Question Book for Time Value of Money

Self-Test Problems

(ST–1) Future Value

Assume that 1 year from now you plan to deposit $1,000 in a savings account that pays a nominal rate of 8%.

a. If the bank compounds interest annually, how much will you have in your account 4 years from now?
b. What would your balance be 4 years from now if the bank used quarterly compounding rather than annual compounding?
c. Suppose you deposited the $1,000 in 4 payments of $250 each at the end of Years 1, 2, 3, and 4. How much would you have in your account at the end of Year 4, based on 8% annual compounding?
d. Suppose you deposited 4 equal payments in your account at the end of Years 1, 2, 3, and 4. Assuming an 8% interest rate, how large would each of your payments have to be for you to obtain the same ending balance as you calculated in part a?

(ST–2) Time Value of Money

Assume that 4 years from now you will need $1,000. Your bank compounds interest at an 8% annual rate.

a. How much must you deposit 1 year from now to have a balance of $1,000 at Year 4?
b. If you want to make equal payments at the end of Years 1 through 4 to accumulate the $1,000, how large must each of the 4 payments be?
c. If your father were to offer either to make the payments calculated in part b ($221.92) or to give you a lump sum of $750 one year from now, which would you choose?
d. If you will have only $750 at the end of Year 1, what interest rate, compounded annually, would you have to earn to have the necessary $1,000 at Year 4?
e. Suppose you can deposit only $186.29 each at the end of Years 1 through 4, but you still need $1,000 at the end of Year 4. What interest rate, with annual compounding, is required to achieve your goal?
f. To help you reach your $1,000 goal, your father offers to give you $400 one year from now. You will get a part-time job and make 6 additional deposits of equal amounts each 6 months thereafter. If all of this money is deposited in a bank that pays 8%, compounded semiannually, how large must each of the 6 deposits be?
g. What is the effective annual rate being paid by the bank in part f?

(ST–3) Effective Annual Rates

Bank A pays 8% interest, compounded quarterly, on its money market account. The managers of Bank B want its money market account's effective annual rate to equal that of Bank A, but Bank B will compound interest on a monthly basis. What nominal, or quoted, rate must Bank B set?

Problems

EASY PROBLEMS 1–8

(4–1) Future Value of a Single Payment

If you deposit $10,000 in a bank account that pays 10% interest annually, how much will be in your account after 5 years?

(4–2) Present Value of a Single Payment

What is the present value of a security that will pay $5,000 in 20 years if securities of equal risk pay 7% annually?

(4–3) Interest Rate on a Single Payment

Your parents will retire in 18 years. They currently have $250,000, and they think they will need $1 million at retirement. What annual interest rate must they earn to reach their goal, assuming they don't save any additional funds?

(4–4) Number of Periods of a Single Payment

If you deposit money today in an account that pays 6.5% annual interest, how long will it take to double your money?

(4–5) Number of Periods for an Annuity

You have $42,180.53 in a brokerage account, and you plan to deposit an additional $5,000 at the end of every future year until your account totals $250,000. You expect to earn 12% annually on the account. How many years will it take to reach your goal?

(4–6) Future Value: Ordinary Annuity versus Annuity Due

What is the future value of a 7%, 5-year ordinary annuity that pays $300 each year? If this were an annuity due, what would its future value be?

(4–7) Present and Future Value of an Uneven Cash Flow Stream

An investment will pay $100 at the end of each of the next 3 years, $200 at the end of Year 4, $300 at the end of Year 5, and $500 at the end of Year 6. If other investments of equal risk earn 8% annually, what is this investment's present value? Its future value?

(4–8)
Annuity Payment and EAR

INTERMEDIATE PROBLEMS
9–29

You want to buy a car, and a local bank will lend you $20,000. The loan would be fully amortized over 5 years (60 months), and the nominal interest rate would be 12%, with interest paid monthly. What is the monthly loan payment? What is the loan's EFF%?

