(Final exam)

4.Solving Inequalities

5.Function

- Relations and function
- Domain and Ranges of functions
- Types and applications of functions
- Inverse function
- Composition of functions

6.Solving systems of linear equations and Partial faction decomposition

7.Curve sketching

Inequalities

Proof of quadratic formula

the equation $ax^2 + bx + c = 0$ is equivalent to

$$a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a} = 0.$$

$$\begin{aligned}
a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a} &= 0 \\
a\left(x + \frac{b}{2a}\right)^2 &= -\frac{4ac - b^2}{4a} \\
\frac{1}{a}\left[a\left(x + \frac{b}{2a}\right)^2\right] &= \frac{1}{a}\left(\frac{b^2 - 4ac}{4a}\right) \\
\left(x + \frac{b}{2a}\right)^2 &= \frac{b^2 - 4ac}{4a^2} \\
x + \frac{b}{2a} &= \pm\sqrt{\frac{b^2 - 4ac}{4a^2}} \qquad \text{extract square roots} \\
x + \frac{b}{2a} &= \pm\frac{\sqrt{b^2 - 4ac}}{2a} \\
x &= -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\end{aligned}$$

- If $b^2 - 4ac < 0$, we cannot factor the polynomial in terms of real numbers. Moreover,
  - if $a > 0$, then we always have $ax^2 + bx + c > 0$, $\forall x \in \mathbb{R}$.
  - if $a < 0$, then we always have $ax^2 + bx + c < 0$, $\forall x \in \mathbb{R}$.

Exercises

1) Given that following function

Total revenue function $=10Q - Q^2$
Total cost function $= 6Q^2 - 7Q + 6$

Find the set of Q-values that will keep the profit positive

2) Find the solution set

(a) $\frac{7}{(x-2)(x-3)} + \frac{9}{x-3} < -1$

(b) $\frac{2(x-3)}{(x-1)(x-10)} \geq \frac{1}{x-2}$

(c) $\left|\frac{3x}{x^2-4}\right| \leq 1$

(d) $\frac{|x-3|x^2}{x^2-1} > \frac{10x}{x^2-1}$

(e) $\left|x^2 - |x-1|\right| \geq 1$

3) Find all possible values of k so that $x \in [-7,1]$ satisfies the following inequality:

$$\frac{1}{2}x^2 + 6kx - 14k^2 \geq 0$$

1. Given that following function

Total revenue function $=10Q - Q^2$
Total cost function $= 6Q^2 - 7Q + 6$

Find the set of Q-values that will keep the profit positive

2. (exercise I) $\left(\frac{x}{(x-3)} - 2\right)\left(\frac{e^x}{\cos(x)+2}\right) \geq 0$

3. $\left(\frac{2(x-3)}{(x-1)(x-10)}\right) \geq \frac{1}{x-2}$

4. Find all possible values of k so that $x \in [-7,1]$ satisfies the following inequality

$$\frac{1}{2}x^2 + 6kx - 14k^2 \geq 0$$

5. $\frac{|x-3|x^2}{x^2-1} \geq \frac{10x}{x^2-1}$

6. (Aj. Supranee) $\left|x^2 - |x-1|\right| \geq 1$

7. (exercise III) Let a and b be real numbers with $|b| < |a| < 1$ and $a > 0$.

(a) Show that $a - 1 < 0 < b + a < 2a < 1 + a$.

(b) Show that $\frac{2a^2-2a}{b+a} > \frac{a^2-1}{2a}$

.

8. (exercise III) $\frac{|x^2-2|+2}{x^2-3|x|+2} < 0$