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EE320 Placement test

1. Attempt all.
2. Submit your work (in .pdf) on the Moodle. The required format of your filename is studentID_PT
3. You will get **TWO** bonus points if you submit this placement test by the deadline.
4. **This placement test is due on Friday 14th, at 11 AM. Late submission will not be accepted.**

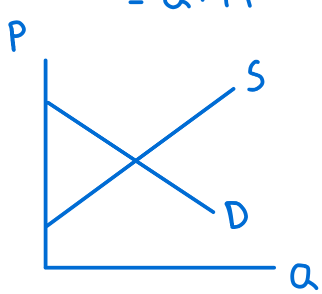
1. Suppose that market demand is given by $P = 10 - Q^2$ and the market supply is given by $Q = a + P$, where P is the unit price, Q is the quantity of output, and a is the coefficient in the supply equation.
   1.1) Graph the market demand and market supply curve in a P-Q diagram. Set the value of a equal to – 14.
   1.2) Solve for the market equilibrium quantity (Q*) and price (P*) when a = -14. Show your work.
   1.3) If "a" increases to -12, what would happen to the market equilibrium quantity and price? State the qualitative predictions without redoing the algebra.

D $P = 10 - Q^2$

1) $Q = a + P$

$\therefore P = Q - a$

$= Q + 14$ S



D = P

2) D = S

$10 - Q^2 = Q + 14$

$0 = Q^2 + Q + 4$

$\dfrac{-a \pm \sqrt{b^2 - 2ac}}{2a}$

$= \dfrac{-1 \pm \sqrt{1 - 2(1)(4)}}{2}$

3) $Q = P - 12$

If a increase, the market equilibrium quantity and price will decrease.

2. Suppose that the revenue function is given by $R(Q) = \ln(Q^2 + 1) + 3\left(\frac{Q}{Q+1}\right), Q \geq 0$. Use the derivative technique and calculate the marginal revenue function. Is the revenue function an increasing or decreasing function?

$$\frac{dR(Q)}{dQ} = \frac{1}{Q^2+1}(2Q) + 3\left[\frac{(Q+1) - Q}{(Q+1)^2}\right]$$

$$= \frac{2Q}{Q^2+1} + \frac{3}{(Q+1)^2}$$

$$MR = \frac{2Q}{Q^2+1} + \frac{3}{(Q+1)^2}$$

$$\frac{d\,MR}{dQ} =$$

3. Suppose that the profit function is given by $\pi(Q) = -\frac{1}{3}Q^3 - Q^2 + 8Q - 1$ where Q is the level of output. Use the calculus and solve for the level of profit-maximizing output. Confirm your answer with the second derivative.

$$\frac{d\pi(Q)}{dQ} = -\frac{3}{3}Q^2 - 2Q + 8 = 0$$

$$-Q^2 - 2Q + 8 = 0$$

$$Q^2 + 2Q - 8 = 0$$

$$(Q+4)(Q-2)$$

$$Q = 2, -4$$

4. Suppose that $A = \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, calculate the following object. Show your work.

4.1 A + B

$m \times n\,[A] \neq m \times n\,[B]$ $\quad$ + ไม่ได้

4.2 A*B

$$AB = \begin{bmatrix} 8(1)+9(4) & 8(2)+9(5) & 8(3)+9(6) \\ 10(1)+11(4) & 10(2)+11(5) & 10(3)+11(6) \end{bmatrix} = \begin{bmatrix} 44 & 61 & 78 \\ 51 & 75 & 96 \end{bmatrix}$$

4.3 det(A)

$$\begin{aligned} \det(A) &= ad-bc \\ &= 8(11)-9(10) \\ &= 88-90 = -2 \end{aligned}$$

4.4 det(B)

$$[(5\times1)+(6\times2)] - [(4\times2)+(5\times3)] = -6$$

4.5 det(C)

$$C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{matrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{matrix}$$

$$\begin{aligned} \det(c) &= [1(5)(9) + 2(6)(7) + 3(4)(8)] - [(7)(5)(3) + 8(6)(1) + (9)(4)(2)] \\ &= (45+84+96)-(85+48+72) \\ &= 225-205 = 20 \end{aligned}$$

5. Suppose that $U(x, y) = x^a y^b + \ln\left(\frac{x}{x+y}\right)$. Use the partial derivative technique, calculate $\frac{\partial U}{\partial x}$ and $\frac{\partial U}{\partial y}$.

$$\frac{\partial U}{\partial x} = a x^{a-1} y^b + \frac{\cancel{x+y}}{x}\left[\frac{(\cancel{x}+y)-\cancel{x}}{(x+y)^{\cancel{2}}}\right]$$

$$= a x^{a-1} y^b + \frac{y}{x(x+y)}$$

$$\frac{\partial U}{\partial y} = x^a b y^{b-1} + \frac{\cancel{x+y}}{\cancel{x}}\left[\frac{(x+y)(0)-\cancel{(x)}(1)}{(x+y)^{\cancel{2}}}\right]$$

$$= x^a b y^{b-1} - \frac{1}{x+y}$$