①

a) What is(are) the endogenous variable(s) in the model? What is(are) the exogenous variable(s) in the model?

$P_x = 8 - bQ_x^d + cP_y; \quad b > 0 \text{ and } c > 0$

$P_x = 20 + dQ_x^S; \quad d > 0$

treated as given

the endogenous variables is P_x, Q^d_x, Q^s_x

the exogenous variables is 8, $-b$, c, 20, d, P_y

$d > 0$; $P_x > 20$

$b > 0$, $c > 0$; $P_x > 8$

b) Specify condition(s) under which the market equilibrium for good x is guaranteed to exist. What restrictions do we need to place on the values of P_y so that the market equilibrium exists?

market of x

$$Q_x^d = \frac{8 - P_x + cP_y}{b} \; ?$$

$$Q_x^s = \frac{-20 + P_x}{d}$$

P_y is an exogenous variables, therefore, we should treated P_y as a fixed number so that market equilibrium exist!

→ complement product x and y using at the same time!

c) Solve for the equilibrium price (P_x^*) and equilibrium quantity (Q_x^*) of the market for good x.

$$Q_x^d = Q_x^s$$

$$\frac{8 - P_x + cP_y}{b} = \frac{-20 + P_x}{d}$$

$$(P_x)^* = -20d - 8b + cbP_y$$

$$(Q_x^*) = \frac{-20 + (-20d - 8b + cbP_y)}{d} \; \#$$

d) Calculate the magnitude of the response of equilibrium quantity to the change in exogenous variable(s).

$$\frac{Q_x^*}{\Delta y} = -20 - 8 = -28 \; \#$$

2. (an old midterm exam question) Consider a market with 10 *identical consumers*. Each of the consumer's demand function is given by:

$$\boxed{P = 10 + k_1 P_x + k_2 Y - Q_j^d,}$$

where P is the unit price of the product sold in this market, Q_j^d is the amount of quantity demanded by the j-th consumer, P_x is the price of product x, and Y is the level of income. Assume further that the industry is controlled by two producers, each of whom has the following supply function:

constant

$$P = 5 + k_3 W + Q_1^s,$$ and

$j = 1, 2, 3, \ldots, 10$

$$P = 20 + k_4 T + 2Q_2^s,$$

constant

where Q_1^s and Q_2^s are the amount of quantity supplied by the first and second producer, respectively. W is the price of gasoline and T is the level of technology. All the parameters are STRICTLY positive.

Use the information given to answer the following questions:

a. Is "product x" considered as a *substitute* product or a *complementary* product?

b. Derive the market demand equation.

Now, I supplement two pieces of information to be used in the remaining parts of this question. That is, I assume that

(i) $0 < (20 + k_4 T) < (5 + k_3 W)$ and (ii) $10 + k_1 P_x + k_2 Y > 5 + k_3 W$

a) From $P = 10 + k_1 P_x + k_2 Y - Q_j^d$

$$Q_j^d = 10 + k_2 Y - P + k_1 P_x$$

because k_1 is positive and an increasing in P_x affect Q_j^d

to increases, so product X is a substitute product

b) $Q_j^d = 10 + k_2 Y - P + k_1 P_X$

$Q_1^d = 10 + k_2 Y - P + k_1 P_X$

$Q_2^d = 10 + k_2 Y - P + k_1 P_X$

$\vdots$

$Q_{10}^d = 10 + k_2 Y - P + k_1 P_X$

$$Q^d = \sum_{i=1}^{10} Q_i^d$$

$$= Q_1^d + Q_2^d + Q_3^d + \ldots + Q_{10}^d$$

$$= 10\left[10 + k_2 Y - P + k_1 P_X\right]$$

$Q^d = 100 + 10k_2 Y - 10P + 10k_1 P_X$ ← The market demand

$P = 5 + k_3 W + Q_1^s,$ (i) $0 < (20 + k_4 T) < (5 + k_3 W)$ and (ii) $10 + k_1 P_x + k_2 Y > 5 + k_3 W$

$P = 20 + k_4 T + 2Q_2^s,$

Find P-intercept ($Q = 0$)

$Q_j^d = 10 + k_2 Y - P + k_1 P_X$

$Q_j^d = 10 + k_1 P_X + k_2 Y - P$

$P = 10 + k_1 P_X + k_2 Y - Q_j^d$

c. What does the first condition mean in terms of the relative cost advantages between the two firms? Given your interpretation, derive the market supply equation.

c) $P = 5 + k_3 W + Q_1^s$

$P - (5 + k_3 W) = Q_1^s$

$Q_1^s = P - (5 + k_3 W)$

because Q_1^s have to higher than zero. $Q_1^s > 0$

so $P - (5 + k_3 W) > 0$

$P > 5 + k_3 W$

$P = 20 + k_4 T + 2Q_2^s$

$$Q_2^s = \frac{P - (20 + k_4 T)}{2}$$

because Q_2^s have to higher than zero. $Q_2^s > 0$

so $\frac{P - (20 + k_4 T)}{2} > 0$

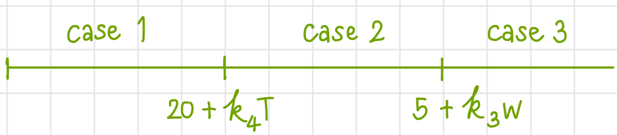
$P - (20 + k_4 T) > 0$

$P > 20 + k_4 T$

Summary : $Q_1^s = P - (5 + k_3 W)$, $P > 5 + k_3 W$

$Q_2^s = \frac{P - (20 + k_4 T)}{2}$, $P > 20 + k_4 T$

due to $0 < 20 + k_4 T < 5 + k_3 W$



case 1 $\quad 0 < P \leq 20 + k_4 T$

$Q^s = Q_1^s + Q_2^s$

$Q^s = 0 + 0 = 0$

case 2 $\quad 20 + k_4 T < P \leq 5 + k_3 W$

$$Q^s = Q_1^s + Q_2^s$$

$$Q^s = 0 + \frac{P - (20 + k_4 T)}{2}$$

$$Q^s = \frac{P - (20 + k_4 T)}{2}$$

case 3 $\quad P < 5 + k_3 W$

$$Q^s = Q_1^s + Q_2^s$$

$$Q^s = \left(P - (5 + k_3 W)\right) + \left(\frac{P - (20 + k_4 T)}{2}\right)$$

$$Q^s = P - 5 - k_3 W + \left(P\frac{1}{2} - 10 + \frac{1}{2} k_4 T\right)$$

$$Q^s = \frac{3}{2} P - k_3 W + \frac{1}{2} k_4 T - 15$$

$$Q^s \begin{cases} 0 & ; \ 0 < P \leq 20 + k_4 T \\ \dfrac{P - (20 + k_4 T)}{2} & ; \ 20 + k_4 T < P \leq 5 + k_3 W \\ \dfrac{3}{2} P - k_3 W + \dfrac{1}{2} k_4 T - 15 & ; \ P > 5 + k_3 W \end{cases}$$

d. Given (i) and (ii), state the condition under which the market ceases to have only *single* firm that stays active in the business?

