

EE325 Section 1 HW 2 Due Thursday February 20th (23:00 hr.),2020

Use 4 decimal places for numerical answers

1. In Table 1.a. X_i is total microeconomics exam point (total points are 100) and Y_i is GPA of each student.

Table 1.a

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

$$\bar{y} = \frac{2.8 + 3.4 + 3.0 + 3.5 + 3.6 + 3.0 + 2.7 + 3.7}{8}$$

$$= \frac{25.7}{8} = 3.2125$$

$$\bar{x} = \frac{63 + 72 + 78 + 81 + 87 + 75 + 75 + 90}{8}$$

$$= \frac{621}{8} = 77.625$$

1.1 Now consider the two-variable $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Use OLS to find the estimator of β_0 and β_1 . (Note: *NIID* = Normally, Identically, and Independently Distributed).

$$\hat{\beta}_1 = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2}$$

$$= \frac{(63 - 77.625)(2.8 - 3.2125) + (72 - 77.625)(3.4 - 3.2125) + (78 - 77.625)(3.0 - 3.2125) + (81 - 77.625)(3.5 - 3.2125) + (87 - 77.625)(3.6 - 3.2125) + (75 - 77.625)(3.0 - 3.2125) + (75 - 77.625)(2.7 - 3.2125) + (90 - 77.625)(3.7 - 3.2125) + (78 - 77.625)(3.0 - 3.2125)}{(63 - 77.625)^2 + (72 - 77.625)^2 + (78 - 77.625)^2 + (81 - 77.625)^2 + (87 - 77.625)^2 + (75 - 77.625)^2 + (75 - 77.625)^2 + (90 - 77.625)^2}$$

$$= \frac{17.4374}{51.875} = 0.3341$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

$$= 3.2125 - (0.3341)(77.625)$$

$$= 0.5655$$

1.2 For each observation i , find \hat{Y}_i and \hat{u}_i . Show that $\sum_{i=0}^N \hat{u}_i = 0$.

$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i \quad \text{and} \quad \hat{u}_i = y_i - \hat{y}_i$$

y_i	x_i	\hat{y}_i	\hat{u}_i
2.8	63	2.7143	0.0857
3.4	72	3.0209	0.3791
3.0	78	3.2253	-0.2253
3.5	81	3.3275	0.1725
3.6	87	3.5319	0.0681
3.0	75	3.1231	-0.1231
2.7	75	3.1231	-0.4231
3.7	90	3.6341	0.0659

$$\text{So, } \sum_{i=0}^N \hat{u}_i = 0$$

1.3 Find $var(\hat{u}_i)$, $var(\hat{\beta}_0)$, $var(\hat{\beta}_1)$

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

$$\bar{x} = \frac{10+12+14+16+18+22+24+26+28+30}{10} = 20$$

$$\bar{y} = \frac{0+2+5+6+7+10+10+15+16+20}{10} = 9.1$$

2.1 From the simple regression model $Y_i = \beta_0 + \beta_1 X_i + u_i$, $u_i \sim NIID(0, \sigma^2)$. Find estimators of β_0 and β_1 from the OLS method and interpret the meaning.

$$\hat{\beta}_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = \frac{394}{440} = 0.8955$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x} = 9.1 - (0.8955)(20) = -8.81$$

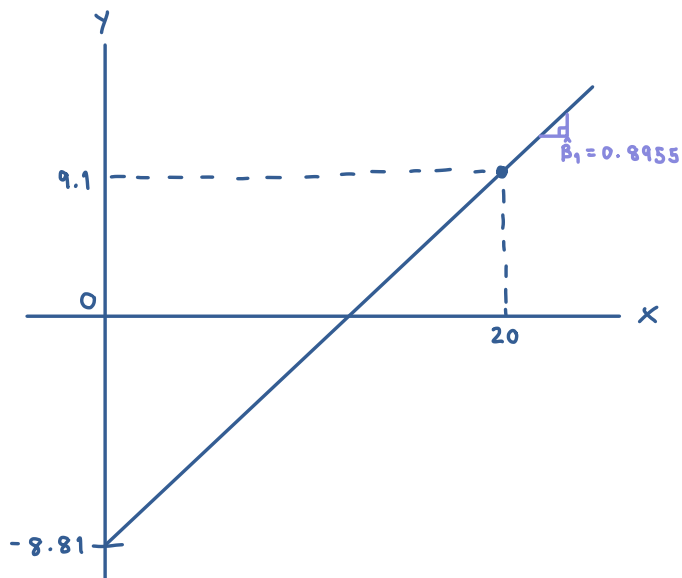
2.2 Find the value of \hat{Y}_i and \hat{u}_i . Show that $\sum_{i=0}^N \hat{u}_i = 0$.

$$\hat{Y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i \quad \text{and} \quad \hat{u}_i = y_i - \hat{Y}_i$$

x_i	y_i	\hat{Y}_i	\hat{u}_i
10	0	0.145	-0.145
12	2	1.936	0.064
14	5	3.727	1.273
16	6	5.518	0.482
18	7	7.309	-0.309
22	10	10.891	-0.891
24	10	12.682	-2.682
26	15	14.473	0.527
28	16	16.264	-0.264
30	20	18.055	1.945

$$\text{So, } \sum_{i=0}^N \hat{u}_i = 0$$

2.3 Plot graph and draw regression line. Does the line pass (\bar{X}, \bar{Y}) ?



2.4 If $X_i = 16$, what is the predicted Y ?

If $x_i = 16$ from 2.2) the predicted $Y (\hat{Y})$ is 5.518

2.5 Find $\text{var}(\hat{u}_i)$, $\text{var}(\hat{\beta}_0)$, $\text{var}(\hat{\beta}_1)$

3. Consider the below regression function:

$$Y_i = \beta_1 X_i + u_i,$$

Where $u_i \sim NIID(0, \sigma^2)$. Find an OLS estimator of β_1 . Then, provide a proof that this is an unbiased estimator. Please state the SLR assumption(s) when used.

$$\begin{aligned} 0 &= \sum_i (\gamma_i - \hat{\beta}_0 - \hat{\beta}_1 x_i) \\ &= \sum_i \gamma_i - n \hat{\beta}_0 - \hat{\beta}_1 \sum_i x_i \\ \hat{\beta}_0 &= \frac{\sum_i \gamma_i}{n} - \hat{\beta}_1 \frac{\sum_i x_i}{n} \\ \hat{\beta}_0 &= \bar{y} - \hat{\beta}_1 \bar{x} = 0 \\ \text{So, } 0 &= \sum_i x_i (\gamma_i - (0) - \hat{\beta}_1 x_i) \\ 0 &= \sum_i x_i \gamma_i - \sum_i x_i \hat{\beta}_1 x_i \\ 0 &= \sum_i x_i \gamma_i - \sum_i \hat{\beta}_1 x_i^2 \\ \hat{\beta}_1 &= \frac{\sum_i x_i (\gamma_i - \hat{\beta}_1 x_i)}{\sum_i x_i^2} \end{aligned}$$