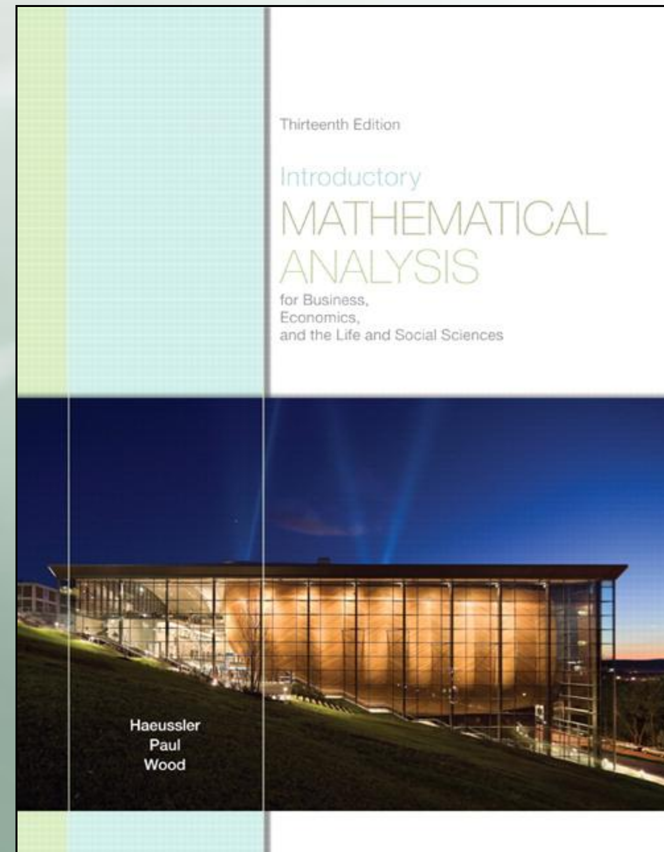
# INTRODUCTORY MATHEMATICAL ANALYSIS

## For Business, Economics, and the Life and Social Sciences

## Chapter 4
## Exponential and Logarithmic Functions

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# Chapter Objectives

- To introduce exponential functions and their applications.
- To introduce logarithmic functions and their graphs.
- To study the basic properties of logarithmic functions.
- To develop techniques for solving logarithmic and exponential equations.

# Chapter Outline

# 4.1 Exponential Functions

- The function *f* defined by $f(x) = b^x$

  where $b > 0$, $b \neq 1$, and the exponent $x$ is any real number, is called an ***exponential function*** with base *b*[1].

**Rules for Exponents**

**1.** $b^x b^y = b^{x+y}$

**2.** $\frac{b^x}{b^y} = b^{x-y}$

**3.** $(b^x)^y = b^{xy}$

**4.** $(bc)^x = b^x c^x$

**5.** $\left(\frac{b}{c}\right)^x = \frac{b^x}{c^x}$

**6.** $b^1 = b$

**7.** $b^0 = 1$

**8.** $b^{-x} = \frac{1}{b^x}$

# Example 1 – Bacteria Growth

*The number of bacteria present in a culture after t minutes is given by* $N(t)=300\left(\frac{4}{3}\right)^{t}$.

*a. How many bacteria are present initially?*

*b. Approximately how many bacteria are present after 3 minutes?*

Solution:

## Example 3 – Graphing Exponential Functions with $0 < b < 1$

*Graph the exponential function* $f(x) = (1/2)^x$.

Solution:

# Properties of Exponential Functions

1. The domain of an exponential function consists of all real numbers.
   The range consists of all positive numbers.
2. The graph of $f(x) = b^x$ has $y$-intercept $(0, 1)$.
   There is no $x$-intercept.
3. If $b > 1$, the graph *rises* from left to right.
   If $0 < b < 1$, the graph *falls* from left to right.
4. If $b > 1$, the graph approaches the $x$-axis as $x$ becomes more and more negative.
   If $0 < b < 1$, the graph approaches the $x$-axis as $x$ becomes more and more positive.