(4–9)
Present and Future Values of Single Cash Flows for Different Periods

Find the following values, *using the equations*, and then work the problems using a financial calculator to check your answers. Disregard rounding differences. (*Hint:* If you are using a financial calculator, you can enter the known values and then press the appropriate key to find the unknown variable. Then, without clearing the TVM register, you can "override" the variable that changes by simply entering a new value for it and then pressing the key for the unknown variable to obtain the second answer. This procedure can be used in parts b and d, and in many other situations, to see how changes in input variables affect the output variable.)

a. An initial $500 compounded for 1 year at 6%
b. An initial $500 compounded for 2 years at 6%
c. The present value of $500 due in 1 year at a discount rate of 6%
d. The present value of $500 due in 2 years at a discount rate of 6%

(4–10)
Present and Future Values of Single Cash Flows for Different Interest Rates

Use both the TVM equations and a financial calculator to find the following values. See the Hint for Problem 4-9.

a. An initial $500 compounded for 10 years at 6%
b. An initial $500 compounded for 10 years at 12%
c. The present value of $500 due in 10 years at a 6% discount rate
d. The present value of $500 due in 10 years at a 12% discount rate

(4–11)
Time for a Lump Sum to Double

To the closest year, how long will it take $200 to double if it is deposited and earns the following rates? [*Notes:* (1) See the Hint for Problem 4-9. (2) This problem cannot be solved exactly with some financial calculators. For example, if you enter PV = –200, PMT = 0, FV = 400, and I = 7 in an HP-12C and then press the N key, you will get 11 years for part a. The correct answer is 10.2448 years, which rounds to 10, but the calculator rounds up. However, the HP-10B gives the exact answer.]

a. 7%
b. 10%
c. 18%
d. 100%

(4–12)
Future Value of an Annuity

Find the *future value* of the following annuities. The first payment in these annuities is made at the *end* of Year 1, so they are *ordinary annuities.* (*Notes:* See the Hint to Problem 4-9. Also, note that you can leave values in the TVM register, switch to Begin Mode, press FV, and find the FV of the annuity due.)

a. $400 per year for 10 years at 10%
b. $200 per year for 5 years at 5%
c. $400 per year for 5 years at 0%
d. Now rework parts a, b, and c assuming that payments are made at the *beginning* of each year; that is, they are *annuities due.*

(4–13) Present Value of an Annuity

Find the *present value* of the following *ordinary annuities* (see the Notes to Problem 4-12).

a. $400 per year for 10 years at 10%
b. $200 per year for 5 years at 5%
c. $400 per year for 5 years at 0%
d. Now rework parts a, b, and c assuming that payments are made at the *beginning* of each year; that is, they are *annuities due.*

(4–14) Uneven Cash Flow Stream

Find the present values of the following cash flow streams. The appropriate interest rate is 8%. (*Hint:* It is fairly easy to work this problem dealing with the individual cash flows. However, if you have a financial calculator, read the section of the manual that describes how to enter cash flows such as the ones in this problem. This will take a little time, but the investment will pay huge dividends throughout the course. Note that, when working with the calculator's cash flow register, you must enter $CF_0 = 0$. Note also that it is quite easy to work the problem with *Excel*, using procedures described in the Chapter 4 ***Tool Kit***.)

Year	Cash Stream A	Cash Stream B
1	$100	$300
2	400	400
3	400	400
4	400	400
5	300	100

b. What is the value of each cash flow stream at a 0% interest rate?

(4–15) Effective Rate of Interest

Find the interest rate (or rates of return) in each of the following situations.

a. You *borrow* $700 and promise to pay back $749 at the end of 1 year.
b. You *lend* $700 and receive a promise to be paid $749 at the end of 1 year.
c. You borrow $85,000 and promise to pay back $201,229 at the end of 10 years.
d. You borrow $9,000 and promise to make payments of $2,684.80 at the end of each of the next 5 years.