Find P-intercept $(Q=0)$

$P = 5 + k_3W + Q_1^s$, : $P = 5 + k_3W$

$P = 20 + k_4T + 2Q_2^s$, : $P = 20 + k_4T$

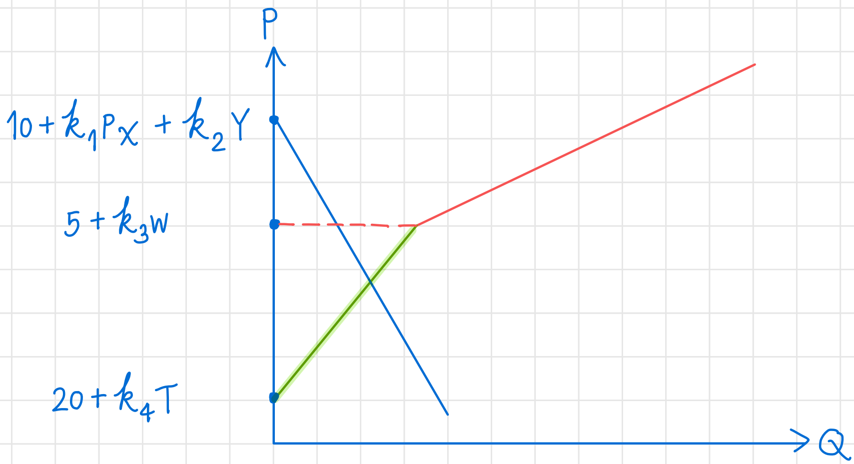
The market demand

$Q^d = 100 + 10k_2Y - 10P + 10k_1P_x$

$0 = 100 + 10k_2Y - 10P + 10k_1P_x$

P-intercept is $P = 10 + k_1P_x + k_2Y$

from (i), (ii)



Due to $20 + k_4T < 5\ k_3W < 10 + k_1P_x + k_2Y$, so P-intercept of the market demand is on the top of the graph, and it affects the market demand to intersect with the market supply that has

$\boxed{\text{only one producer stays active in the } 20 + k_4T < P \leqslant 5 + k_3W.}$

From the market supply in c.) we found that if $P > 5 + k_3 w$ is the price which is two firms can stay active in the business, but if the price decreases till $20 + k_4 T < P \leqslant 5 + k_3 w$, it will has only one firm stays active in the business.

Continue with the information given above, but now consider a specific case where the value of coefficients and exogenous variables are given in the following table:

Coefficients	k_1	k_2	k_3	k_4
Value	2	2	3	1

Variables	Y	P_x	w	T
Value	5	10	10	5

e. Solve for the equilibrium price *and* quantity.

From the market demand equation

$$Q^d = 100 + 10k_2Y - 10P + 10k_1P_x$$

$$Q^d = 100 + 10(2)(5) - 10P + 10(2)(10)$$

$$Q^d = 100 + 100 - 10P + 200$$

$$Q^d = 400 - 10P$$

because $Q^d > 0$

$400 - 10P > 0$

$400 > 10P$

$40 > P$

$P < 40$ so $Q^d = 400 - 10P, \ P < 40$ ____ 1

From the market supply

$$Q^s \begin{cases} 0 & ; \ 0 < P \leq 20 + k_4 T \\ \dfrac{P - (20 + k_4 T)}{2} & ; \ 20 + k_4 T < P \leq 5 + k_3 W \\ \dfrac{3}{2}P - k_3 W + \dfrac{1}{2} k_4 T - 15 & ; \ P > 5 + k_3 W \end{cases}$$

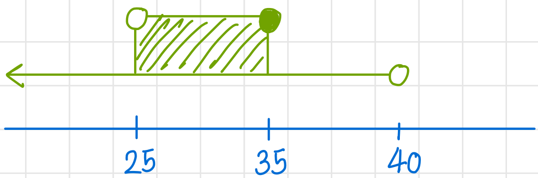
$$Q^s \begin{cases} 0 & ; \ 0 < P \leq 20 + (1)(5) \\ \dfrac{P - (20 + (1)(5))}{2} & ; \ 20 + (1)(5) < P \leq 5 + (3)(10) \\ \dfrac{3}{2}P - (3)(10) + \dfrac{1}{2}(1)(5) - 15 & ; \ P > 5 + (3)(10) \end{cases}$$

$$Q^s = \begin{cases} 0 & ; \; 0 < P \leq 25 \\ \frac{1}{2}P - \frac{25}{2} & ; \; 25 < P \leq 35 \\ \frac{3}{2}P - \frac{95}{2} & ; \; P > 35 \end{cases}$$

The equilibrium occurs when $Q^d = Q^s$

<u>case 1</u> $Q^d = 400 - 10P \quad ; \; P < 40$

$Q^s = \frac{1}{2}P - \frac{25}{2} \quad ; \; 25 < P \leq 35$



$$Q^d = Q^s$$

$$400 - 10P = \frac{1}{2}P - \frac{25}{2}$$

$$400 + \frac{25}{2} = \frac{1}{2}P + 10P$$

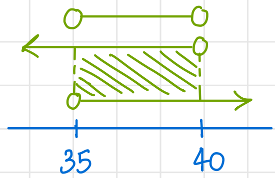
$$400 + 12.5 = 0.5P + 10P$$

$$412.5 = 10.5P$$

$$P = 39.2857$$

cannot because → $25 < \underline{\underline{P}} \leq 35$

case 2 $\quad Q^d = 400 - 10P \;;\; P < 40$



$$Q^s = \frac{3}{2}P - \frac{95}{2} \;;\; P > 35$$

$$Q^d = Q^s$$

$$400 - 10P = \frac{3}{2}P - \frac{95}{2}$$

$$400 + \frac{95}{2} = \frac{3}{2}P + 10P$$

$$400 + 47.5 = 1.5P + 10P$$

$$447.5 = 11.5P$$

$$P^* = 38.9130$$ ✓✓ due to $35 < P < 40$

plug in $P = 38.9130$ into $Q^d = 400 - 10P$

$$Q^d = 400 - 10(38.9130)$$

$$Q^d = 400 - 389.130$$

$$Q^* = 10.870$$

The equilibrium price = \$ 38.9130

The equilibrium quantity = 10.870 unit

f. Suppose that the government provides a subsidy of \$5 for each unit of output that the consumers have purchased. Calculate the benefit that consumers and each of the two producers receive under the subsidy program.

When the government provides a subsidy of \$5 for each unit

from case 2 $\quad Q^d = 400 - 10P^d$ ____ (1) $\quad$ when P^d = price per unit of consumer

$Q^s = \frac{3}{2}P^s - \frac{95}{2}$ ____ (2) $\quad$ P^s = price per unit of producer

and $P^s = P^d$ + subsidy

$P^s = P^d + 5$ ____ (3)

plug in P^s into (2)

$Q^s = \frac{3}{2}P^s - \frac{95}{2}$

$Q^s = \frac{3}{2}(P^d + 5) - \frac{95}{2}$

$Q^s = \frac{3}{2}P^d + \frac{15}{2} - \frac{95}{2}$

$Q^s = \frac{3}{2}P^d - 40$ ____ (4)

The market equilibrium occurs when

from (1) $\quad Q^d = Q^s \quad$ from (4)

$400 - 10P^d = \frac{3}{2}P^d - 40$

$400 + 40 = \frac{3}{2}P^d + 10P^d$

$$440 = 11.5\,P^d$$

$$\boxed{38.2609 = P^d}$$

plug in P^d into 3

$$P^s = P^d + 5$$

$$P^s = 38.2609 + 5$$

$$\boxed{P^s = 43.2609}$$

plug in P^d into 4 to find quantity

$$Q^s = \frac{3}{2}P^d - 40$$

$$Q^s = \frac{3}{2}(38.2609) - 40$$

$$\boxed{Q^* = 17.3914}$$

The benefit that consumers and each of the two producers receive

For consumers, price per unit that consumers have to pay before the subsidy is \$ 38.9130, but the price per unit after the subsidy is \$ 38.2609, so the benefit that the consumers received will be \$ 0.6521 per unit.

For producers, price per unit that the producers sell before the subsidy is \$ 38.9130, but the price per unit after the subsidy is \$ 43.2609, so the benefit that the producers received will be \$ 6.5214 per unit.

3. Consider a simple macroeconomics model given below

$C = 0.3Y_d - k_1 r;$ $\quad k_1 > 0.$

$I = 0.5Y - k_2 r;$ $\quad k_2 > 0.$

$Y_d = Y - T$

$G = G_0$

$T = T_0$

$M^d = k_3 Y - k_4 r;$ $\quad k_3 > 0 \;\; and \;\; k_4 > 0.$

$M^s = M_0$

where $Y = GDP,\;\; C = consumption,\;\; I = investment,\;\; G = government\ purchase,\;\; T = tax,\;\; r = interest\ rate,\;\; M^d = money\ demand,\ and\ M^s = money\ supply$

a. Write the system of linear equations in matrix form with two variables included, namely Y and r.

A) goods market

$Y = C + I + G$

$Y = 0.3(Y - T) - k_1 r + 0.5Y - k_2 r + G_0$

$Y = 0.3Y - 0.3T - k_1 r + 0.5Y - k_2 r + G_0$

$Y = \frac{1}{0.2} \cdot (-0.3T - k_1 r - k_2 r + G_0)$

$Y = -1.5T_0 - 5k_1 r - 5k_2 r + 5G_0$

$Y = -1.5T_0 - 5r(k_1 + k_2) + 5G_0$

$Y + r(5k_1 + 5k_2) = -1.5T_0 + 5G_0$

→ Money market

$M^d = M^s$

$k_3 Y - k_4 r = M_0$

$$\begin{bmatrix} 1 & 5k_1 + 5k_2 \\ k_3 & -k_4 \end{bmatrix} \begin{bmatrix} Y \\ r \end{bmatrix} = \begin{bmatrix} -1.5T_0 + 5G_0 \\ M_0 \end{bmatrix}$$

b. Solve for the equilibrium solution (Y^*, r^*)

$$\frac{M^S}{P} = L^d$$
$$M^S = M^d$$
$$M_0 = k_3 Y - k_4 r$$
$$r^* = \frac{k_3 Y - M_0}{k_4}$$

Equilibrium: $Y = -1.5T + 5G_0 - 5 \cdot \overbrace{\left(\frac{k_3 Y - M_0}{k_4}\right)}^{r^*} \cdot (k_1 + k_2)$

$$= -1.5T + 5G_0 - 5k_1\left(\frac{k_3 Y - M_0}{k_4}\right) - 5k_2\left(\frac{k_3 Y - M_0}{k_4}\right)$$

$$= -1.5T + 5G_0 - \frac{5k_1k_3Y + 5k_1M_0}{k_4} - \frac{5k_2k_3Y + 5k_2M_0}{k_4}$$

$$= -1.5T + 5G_0 - \frac{5k_1k_3Y}{k_4} + \frac{5k_1M_0}{k_4} - \frac{5k_2k_3Y}{k_4} + \frac{5k_2M_0}{k_4}$$

$$Y + \frac{5k_1k_3Y}{k_4} + \frac{5k_2k_3Y}{k_4} = -1.5T + 5G_0 + \frac{5k_1M_0}{k_4} + \frac{5k_2M_0}{k_4}$$

$$Y\left(1 + \frac{5k_1k_3}{k_4} + 5\frac{k_2k_3}{k_4}\right) = -1.5T + 5G_0 + \frac{5k_1M_0}{k_4} + \frac{5k_2M_0}{k_4}$$

$$Y^* = \frac{-1.5T + 5G_0 + \frac{5k_1M_0}{k_4} + \frac{5k_2M_0}{k_4}}{1 + \frac{5k_1k_3}{k_4} + \frac{5k_2k_3}{k_4}}$$

$$Y^* = \frac{\frac{-1.5Tk_4 + 5G_0k_4 + 5k_1M_0 + 5k_2M_0}{k_4}}{\frac{k_4 + 5k_1k_3 + 5k_2k_3}{k_4}}$$

$$= \frac{-1.5Tk_4 + 5G_0 + 5k_1M_0 + 5k_2M_0}{k_4} \times \frac{k_4}{k_4 + 5k_1k_3 + 5k_2k_3}$$

$$Y^* = \frac{-1.5Tk_4 + 5G_0k_4 + 5k_1M_0 + 5k_2M_0}{k_4 + 5k_1k_3 + 5k_2k_3}$$

c. Has fiscal policy, for example, the change in government purchase, become *more effective* if both k_1 and k_2 are getting bigger? Show your numerical result, and explain your result with some economic intuitions. (Hint: how do we measure the effectiveness of a policy?)

$$Y^* = \frac{-1.5Tk_4 + 5G_0k_4 + 5k_1M_0 + 5k_2M_0}{k_4 + 5k_1k_3 + 5k_2k_3}$$

$$\frac{\partial Y^*}{\partial G_0} = \frac{5k_4}{k_4 + 5k_1k_3 + 5k_2k_3}$$

If k_1 and k_2 is getting larger, Y^* will fall.

So, No it hasn't fiscal policy become more efficitive.

4. Let the demand function be $P = 14 - 3Q$ and the supply function be $P = 4 + 2Q$. Suppose that the government imposes tax by \$t per unit of output. This tax is assumed to impose on consumer. Answer the following questions.

a. Find the equilibrium under pre-tax situation. That is, when "t" is set to equal to zero.

Demand = supply

$$14 - 3Q = 4 + 2Q$$

$$Q^* = 2$$

$$14 - 3(Q) = 14 - 3(2)$$

$$P^* = 8$$

b. State the condition that links between consumer's and producer's price

Demand

$$P = 14 - 3Q$$

$$Q^d = \frac{14 - P}{3}$$

$$Q^d = \begin{cases} 0, & P \geq 14 \\ \frac{14-P}{3}, & 0 \leq P < 14 \end{cases}$$

Supply

$$P = 4 + 2Q^S$$

$$Q^S = \frac{P - 4}{2}$$

$$Q^S = \begin{cases} 0, & 0 \leq P < 4 \\ \frac{P-4}{2}, & P > 4 \end{cases}$$

c. Find the equilibrium after tax. (Hint: your solution should be written in terms of "t".)

Tax on consumer $\rightarrow P^d = P^S + t$

$$4 + 2Q + t = 14 - 3Q$$

$$5Q + t = 10$$

$$t = 10 - 5Q$$

$\because Q^*$ after tax $= \frac{10-t}{5}$ —* sub to demand function b/f tax

$$P^{d*} = 14 - \left(\frac{10-t}{5} \times 3\right)$$

$$= 14 - \frac{30 + 3t}{5}$$

P^{d*} after tax $= \frac{40+3t}{5} = 8 + \frac{3t}{5}$

$\because Q^*$ after tax $= \frac{10-t}{5}$ —* sub to supply function b/f tax

P^{S*} a/f tax $= 4 + 2\left(\frac{10-t}{5}\right)$

$$= 8 - \frac{2t}{5}$$

d. Calculate the consumers' and producers' burden. Which group is paying more for the tax under the equilibrium?

Tax burden $= P^d$ alf tax $+ P^s$ alf tax

$$= (8+\frac{3}{5}t) + (8-\frac{2}{5}t) = 16+\frac{t}{5}$$

producer's burden $= \frac{2}{5} \times 100 = 40\%$

consumer's burden $= \frac{3}{5} \times 100 = 60\%$

$\therefore$ consumer paying more for tax under the equilibrium.

e. Find the expression of the revenue that the government can collect from the market under the equilibrium.

Tax revenue = (\$ t / unit) × (unit output sold)

$$= t \times \left(\frac{10-t}{5}\right)$$

$$= \frac{10t - t^2}{5}$$

f. If the government were to collect tax so that total revenue is maximized, what is the appropriate level of unit tax, "t", that it should impose to the market?

$$P^d = a - bQ^d = 14 - 3Q^d$$

$$P^s = c + dQ^s = 4 + 2Q^s$$

$$t^* = \frac{-\left(\frac{a-c}{b+d}\right)}{2\left(\frac{-1}{b+d}\right)}$$

* t maximizes T(t) *

$$= \frac{-\left(\frac{14-4}{-3+2}\right)}{2\left(\frac{-1}{3+2}\right)} = 5$$