# Example 5 – Graph of a Function with a Constant Base

Graph $y = 3^{x^2}$.

Solution:

## Compound Interest

- The compound amount *S* of the principal *P* at the end of *n* years at the rate of *r* compounded annually is given by $S = P(1+r)^n$.

# Example 7 – Population Growth

*The population of a town of* 10,000 *grows at the rate of* 2% *per year. Find the population three years from now.*

Solution:

# Example 9 – Population Growth

*The projected population P of a city is given by* $P = 100,000e^{0.05t}$ *where t is the number of years after* 1990. *Predict the population for the year* 2010.

Solution:

# Example 11 – Radioactive Decay

*A radioactive element decays such that after t days the number of milligrams present is given by*

$$N = 100e^{-0.062t}.$$

*a. How many milligrams are initially present?*

Solution:

*b. How many milligrams are present after 10 days?*

Solution:

# 4.2 Logarithmic Functions

- $y = \log_b x$ if and only if $b^y = x$.
- Fundamental equations are $\log_b b^x = x$ and $b^{\log_b x} = x$

## Example 1 – Converting from Exponential to Logarithmic Form

| | *Exponential Form* | | *Logarithmic Form* |
|---|---|---|---|
| a. Since | $5^2 = 25$ | it follows that | $\log_5 25 = 2$ |
| b. Since | $3^4 = 81$ | it follows that | $\log_3 81 = 4$ |
| c. Since | $10^0 = 1$ | it follows that | $\log_{10} 1 = 0$ |

# Example 3 – Graph of a Logarithmic Function with $b > 1$

*Sketch the graph of* $y = \log_2 x$.

Solution:

# Example 5 – Finding Logarithms

a. *Find log 100.*

b. *Find ln 1.*

c. *Find log 0.1.*

d. *Find ln* $e^{-1}$.

d. *Find* $\log_{36} 6$.

- *If a radioactive element has decay constant* $\lambda$, *the half-life of the element is given by*

$$T = \frac{\ln 2}{\lambda}$$

## Example 7 – Finding Half-Life

*A 10-milligram sample of radioactive polonium* 210 (which is denoted $^{210}$Po) *decays according to the equation. Determine the half-life of* $^{210}$Po.

Solution:

# 4.3 Properties of Logarithms

- Properties of logarithms are:

1. $\log_b(mn) = \log_b m + \log_b n$
2. $\log_b \frac{m}{n} = \log_b m - \log_b n$
3. $\log_b m^r = r \log_b m$
4. $\log_b \frac{1}{m} = -\log_b m$
5. $\log_b 1 = 0$
6. $\log_b b = 1$
7. $\log_b m = \frac{\log_a m}{\log_a b}$

## Example 1 – Finding Logarithms

# Example 3 – Writing Logarithms in Terms of Simpler Logarithms

# Example 5 – Simplifying Logarithmic Expressions

# Example 7 – Evaluating a Logarithm Base 5

*Find* $\log_5 2$.

Solution:

# 4.4 Logarithmic and Exponential Equations

- A **logarithmic equation** involves the logarithm of an expression containing an unknown.
- An **exponential equation** has the unknown appearing in an exponent.

# Example 1 – Oxygen Composition

*An experiment was conducted with a particular type of small animal. The logarithm of the amount of oxygen consumed per hour was determined for a number of the animals and was plotted against the logarithms of the weights of the animals. It was found that*

$$\log y = \log 5.934 + 0.885 \log x$$

*where y is the number of microliters of oxygen consumed per hour and x is the weight of the animal (in grams). Solve for y.*

# Solution:

# Example 3 – Using Logarithms to Solve an Exponential Equation

$\mathit{Solve}\; 5+(3)4^{x-1}=12.$

Solution:

# Example 5 – Predator-Prey Relation

*In an article concerning predators and prey, Holling refers to an equation of the form* $y = K(1 - e^{-ax})$ *where x is the prey density, y is the number of prey attacked, and K and a are constants. Verify his claim that*

$$\ln \frac{K}{K - y} = ax$$

Solution: