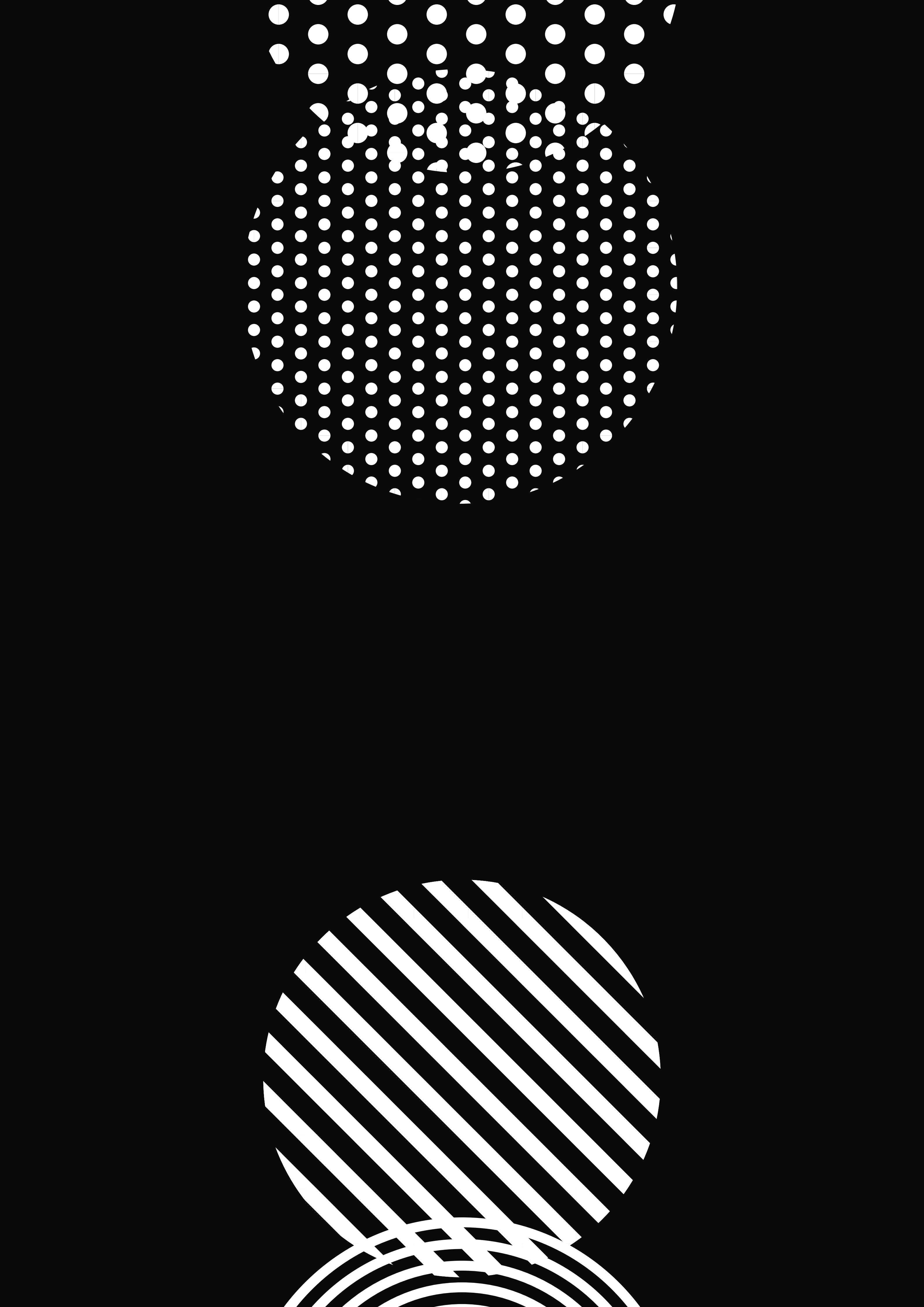


a.) Find β_1 , β_2

Ans: $\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$

$$\hat{\beta}_2 = \frac{\sum (x_i - \bar{x})(Y_i - \bar{Y})}{\sum (x_i - \bar{x})^2}$$

$$\hat{\beta}_1 = 21.03 - (-0.15854)(12.2) = 22.9642$$

$$\hat{\beta}_2 = \frac{-174.2}{1098.8} = -0.15854$$

This mean that $\hat{Y}_i = 22.9642 - 0.15854X + \hat{u}$

The interception of Y is at 22.9642 and when X is increase by 1 unit Y will decrease by 0.15854 unit.

b.) $$r^2 = \frac{ESS}{TSS} = \frac{\sum (\hat{Y}_i - \bar{Y})^2}{\sum (Y_i - \bar{Y})^2}$$

$$r^2 = \frac{(\sum x_i y_i)^2}{\sum x_i^2 \sum y_i^2}$$

$$r^2 = 1 - \frac{RSS}{TSS} = 1 - \frac{\sum \hat{u}_i^2}{\sum (Y_i - \bar{Y})^2}$$

$$RSS = \sum \hat{u}_i^2$$

$$r^2 = 1 - \frac{\sum \hat{u}_i^2}{\sum (Y_i - \bar{Y})^2} = 1 - \left(\frac{873.14}{882.97}\right) = 0.011133$$

C.) $X_i = 5$

$$\hat{Y}_i = 22.9642 - 0.15854X + \hat{u}$$

when $X_i = 5$ $\hat{Y}_i$ will decrease by 0.7927 unit

D.) Find estimators of $var(u_i)$, $var(\hat{\beta}_1)$, $var(\hat{\beta}_2)$

$$Var(u_i) = \frac{\sum_i (Y_i - \hat{Y})^2}{n-k} = \hat{\sigma}^2$$

$$Var(\hat{\beta}_1) = \frac{\sum_i X_i^2}{n\sum_i x_i^2}\sigma^2 \rightarrow \frac{\sum_i X_i^2}{n\sum_i x_i^2} \cdot \frac{\sum \hat{u}_i^2}{n-k} = \sigma^2_{\hat{\beta}_1}$$

$$Var(\hat{\beta}_2) = \frac{\sum \hat{u}_i^2}{(n-k)(\sum_i x_i^2)} = \sigma^2_{\hat{\beta}_2}$$

e.) $\alpha = 0.05$ $\quad CI = 95\%$

$$\hat{Y}_i = \underset{(1.01953)}{22.9642} - \underset{(0.07486)}{0.15854}X + \hat{u}$$

$$\sigma^2_{\hat{\beta}_1} := \frac{\sum x_i^2}{n \sum x_i^2}\sigma^2 \qquad ; \qquad \hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-k} = \frac{\sum (Y_i - \hat{Y})^2}{n-k}$$

$$\sigma^2_{\hat{\beta}_2} := \frac{\sigma^2}{\sum x_i^2}$$

$$\sigma^2_{\hat{\beta}_1} = \frac{(5564)}{(30)(5564)} \cdot \frac{(873.14)}{30-2} = 1.03945$$

$$\sigma^2_{\hat{\beta}_2} = \frac{873.14}{(28)(5564)} = 0.00560452$$

$H_0: \beta_1 = 0$

$H_1: \beta_1 \neq 0$

$$t_{\frac{0.05}{2}} = t_{0.025}$$

$n = 30$

$k = 2$

$df = 28$

$$t_{0.025} = 2.048$$

$$t_{cal} = \frac{\hat{\beta}_1 - \beta_1}{\sigma_{\hat{\beta}_1}} = \frac{22.9642 - 0}{1.01953} = 22.5243$$

$$\hat{\beta}_1 \pm t_{0.025} \cdot \sigma_{\hat{\beta}_1} \longrightarrow$$

<u>upper limit</u>

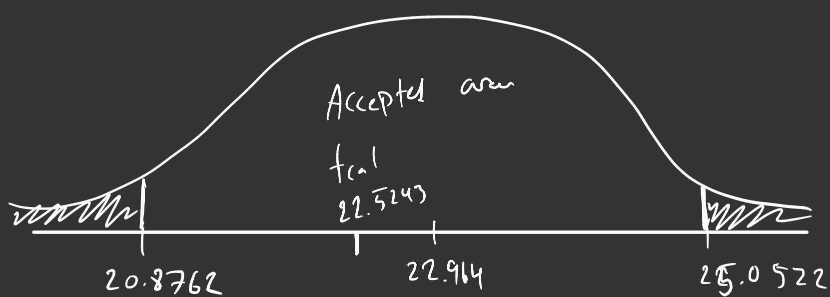
$22.9642 + (2.048 \cdot 1.01953)$

$= 25.0522$

<u>lower limit</u>

$22.9642 - (2.048 \cdot 1.01953)$

$= 20.87620256$



$H_0 : \beta_2 = 0$

$H_1 : \beta_2 \neq 0$

$\hat{\beta}_2 \pm \left(t_{0.025} \cdot \sigma_{\hat{\beta}_2}\right)$ $\qquad t_{0.025} = 2.048$ $\qquad n = 30$, $k = 2$, $df = 28$