(4–16) Future Value for Various Compounding Periods

Find the amount to which $500 will grow under each of the following conditions.

a. 12% compounded annually for 5 years
b. 12% compounded semiannually for 5 years
c. 12% compounded quarterly for 5 years
d. 12% compounded monthly for 5 years

(4–17) Present Value for Various Compounding Periods

Find the present value of $500 due in the future under each of the following conditions.

a. 12% nominal rate, semiannual compounding, discounted back 5 years
b. 12% nominal rate, quarterly compounding, discounted back 5 years
c. 12% nominal rate, monthly compounding, discounted back 1 year

(4–18) Future Value of an Annuity for Various Compounding Periods

Find the future values of the following ordinary annuities.

a. FV of $400 each 6 months for 5 years at a nominal rate of 12%, compounded semiannually
b. FV of $200 each 3 months for 5 years at a nominal rate of 12%, compounded quarterly

c. The annuities described in parts a and b have the same total amount of money paid into them during the 5-year period, and both earn interest at the same nominal rate, yet the annuity in part b earns $101.75 more than the one in part a over the 5 years. Why does this occur?

(4–19) Effective versus Nominal Interest Rates

Universal Bank pays 7% interest, compounded annually, on time deposits. Regional Bank pays 6% interest, compounded quarterly.

a. Based on effective interest rates, in which bank would you prefer to deposit your money?
b. Could your choice of banks be influenced by the fact that you might want to withdraw your funds during the year as opposed to at the end of the year? In answering this question, assume that funds must be left on deposit during an entire compounding period in order for you to receive any interest.

(4–20) Amortization Schedule

a. Set up an amortization schedule for a $25,000 loan to be repaid in equal installments at the end of each of the next 5 years. The interest rate is 10%.
b. How large must each annual payment be if the loan is for $50,000? Assume that the interest rate remains at 10% and that the loan is still paid off over 5 years.
c. How large must each payment be if the loan is for $50,000, the interest rate is 10%, and the loan is paid off in equal installments at the end of each of the next 10 years? This loan is for the same amount as the loan in part b, but the payments are spread out over twice as many periods. Why are these payments not half as large as the payments on the loan in part b?

(4–21) Growth Rates

Sales for Hanebury Corporation's just-ended year were $12 million. Sales were $6 million 5 years earlier.

a. At what rate did sales grow?
b. Suppose someone calculated the sales growth for Hanebury in part a as follows: "Sales doubled in 5 years. This represents a growth of 100% in 5 years; dividing 100% by 5 results in an estimated growth rate of 20% per year." Explain what is wrong with this calculation.

(4–22) Expected Rate of return

Washington-Pacific invested $4 million to buy a tract of land and plant some young pine trees. The trees can be harvested in 10 years, at which time W-P plans to sell the forest at an expected price of $8 million. What is W-P's expected rate of return?

(4–23) Effective Rate of Interest

A mortgage company offers to lend you $85,000; the loan calls for payments of $8,273.59 at the end of each year for 30 years. What interest rate is the mortgage company charging you?

(4–24) Required Lump-Sum Payment

To complete your last year in business school and then go through law school, you will need $10,000 per year for 4 years, starting next year (that is, you will need to withdraw the first $10,000 one year from today). Your rich uncle offers to put you through school, and he will deposit in a bank paying 7% interest a sum of money that is sufficient to provide the 4 payments of $10,000 each. His deposit will be made today.

a. How large must the deposit be?
b. How much will be in the account immediately after you make the first withdrawal? After the last withdrawal?

(4–25) Repaying a Loan

While Mary Corens was a student at the University of Tennessee, she borrowed $12,000 in student loans at an annual interest rate of 9%. If Mary repays $1,500 per year, then how long (to the nearest year) will it take her to repay the loan?

(4–26) Reaching a Financial Goal

You need to accumulate $10,000. To do so, you plan to make deposits of $1,250 per year—with the first payment being made a year from today—into a bank account that pays 12% annual interest. Your last deposit will be less than $1,250 if less is needed to round out to $10,000. How many years will it take you to reach your $10,000 goal, and how large will the last deposit be?

(4–27) Present Value of a Perpetuity

What is the present value of a perpetuity of $100 per year if the appropriate discount rate is 7%? If interest rates in general were to double and the appropriate discount rate rose to 14%, what would happen to the present value of the perpetuity?

(4–28) PV and Effective Annual Rate

Assume that you inherited some money. A friend of yours is working as an unpaid intern at a local brokerage firm, and her boss is selling securities that call for 4 payments of $50 (1 payment at the end of each of the next 4 years) plus an extra payment of $1,000 at the end of Year 4. Your friend says she can get you some of these securities at a cost of $900 each. Your money is now invested in a bank that pays an 8% nominal (quoted) interest rate but with quarterly compounding. You regard the securities as being just as safe, and as liquid, as your bank deposit, so your required effective annual rate of return on the securities is the same as that on your bank deposit. You must calculate the value of the securities to decide whether they are a good investment. What is their present value to you?

(4–29) Loan Amortization

Assume that your aunt sold her house on December 31, and to help close the sale she took a second mortgage in the amount of $10,000 as part of the payment. The mortgage has a quoted (or nominal) interest rate of 10%; it calls for payments every 6 months, beginning on June 30, and is to be amortized over 10 years. Now, 1 year later, your aunt must inform the IRS and the person who bought the house about the interest that was included in the two payments made during the year. (This interest will be income to your aunt and a deduction to the buyer of the house.) To the closest dollar, what is the total amount of interest that was paid during the first year?

Challenging Problems 30–34

(4–30) Loan Amortization

Your company is planning to borrow $1 million on a 5-year, 15%, annual payment, fully amortized term loan. What fraction of the payment made at the end of the second year will represent repayment of principal?

(4–31) Nonannual Compounding

a. It is now January 1. You plan to make a total of 5 deposits of $100 each, one every 6 months, with the first payment being made *today*. The bank pays a nominal interest rate of 12% but uses *semiannual* compounding. You plan to leave the money in the bank for 10 years. How much will be in your account after 10 years?
b. You must make a payment of $1,432.02 in 10 years. To get the money for this payment, you will make 5 equal deposits, beginning today and for the following 4 quarters, in a bank that pays a nominal interest rate of 12% with *quarterly compounding*. How large must each of the 5 payments be?

(4–32) Nominal Rate of return

Anne Lockwood, manager of Oaks Mall Jewelry, wants to sell on credit, giving customers 3 months to pay. However, Anne will have to borrow from her bank to carry the accounts receivable. The bank will charge a nominal rate of 15% and will

compound monthly. Anne wants to quote a nominal rate to her customers (all of whom are expected to pay on time) that will exactly offset her financing costs. What nominal annual rate should she quote to her credit customers?

(4–33) Required Annuity Payments

Assume that your father is now 50 years old, that he plans to retire in 10 years, and that he expects to live for 25 years after he retires—that is, until age 85. He wants his first retirement payment to have the same purchasing power at the time he retires as $40,000 has today. He wants all of his subsequent retirement payments to be equal to his first retirement payment. (Do not let the retirement payments grow with inflation: Your father realizes that the real value of his retirement income will decline year by year after he retires.) His retirement income will begin the day he retires, 10 years from today, and he will then receive 24 additional annual payments. Inflation is expected to be 5% per year from today forward. He currently has $100,000 saved up; and he expects to earn a return on his savings of 8% per year with annual compounding. To the nearest dollar, how much must he save during each of the next 10 years (with equal deposits being made at the end of each year, beginning a year from today) to meet his retirement goal? (*Note:* Neither the amount he saves nor the amount he withdraws upon retirement is a growing annuity.)

(4–34) Growing Annuity Payments

You want to accumulate $1 million by your retirement date, which is 25 years from now. You will make 25 deposits in your bank, with the first occurring *today*. The bank pays 8% interest, compounded annually. You expect to get annual raises of 3%, which will offset inflation, and you will let the amount you deposit each year also grow by 3% (i.e., your second deposit will be 3% greater than your first, the third will be 3% greater than the second, etc.). How much must your first deposit be if you are to meet your goal?