dfrac{1.7614}{440} = 0.0004$

3. Consider the below regression function: consider the two-variable model

$$Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)$$

Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator.

Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

$$\bar{Y} = \beta_1 + \beta_2 \bar{X} + u \quad — ①$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$$

sub ① ; $\hat{\beta}_1 = \beta_1 + \beta_2 \bar{X} + \bar{u} - \hat{\beta}_2 \bar{X}$

$$\hat{\beta}_1 = \beta_1 + (\beta_2 - \hat{\beta}_2)\bar{X} + u$$

$$E(\hat{\beta}_1 | X) = E(\beta_1 | X) + \bar{X} E[(\beta_2 - \hat{\beta}_2) | X] + E(\cancel{u} | X) \quad 0 \ (3)$$

$$E(\hat{\beta}_1 | X) = \beta_1 + \bar{X}(\beta_2 - \underbrace{E(\hat{\beta}_2 | X)}_{\beta_2})$$

$$E(\hat{\beta}_1 | X) = \beta_1$$