$t_{cal} = \dfrac{\hat{\beta}_2 - \beta_2}{\sigma_{\hat{\beta}_2}} = \dfrac{-0.15854 - 0}{0.07486}$

$t_{cal} = 2.1178$

upper limit

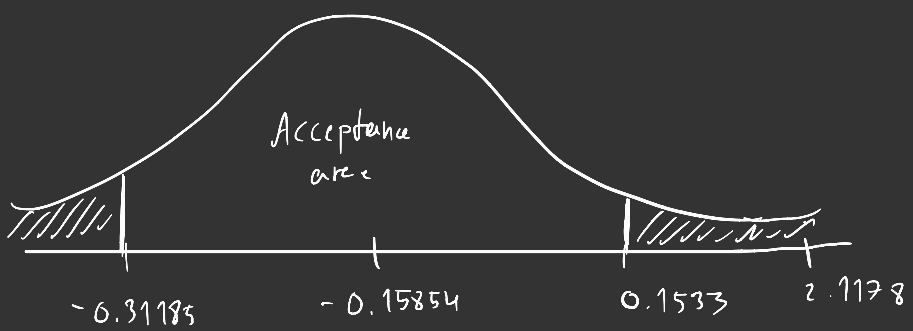
$-0.15854 + (2.048 \cdot 0.07486)$

$= 0.1533$

lower limit

$-0.15854 - (2.048 \cdot 0.07486)$

$= -0.31185$



Answer : At 95 percent confident interval, we can conclude that

for β_1 we can not reject the null hypothesis

but for the β_2 we can reject the null hypothesis

F.) $\hat{Y}_i = 22.9642 - 0.15854X + \hat{u}$

$\quad\quad\;(1.01953) \quad\; (0.07486)$

$\alpha = 0.01$

$CI = 99\%$

$t_{\frac{0.01}{2}} = t_{0.005}$

$t_{cal} =$ $t_{cal,\hat{\beta}_1} = 22.5243$

$t_{cal,\hat{\beta}_2} = 2.1178$

$t_{0.005} = 2.763$

$n = 30$

$k = 2$

$df = 28$

$$\boxed{\hat{\beta} \pm (t_{0.025} \cdot \sigma_{\hat{\beta}_2})}$$

$H_0 : \beta_1 = 0$

$H_1 : \beta_1 \neq 0$

upper limit

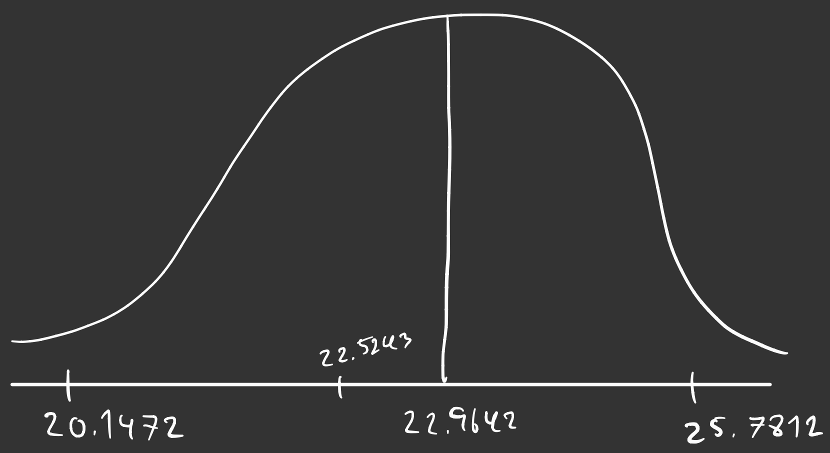
$22.9642 + (2.763 \cdot 1.01953)$

$= 25.7812$

lower limit

$22.9642 - (2.763 \cdot 1.01953)$

$= 20.1472$



$H_0 : \beta_2 = 0$

$H_0 : \beta_2 \neq 0$

upper limit

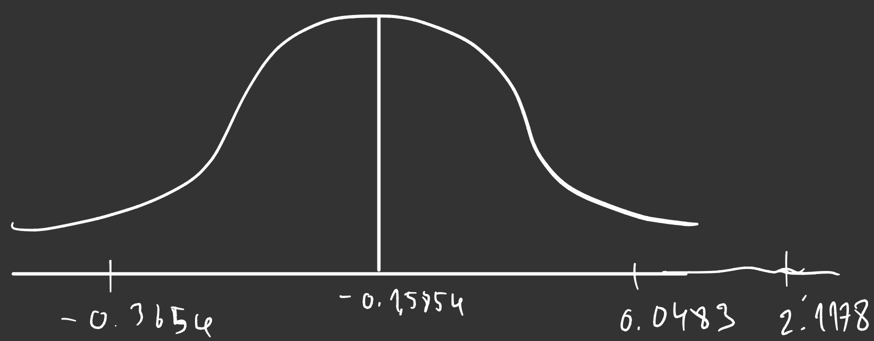
$-0.15854 + (2.763 \cdot 0.07486)$

$= 0.0483$

lower limit

$-0.15854 - (2.763 \cdot 0.07486)$

$= -0.3654$



In concludsion, for β_1 we accept the null hypothesis but for β_2 we reject the null hypothesis with 99.9 Percent confident

(2) $\hat{Y}_i = 7836 - 502.4X_i$
$\quad\;\; (52) \quad\;\; (411.8)$

$n = 11 \qquad df = 9$

$k = 2$

(a) Yes, because when the car get older and older the quality of it Machine part Should decrese in quality hence, when it get older the market Price Should be fall down.

(b) on the average this car price in the market should be

$\widehat{\text{Market price}} = 7{,}836 - 502.4(5) = 5324$ USD

$\alpha = 0.05 \qquad t_{\frac{0.05}{2}} \rightarrow \boxed{t_{0.025} = 2.262}$

$$P\left[\hat{\beta}_2 - (t_{\frac{\alpha}{2}} \cdot \sigma_{\hat{\beta}_2}) \leq \beta_2 \leq \hat{\beta}_2 + (t_{\frac{\alpha}{2}} \cdot \sigma_{\hat{\beta}_2})\right]$$

$$= P\left[502.4 - (2.262 \cdot 411.8) \leq 502.4 \leq 502.4 + (2.262 \cdot 411.8)\right]$$

$$= P\left[-429.4916 \leq 502.4 \leq 1433.4916\right] \times 5$$

$$= P\left[-2147.458 \leq 2512 \leq 7{,}167.458\right]$$

when passing 5 year the price of this car should be in between -2147.458 and 7167.458 USD with 95 percent confident

(c) $\hat{Y}_i = 7836 - 502.4x_i$ $\quad n = 11 \quad df = 9$

$(52) \quad (411.8)$

$k = 2$

$\times 10$

$\rightarrow \hat{Y}_i = 78360 - 5024\, x_i$

$$\sum (x_i - \bar{x})^2 = 78.73$$

$$\sum |x_i - \bar{x}| = \sqrt{78.73} = 8.8730$$

$$\sum x_i - \bar{x} = 8.8730$$

Since $\bar{x} = 7.45$ so $\sum x_i = 8.8730 - 7.45$

$$\sum x_i = 1.423$$

$$\sum x_i^2 = 2.0250$$

$$\sigma^2_{\hat{\beta}_1} = \frac{2.0250}{11 \cdot 2.0250} \cdot 212{,}877 = 19352.4546$$

$$\sigma^2_{\hat{\beta}_2} = \frac{212{,}877}{2.0250} = 105124.4444$$

$$\boxed{\begin{aligned} \sigma_{\hat{\beta}_1} &= 139.1131 \\ \sigma_{\hat{\beta}_2} &= 324.2290 \end{aligned}}$$

$\uparrow$

Ansewe: ① se of $\hat{\beta}_1$ and $\hat{\beta}_2$

② $\hat{Y} = 78360 - 5024\, x_i$

(D) elasticity

$$\hat{Y}_i = \underset{(52)}{7836} - \underset{(411.8)}{502.4} x_i$$

$$\frac{\partial \ln \hat{Y}_i}{\partial \ln x_i} = -502.4 = \frac{\frac{d \hat{Y}_i}{\hat{Y}_i}}{\frac{d x_i}{x_i}} = \frac{d \hat{Y}_i}{d x_i} \cdot \frac{x_i}{\hat{Y}_i}$$

$$= \underbrace{\frac{\% \Delta \text{ Market Price}}{\% \Delta \text{ year}}}_{\downarrow} \cdot \frac{\text{year}}{\text{Market price}}$$

$$= -502.4 \cdot \frac{10}{2812}$$

$7836 - 502.4(10)$

$$= -1.7866$$

Answer: when the car age increase by 1 unit market price will decrease by 1.7866 percent.