4 Testing Hypotheses about a Single Linear Combination of the Parameter

Consider

$$\log(wage) = \beta_0 + \beta_1 jc + \beta_2 univ + \beta_3 \exp er + u$$

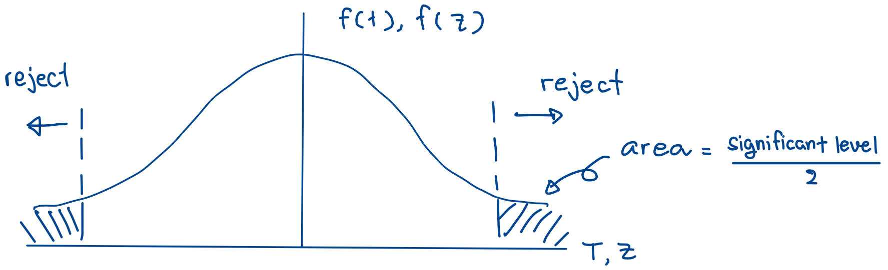
where jc = number of years attending a two-year college
$univ$ = number of years at a four-year college
$\exp er$ = months in the workforce.
We want to test whether $\beta_1 = \beta_2$. → if the returns from 1 more year of education of junior college is the same as that of the university.

$H_0: \beta_1 = \beta_2 \Rightarrow H_0: \beta_1 - \beta_2 = 0$

against

$H_a: \beta_1 \neq \beta_2 \Rightarrow H_a: \beta_1 - \beta_2 \neq 0$

2-tailed test



$$t = \frac{(\hat\beta_1 - \hat\beta_2) - 0}{s.e.(\hat\beta_1 - \hat\beta_2)}$$

we compute this t-statistic and compare with the critical value.

where $s.e.(\hat\beta_1 - \hat\beta_2) = \sqrt{\widehat{var}(\hat\beta_1 - \hat\beta_2)}$

$$= \sqrt{\widehat{var}(\hat\beta_1) + \widehat{var}(\hat\beta_2) - 2\,\widehat{cov}(\hat\beta_1, \hat\beta_2)}$$

not very straight forward to calculate

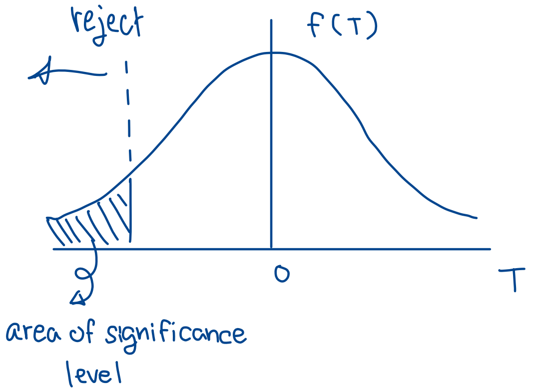
→ we use variable transformation trick → see notes.

another possible hypothesis test (one-tailed alternative)

$H_0: \beta_1 = \beta_2 \Rightarrow H_0: \beta_1 - \beta_2 = 0$

$H_a: \beta_1 < \beta_2 \Rightarrow H_a: \beta_1 - \beta_2 < 0$

- It is assume that β_1 would not be more than β_2 (returns to a 2-year college would never be more than returns to university education)

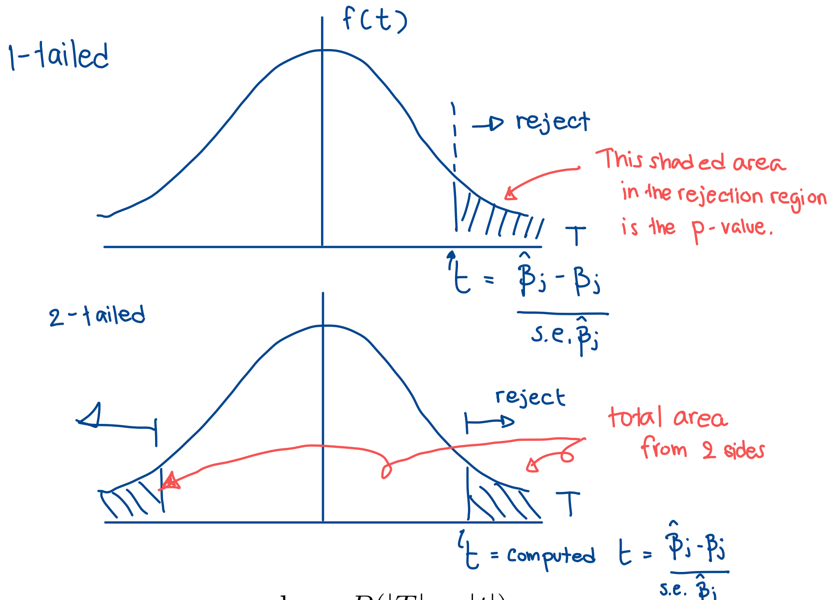


$$t = \frac{(\hat{\beta}_1 - \hat{\beta}_2) - 0}{s.e.\,(\hat{\beta}_1 - \hat{\beta}_2)}$$

* Then, go to the extra notes

5 Computing p-Values for t-Tests

- What is the significance level given the computed t-statistics?



- p-value : $P(|T| > |t|)$

T = t-distributed random variable with d.f = n-k-1

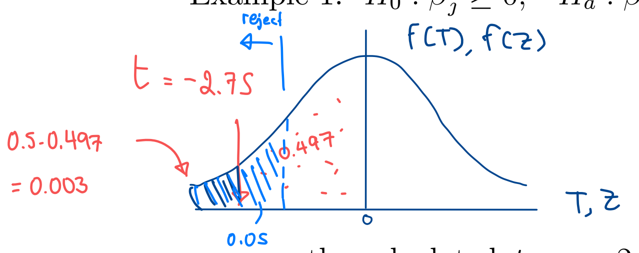
t = computed t-statistic

→ p-value = probability that a random T value will be greater (in the absolute term (| - |) than our t in the hypothesis testing.

Example 1: $H_0 : \beta_j \geq 0$, $H_a : \beta_j < 0$, d.f.= 140. → z - table

→ p-value = what should be the significant level, given the critical value of −2.75 ??

↳ find the shaded area ?



suppose the calculated $t_{\hat{\beta}_j} = -2.75$

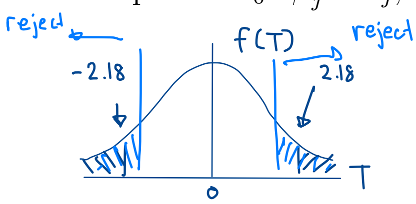
↳ $t_{\hat{\beta}_j} = \frac{(\hat{\beta}_j - \beta_j)}{s.e.(\hat{\beta}_j)}$

- From the z-table, the value -2.75 corresponds to area = 0.003.
- Thus, p-value = 0.003.
- Would we reject H_0 if we use the significance level = 5%?

☆ RULE ! we reject H_0 if p-value < sig. level

Example 2: $H_0 : \beta_j = a_j$, $H_a : \beta_j \neq a_j$, d.f.= 18.

↖ use t-table



suppose the calculated $t_{\hat{\beta}_j} = -2.18$

- From the t-table, the value -2.18 corresponds to area = 0.02 to 0.05.
- Thus, p-value = is between 0.02-0.05.
- Would we reject H_0 if we use the significance level = 5%?

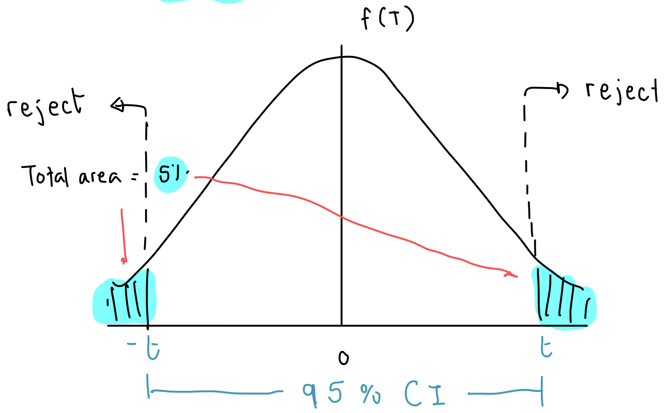
Yes, reject H_0 because the area is less than 0.05 or p-value < 0.05 .

6 Confidence Intervals (CI)

- Confidence Intervales for the POPULATION PARAMETER (β_j)

→ The range of values that would capture the true β_j at a 95% chance

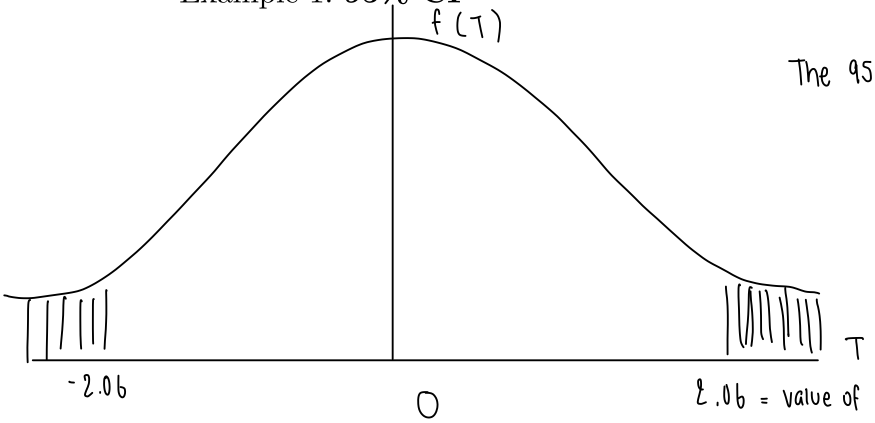
- A 95% CI of β_j is given by



CI => $\hat{\beta}_j \pm C \times s.e.(\hat{\beta}_j)$

C is the 97.5 percentile in the t-distribution with n-k-1 d.f

Example 1: **95% CI**

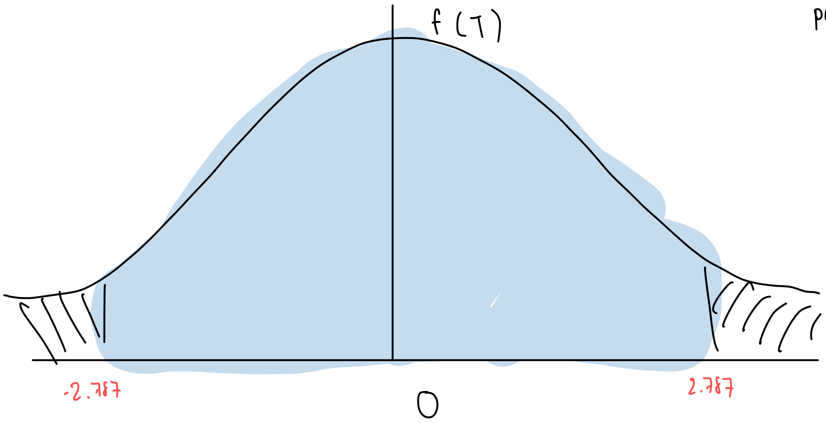


$= $ value of 97.5^{th} percentile in the T_{25} distribution

The 95% CI for $\hat{\beta}_j = [\hat{\beta}_j - 2.06 \cdot s.e.(\hat{\beta}_j),\ \hat{\beta}_j + 2.06 = s.e.(\hat{\beta}_j)]$

Example 2: **99% CI** d.f 25

What area or pctile $= 1 - 0.005 = 0.995$



The 99% CI for $\hat{\beta}_j$: $[\hat{\beta}_j - 2.787 \cdot se\ \hat{\beta}_j,\ \hat{\beta}_j + 2.787 \cdot se\ \hat{\beta}_j]$

7 Testing Multiple Linear Restrictions: The F-test

Suppose the model is specified by

$$\begin{aligned} y &= \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + u \\ H_0 &: \beta_2 = 0 \text{ and } \beta_3 = 0 \\ H_a, H_1 &: H_0 \text{ is not true} \end{aligned}$$

→ want to test if x_1 and x_2 both have no impact on y.

We can use the F-test to test this type of "multiple hypotheses".

1. Our full model is called the "unrestricted" model (ur). Suppose it can be expressed as:

   Big model

   $Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + u$ is true ↳ reject H_0

   $$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + ... + \beta_k x_k + u$$

2. The model which takes out x (which we think its associated $\beta = 0$) is called the restricted model (r).

   Small model

   $Y = \beta_0 + \beta_1 X_1 + u$ is true ↳ do not reject H_0

- Suppose there are "q" number of β that we would like to perform a joint-test of $= 0$

e.g. in this model $q = 2$

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \dots + \beta_{K-q} X_{K-q} + u$$

$H_0: \beta_{K-q+1} = \beta_{K-q+2} = \dots = \beta_K = 0$

(the last q βs $= 0$)

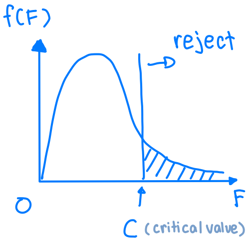
H_a: H_0 is not true.

$$y = \overbrace{\underbrace{\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_{K-q} X_{K-q}}_{(r)} + \beta_{K-q+1} X_{K-q+1} + \beta_{K-q+2} X_{K-q+2} + \dots + \beta_K X_K + u}_{(ur)}$$

$$F \equiv \frac{(SSR_r - SSR_{ur})/q}{SSR_{ur}/(n-K-1)}$$

This is always positive (+) b/c $SSR_{ur} < SSR_r$. Everytime you add 1 more X, the model will be better explained.

d.f. of the "ur" model



- So, if every time you add 1 more variable, the SSR ↓ and R^2 ↑, why don't we just keep the additional X in the model?

  ↳ B/c every time we add 1 more X, var($\hat{\beta}_s$) will increase, making the prediction of β less precise. So, we only keep the additional X_s if it/they can improve the model enough.

  ↳ can ↓ SSR (↑ R^2) enough.

  can significance ↓ SSR and ↑ R^2

$H_0: \beta_2 = \beta_3 = \dots = 0$

H_a: H_0 is not true

$F \sim F_{q, n-K-1}$ ← d.f. of the ur model; q = # of joint hypotheses being tested

We reject H_0 of jointly no effect if $F > C$

3. Some useful facts

① $R^2_{ur} > R^2_r$ because any additional X would increase R^2 (improve fit).

↳ $SSR_{ur} < SSR_r$

② By including more X, the model is certainly explained. However, we would like to reject H_0 if the inclusion of extra variables does not improve the model enough.

4. Other ways to calculate the F-statistics:

↳ From $R^2 = 1 - \frac{SSR}{SST}$ SSR → RSS, SST → TSS

We have $F = \frac{(R^2_{ur} - R^2_r)/q}{(1-R^2_{ur})/(n-k-1)}$

q: # of β that are set to "0".
n: # of obs.
k: # of slope β.
1: intercept.

⇒ If we want to test the overall significance of the model

$H_0: \beta_1 = \beta_2 = \beta_3 = \dots = \beta_k = 0$, H_a = otherwise

$$F \equiv \frac{R^2/k}{(1-R^2)/(n-k-1)}$$

R^2 of the model ≈ UR
the "r" model has no X at all.

Example: Suppose we are interested in understanding the determinant of a baseball player's salary.

If we want to test whether performance has any impact on salary

y *salary* = season salary
r, ur: *years* = years in major leagues
r, ur: *gamesyr* = games per year in the league
ur: *bavg* = career batting average
ur: *hrunsyr* = homeruns per year
ur: *rbisyr* = runs batted in per year

$H_0: \beta_{bavg} = \beta_{hrunsyr} = \beta_{rbisyr} = 0$

H_a: otherwise is true

- the unrestricted model (ur) is defined by ur model

y X

. regress log_salary years gamesyr bavg hrunsyr rbisyr

Source	SS	df	MS
Model	308.989208	5	61.7978416
Residual	183.186327	347	.527914487
Total	492.175535	352	1.39822595

Number of obs	=	353
F(5, 347)	=	117.06
Prob > F	=	0.0000
R-squared	=	0.6278
Adj R-squared	=	0.6224
Root MSE	=	.72658

$k=5$

$q=3$

log_salary	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
years 1	.0688626	.0121145	5.68	0.000	.0450355	.0926898
gamesyr 2	.0125521	.0026468	4.74	0.000	.0073464	.0177578
bavg 3	.0009786	.0011035	0.89	0.376	-.0011918	.003149
hrunsyr 4	.0144295	.016057	0.90	0.369	-.0171518	.0460107
rbisyr 5	.0107657	.007175	1.50	0.134	-.0033462	.0248776
_cons	11.19242	.2888229	38.75	0.000	10.62435	11.76048

when considering each of the performance x one-by-one, none of them has a significant impact at 5%

- the restricted model (r) is defined by

y X

. regress log_salary years gamesyr

Source	SS	df	MS
SSE Model	293.864058	2	146.932029
SSR Residual	198.311477	350	.566604221
SST Total	492.175535	352	1.39822595

Number of obs	=	353
F(2, 350)	=	259.32
Prob > F	=	0.0000
R-squared	=	0.5971
Adj R-squared	=	0.5948
Root MSE	=	.75273

log_salary	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
years	.071318	.012505	5.70	0.000	.0467236	.0959124
gamesyr	.0201745	.0013429	15.02	0.000	.0175334	.0228156
_cons	11.2238	.108312	103.62	0.000	11.01078	11.43683

Now, our H_0 and H_a becomes

Do not have to submit

Homework:

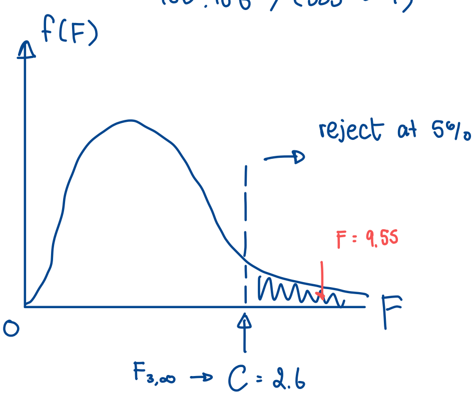
$$F \equiv \frac{(R^2/q)}{(1-R^2)/n-k-1} = ??$$

$$F \equiv \frac{(SSR_r - SSR_{ur})/q}{SSR_{ur}/(n-k-1)}$$

constraint

$$= \frac{(198.311 - 183.186)/3}{183.186\ /(353-5-1)} \approx 9.55$$

- But when performing an F-test, performance have joint impact.



Let's use 5% level of sig.

Since $F = 9.55 > 2.6$, we reject H_0 at 5% level and conclude that performances has joint effect on salary.

8 How the Hypothesis Testing is done in Practice

1. Check the values of $t - statistic$ reported by the statistical software (i.e. STATA, SPSS, SAS)

   $\Rightarrow$ These $t - statistics$ are to test $H_0 : \beta_i = 0$

   z-table

   $\Rightarrow$ If the d.f. > 30, then when $t \;\; > 1.96$, we can reject H_0 with 5% sig level

   $\Rightarrow$ **When** $t \;\; >$ **1.96**, we can say that β_i is **statistically significant** at 5% level. (value of $\beta_i \neq 0$)

   $\Rightarrow$ **When** $t \;\; <$ **1.96** we can say that β_i is **not statistically significant** at 5% level.

   $\Rightarrow$ If $t \;\; < 1.96$ we can drop x_i from the model

   $\Rightarrow$ After we drop x_i, we estimate the new regression function and obtain a new set of $\hat{\beta}$.

2. We can also perform other hypothesis testings of interest.

   e.g. $H_0 : \beta_i = \beta_j$

   or $\quad H_0 : \beta_i = 5 \quad$ etc.

   or perform an F-test for testing multiple linear restrictions

3. Usually, in economics, the estimation results are reported using this form

Dependent Variable: log(*salary*)			
Independent Variables	**(1)**	**(2)**	**(3)**
log(*sales*)	.224 (.027)	.158 (.040)	.188 (.040)
log(*mktval*)	——	.112 (.050)	.100 (.049)
profmarg	——	−.0023 (.0022)	−.0022 (.0021)
ceoten	——	——	.0171 (.0055)
comten	——	——	−.0092 (.0033)
intercept	4.94 (0.20)	4.62 0.25)	4.57 (0.25)
Observations	177	177	177
R-squared	.281	.304	.353

Sales

Company Performance

CEO Characteristics

like a simple regression with 1 x.

Multiple Regression Analysis : Further Issues

1 Data scaling on OLS statistics

When we change the unit of measurment of a variable, the value of estimators would chance accordingly. For example

$$\widehat{bweght} = \widehat{\beta}_0 + \widehat{\beta}_1 cigs + \widehat{\beta}_2 fa\min c,$$

where
$bwght =$ child birth weight, in grams.
$cigs =$ number of cigarettes smoked by the mother while pregnant, per day.
$fa\min c =$ annual family income, in thousands of dollars.

▷ What is we use bweight in kilograms?

1 kg. = 1,000 g.

$$\widehat{bweight}_{kg} = \frac{\widehat{bweight}_g}{1,000} = \frac{\hat{\beta}_0}{1,000} + \frac{\hat{\beta}_1}{1,000} cigs + \frac{\hat{\beta}_2}{1,000} faminc$$

$$= \hat{\alpha}_0 + \hat{\alpha}_1 cigs + \hat{\alpha}_2 faminc$$

$$\rightarrow \hat{\alpha}_0 = \frac{\hat{\beta}_0}{1000}, \quad \hat{\alpha}_1 = \frac{\hat{\beta}_1}{1000}, \quad \hat{\alpha}_2 = \frac{\hat{\beta}_2}{1000}$$

▷ What if we use faminc in USD (instead of 1000 USD)

$$bweight_g = \hat{\beta}_0 + \hat{\beta}_1 cigs + \frac{\hat{\beta}_2}{1000} faminc_{USD}$$

$$= \hat{\beta}_0 + \hat{\beta}_1 cigs + \hat{\theta}_2 faminc_{USD}$$

The value of this variable is going to be 1000 times larger than faminc

$$\rightarrow \hat{\theta}_2 = \frac{\hat{\beta}_2}{1000}$$

in other words $\hat{\theta}_2$ = impact of 1 USD ↑ in income
$\hat{\beta}_2$ = — 1,000 USD ↑ in income

▷ What if we use bweight in kg & income in THB

$$bweight_{kg} = \frac{\hat{\beta}_0}{1000} + \frac{\hat{\beta}_1}{1000} cigs + \frac{\left(\frac{\hat{\beta}_2}{1,000}\right)}{30,000} faminc_{THB}$$

This value is going to be 30,000 times more than faminc

$\frac{d\ln x}{dx} : \frac{1}{x} \rightarrow \boxed{d\ln(x) = \frac{1}{x}dx}$

2 More on functional forms

- Logarithmic Functional Form

usually mean natural log ($\ln$)

$$\log(y) = \beta_0 + \beta_1 \log(x_1) + \beta_2 x_2 + u$$

$\Delta y = y_1 - y_2$

$\Delta x_1 = x_{11} - x_{12}$

$$\beta_1 = \frac{d\log(y)}{d\log(x_1)} = \frac{\frac{1}{y}dy}{\frac{1}{x_1}dx_1} = \frac{\frac{1}{y}\Delta y}{\frac{1}{x_1}\Delta x_1} = \frac{100 \times \frac{1}{y}\Delta y}{100 \times \frac{1}{x_1}\Delta x_1} = \frac{\%\Delta y}{\%\Delta x}$$

with the log y & log x format, the coefficient is going to be the elasticity! (x_1 elasticity of y)
(price) (demand)

$$\beta_2 = \frac{d\log(y)}{dx_2} = \frac{\frac{1}{y}dy}{dx_2} = \frac{\frac{1}{y}\Delta y}{\Delta x_2}$$

→ if we want the upper term to be % change, then $100\beta_2 = \frac{100\,\frac{1}{y}\Delta y}{\Delta x_2}$

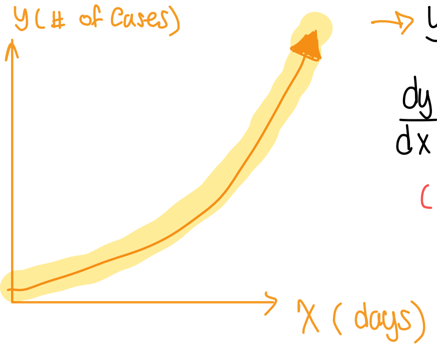
$$100\beta_2 = \frac{\%\Delta y}{\Delta x_2}$$

$100\beta_2 = \%\Delta$ in y given that x_2 increases by 1 unit.

- Models with Quadratics (squares)

→ capture increasing / decreasing marginal effects (slope of the relationship between x & y is not constant.)

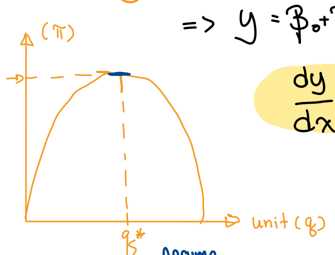
COVID-19 EXAMPLE



→ $y = \beta_0 + \beta_1 x + \beta_2 x^2 + u$

$\frac{dy}{dx} = \beta_1 + 2\beta_2 x$
(+) (+) days

Decreasing returns.



$\Rightarrow y = \beta_0 + \beta_1 x + \beta_2 x^2 + u$

$\frac{dy}{dx} = \beta_1 + 2\beta_2 x$
(+) (−)

Assume $MC = 10$

Demand : $P = 100 - q$

$\pi = (P - MC)q$;

$\pi = (100 - q - 10)q$

F.O.C. $\frac{\partial\pi}{\partial q} = 0 = 90 - 2q$

β_1 is positive

β_2 is (−)

Example : Effects of Pullution on Housing Prices

$$\log(price) = \beta_0 + \underline{\beta_1} \log(nox) + \underline{\beta_2} \log(dist) + \underline{\beta_3} rooms + \underline{\beta_4} room^2 + \underline{\beta_5} stratio + u$$

where

price = housing price

nox = level of pollution

dist = distance from downtown

rooms = number of rooms

stratio = average student per teacher ratio

In the US or many other countries, students can apply to schools in the area without having to take any test. So, the lower stratio, the better the school

The estimation result is given by

```
regress  lprice lnox dist rooms rooms_sq stratio

      Source |       SS       df       MS              Number of obs =     506
-------------+------------------------------           F(  5,   500) =  155.62
       Model |  51.4933152     5  10.298663            Prob > F      =  0.0000
    Residual |  33.0889098   500  .06617782            R-squared     =  0.6088
-------------+------------------------------           Adj R-squared =  0.6049
       Total |   84.582225   505  .167489554           Root MSE      =  .25725

------------------------------------------------------------------------------
      lprice |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        lnox | β1 -.9767545   .0995938    -9.81   0.000    -1.172429   -.7810806
        dist | β2 -.0321972   .0094013    -3.42   0.001     -.050668   -.0137264
       rooms | β3 -.5528032   .1612965    -3.43   0.001    -.8697056   -.2359007
    rooms_sq | β4  .0624697   .0124867     5.00   0.000     .0379368    .0870025
     stratio | β5 -.0486667   .0058131    -8.37   0.000    -.0600879   -.0372455
       _cons |   13.59154   .5650901    24.05   0.000     12.4813     14.70178
------------------------------------------------------------------------------
```

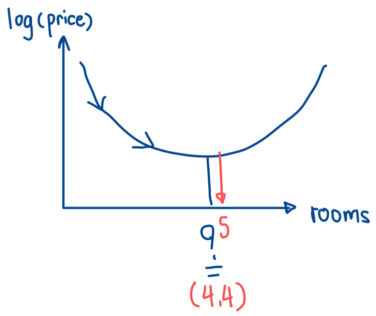
log(price) → lprice

log(nox) → lnox

$|t| > 1.96$ ↑ all < 0.05

Consider the effect of "room" → all variables are significant

$$\frac{d\log(price)}{d\ rooms} = \beta_3 + 2\beta_4\, rooms = -0.553 + 2(0.062)\cdot rooms$$



✡ At how many rooms does 1 additional room has a positive impact on log(price)??

$0 = -0.553 + 2(0.062)\cdot rooms$

$rooms = 4.4$

ANS at 4.4 rooms or more

at → 5 rooms or more. #

What wold be the % change in price when the number of room increases from 5 to 6?

- $$\frac{d\log(price)}{d\ rooms} = -0.553 + 2(0.062)\cdot rooms$$

$$\frac{100\cdot\frac{1}{price}\,d\,price}{d\ rooms} = 100(-0.553 + 2(0.062))$$

$$= 100 \times 0.067 = 6.7\%\ \text{increase.}$$

What about % in price when # rooms increases from 5 to 7?

$\%\Delta\ price = 100(-0.553 + 2(0.062)\cdot 6)$

$= 19.1\%$

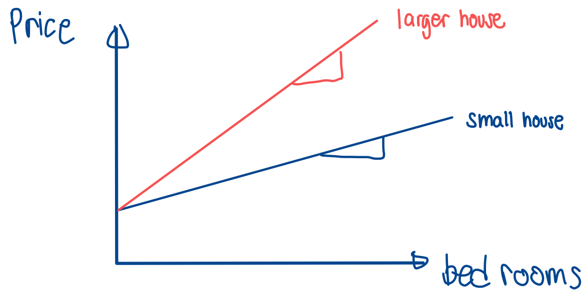
total % Δ in price when # rooms ↑ from 5 to 7 is $6.7 + 19.1 = 25.8\%$

3 Models with Interaction Terms → Used when the impact of 1 variable depends on the value (level) of another variable.

Consider

$$price = \beta_0 + \beta_1 \underset{x_1}{sqrft} + \beta_2 \underset{x_2}{bdrms} + \overbrace{\beta_3 \underset{x_1 \cdot x_2}{sqrft \times bdrms}}^{X_3} + \beta_4 \underset{x_2}{bthrms} + u$$

where
$price$ = housing price
$sqrft$ = house size (square feet)
$bdrms$ = number of bedrooms
$bthrms$ = number of bathrooms



$$\frac{d\,price}{d\,bdrms} = \beta_2 + \beta_3 sqrft$$

→ if $\beta_2 > 0$ then, an additional bedroom would increase price more for a larger house !

4 More on the Goodness-of-Fit and Selection of Regressors

- Adding more regressors ALWAYS improve fit → R^2 always ↑

▷ But we lose the "degree of freedom"
(d.f. = free data point used to estimate the parameter)
↪ 1 data point is sacrified every time we estimate a parameter.

▷ Using R^2 would not punish "having too many regressors."

▷ We use adjusted-R^2 or $\bar{R}^2$ when we want to punish adding too many regressors.

$$R^2 = 1 - \frac{SSR}{SST} = 1 - \frac{SSR/\cancel{n}}{SST/\cancel{n}}$$

$$\text{adj. } R^2 = \boxed{\frac{1 - SSR/(n-k-1)}{1 - SST/(n-1)}}$$

▷ If we have more k, d.f. $= n-k-1$ ↓,
$SSR/(n-k-1)$ ↑, adj R^2 ↓

Using adjusted R-squared to choose between non-nested models (one model is not a subset of another).

Consider Model 1

$$\begin{aligned}\widehat{salary} &= \begin{matrix} 830.63 & +0.0163sales & +19.63roe \\ (223.90) & (0.0089) & (11.08) \end{matrix} \\ n &= 209, \quad R^2 = 0.029, \quad \bar{R}^2 = 0.020\end{aligned}$$

Consider Model 2

$$\begin{aligned}\log(\widehat{salary}) &= \begin{matrix} 4.36 & +0.2751\log(sales) & +0.0179roe \\ (0.29) & (0.033) & (0.004) \end{matrix} \\ n &= 209, \quad R^2 = 0.282, \quad \bar{R}^2 = 0.275\end{aligned}$$

27.5 % of variation in y is explained.
So, this model is better!

Multiple Regression Analysis with Qualitative Information:

1 Outline

- Describing qualitative information
- Using a single dummy independent variable
- Using dummy variables for multiple categories
- Interactions involving dummy variables
- A binary dependent variable (Y variable): The linear probability model

2 Describing Qualitative Information

- "Female" and "Married" are qualitative variable.
- We arbitarily assign a dummy variable to decribe them.

$$famale = \begin{cases} 1 & \text{if female} \\ 0 & \text{otherwise (or if male)} \end{cases}$$
$$married = \begin{cases} 1 & \text{if married} \\ 0 & \text{otherwise (of if single)} \end{cases}$$

TABLE 7.1

A Partial Listing of the Data in WAGE1.RAW

person	wage	educ	exper	female	married
1	3.10	11	2	1	0
2	3.24	12	22	1	1
3	3.00	11	2	0	0
4	6.00	8	44	0	1
5	5.30	12	7	0	1
.	.	.	.	.	.
525	11.56	16	5	0	1
526	3.50	14	5	1	0

3 Models with a single dummy independent variable

Consider

$$wage = \beta_0 + \delta_0 female + \beta_1 educ + u. \quad (1)$$

where

$$famale = \begin{cases} 1 & \text{if female} \\ 0 & \text{otherwise (or if male)} \end{cases}$$

In this case, the δ_0 notation is used to highlight the interpretation of the parameters multiplying dummy variables. In other cases, we can use any notation that is the most convenient.

① $E(wage \mid female, educ) = E(\beta_0 + \delta_0 female + \beta_1 educ + u \mid female, educ)$

$= \beta_0 + \delta_0 female + \beta_1 educ + \underbrace{E(u \mid female, educ)}_{=0}$

$= \beta_0 + \delta_0 female + \beta_0 educ$ (assm MLR1-4 holds)

② Thus

♀ : $E(wage \mid female = 1, educ) = \beta_0 + \delta_0(1) + \beta_0 educ = \beta_0 + \delta_0 + \beta_1 educ$

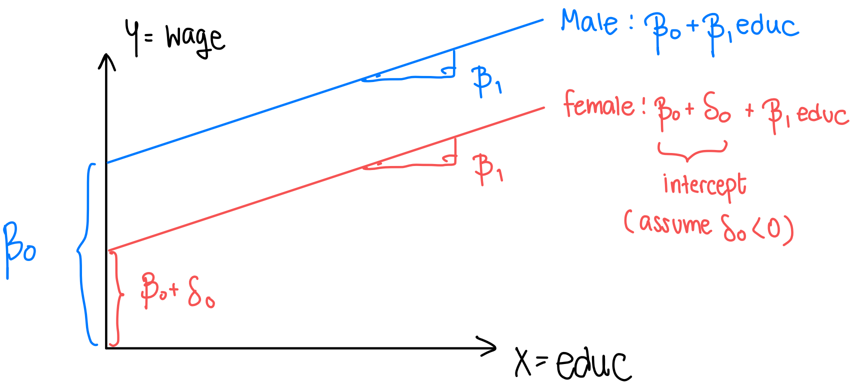
♂ : $E(wage \mid female = 0, educ) = \beta_0 + \delta_0(0) + \beta_1 educ = \beta_0 + \beta_1 educ$

$\delta_0 = E(wage \mid female = 1, educ) - E(wage \mid female = 0, educ)$

or $\delta_0 = E(wage \mid female, educ) - E(wage \mid male, educ)$

* given the same value of educ (same education level),

δ_0 is the difference in the expected wage of females and males.



By the way we model this regression function, "female" is going to give a constant impact on wage, regardless of the level of education.

4 It is not possible to include all of the dummy alternatives in the same model (as long as there is an intercept in the model).

- If we include all alternatives of a dummy variable in the same model, we will face the "perfect collinearity" problem.

For example:

$$wage = \beta_0 X_0 + \delta_0 \underbrace{female}_{(X_1)} + \beta_1 \underbrace{educ}_{(X_2)} + \delta_1 \underbrace{male}_{(X_3)} + u$$

intercept × 1

Id.	female	male	X_0
1	1	0	1
2	1	0	1
3	0	1	1
4	0	1	1
5	0	1	1

$$X_0 = X_1 + X_3$$
$$1 = female + male$$
$$female = male + 1$$

or

If there are "n" categories, we omit "1" category to avoid multi collinearity.

$$1 = winter + spring + summer + fall$$
$$winter = 1 - spring - summer - fall$$

$winter = \begin{cases} 1 & \text{if winter} \\ 0 & \text{otherwise} \end{cases}$

$spring = \begin{cases} 1 & \text{if spring} \\ 0 & \text{otherwise} \end{cases}$

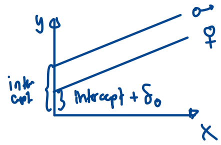
etc.

id	winter	spring	summer	fall	X_0
1	1	0	0	0	1
2	1	0	0	0	1
3	0	0	1	0	1
4	.	.	.	.	1
5	.	.	.	.	1
⋮	⋮	⋮	⋮	⋮	⋮

- At least one alternative has to be dropped. We treat the dropped alternative as the "BASE GROUP" or "BASELINE" or "BENCHMARK GROUP".

in this case, male

$\begin{cases} 1 & \text{if female} \\ 0 & \text{if male} \end{cases}$ $\begin{cases} 1 & \text{if male} \\ 0 & \text{if female} \end{cases}$




```
. regress lwage female male married educ exper
note: male omitted because of collinearity

      Source |       SS           df       MS      Number of obs   =       526
-------------+----------------------------------   F(  4,    521)   =     75.27
       Model |  54.3265253         4  13.5816313   Prob > F        =    0.0000
    Residual |  94.0032262       521  .180428457   R-squared       =    0.3663
-------------+----------------------------------   Adj R-squared   =    0.3614
       Total |  148.329751       525  .28253286    Root MSE        =    .42477

------------------------------------------------------------------------------
       lwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      female |  -.3251146   .0377061    -8.62   0.000    -.3991892     -.25104
        male |          0  (omitted)
     married |   .1380145   .0411197     3.36   0.001     .0572338    .2187953
        educ |   .0872644   .0071554    12.20   0.000     .0732075    .1013213
       exper |   .0076213   .0015314     4.98   0.000     .0046129    .0106297
       _cons |   .4690918   .1040575     4.51   0.000      .264668    .6735156
------------------------------------------------------------------------------
```

Female workers are expected to have less wage compared to male workers.

5 Using dummy variables for multiple categories

Case 1 We can use many dummy variables in the same model

Consider a model which includes 2 dummy variables– *female* and *married*.

$\begin{cases} 1 & \text{if female} \\ 0 & \text{if otherwise} \end{cases}$ $\begin{cases} 1 & \text{if married} \\ 0 & \text{otherwise} \end{cases}$

$$\log(wage) = \beta_0 + \delta_0 female + \delta_1 \underline{married} + \beta_1 educ + \beta_2 \exp er + \beta_3 \exp er^2 + \beta_4 tenure + \beta_5 tenure^2 + u.$$

```
regress lwage female married educ exper expersq tenure tenursq
```

Source	SS	df	MS
Model	65.6482326	7	9.37831895
Residual	82.6815188	518	.159616832
Total	148.329751	525	.28253286

Number of obs =	526
F(7, 518) =	58.76
Prob > F =	0.0000
R-squared =	0.4426
Adj R-squared =	0.4351
Root MSE =	.39952

lwage	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
female	-.2901838	.0361121	-8.04	0.000	-.3611279	-.2192396
married	.0529219	.0407561	1.30	0.195	-.0271456	.1329894
educ	.0791547	.0068003	11.64	0.000	.0657952	.0925143
exper	.0269535	.0053258	5.06	0.000	.0164907	.0374163
expersq	-.0005399	.0001122	-4.81	0.000	-.0007603	-.0003196
tenure	.0312962	.0068482	4.57	0.000	.0178426	.0447499
tenursq	-.0005744	.0002347	-2.45	0.015	-.0010355	-.0001134
_cons	.4177837	.0988662	4.23	0.000	.2235557	.6120116

$\hat{\beta}$

2) δ_1 measures the impact of be married. (marriage premium) But since $|t| < 1.96$ or $p > 0.05$, we do not reject H_0 of no impact.

Comments:

1) δ_0 measures the expected difference between female & male workers given the same martial status and other factors.

$$\frac{d\log(wage)}{d\,female} = \frac{\frac{1}{wage}\,d\,wage}{\partial\,female} = -0.29$$

$$\frac{100 \cdot \frac{1}{wage}\,d\,wage}{\partial\,female} = 100 \cdot -0.29$$

$$\frac{\%\Delta\,wage}{\partial\,female} = 29.02\,\%$$

- female workers are expected to earn less than male workers by 29.02%, holding other factors the same.

	♀	♂
marr	marr fem	marr male
sing	sing fem	sing male

basecase

Consider a model which includes dummy variables for each gender/marital status combination– *marrmale*, *marrfem* and *singfem*. (or sing male ← used as the base case)

$$\log(wage) = \beta_0 + \delta_0 marrmale + \delta_1 marrfem + \delta_3 singfem + \beta_1 educ + \beta_2 exper + \beta_3 exper^2 + \beta_4 tenure + \beta_5 tenure^2 + u. \quad (8.1)$$

```
regress lwage marrmale marrfem singfem educ exper expersq tenure tenursq
```

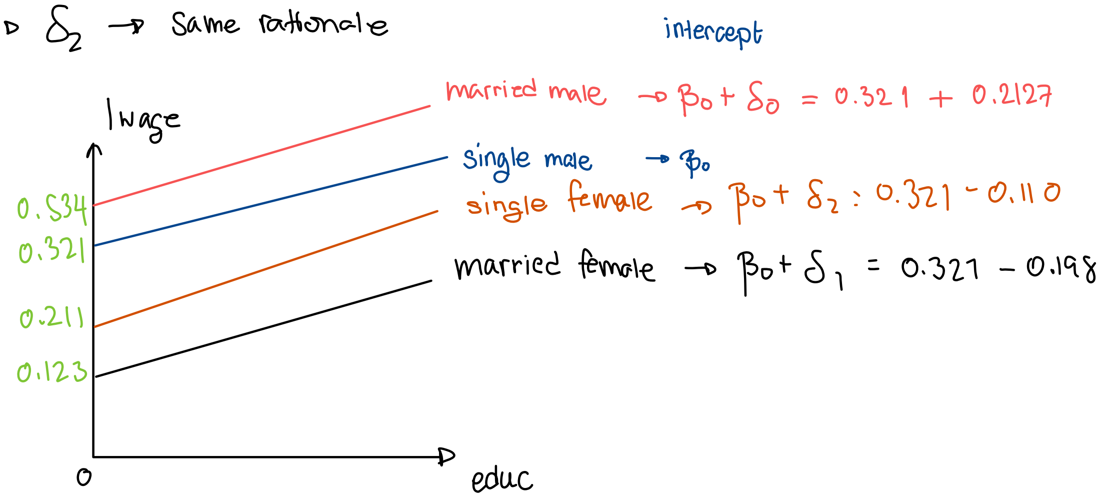
Source	SS	df	MS
Model	68.3617623	8	8.54522029
Residual	79.9679891	517	.154676961
Total	148.329751	525	.28253286

Number of obs =	526
F(8, 517) =	55.25
Prob > F =	0.0000
R-squared =	0.4609
Adj R-squared =	0.4525
Root MSE =	.39329

	lwage	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
δ_0	marrmale	.2126757	.0553572	3.84	0.000	.103923	.3214284
δ_1	marrfem	-.1982676	.0578355	-3.43	0.001	-.311889	-.0846462
δ_2	singfem	-.1103502	.0557421	-1.98	0.048	-.219859	-.0008414
$\hat{\beta}$	educ	.0789103	.0066945	11.79	0.000	.0657585	.092062
	exper	.0268006	.0052428	5.11	0.000	.0165007	.0371005
	expersq	-.0005352	.0001104	-4.85	0.000	-.0007522	-.0003183
	tenure	.0290875	.006762	4.30	0.000	.0158031	.0423719
	tenursq	-.0005331	.0002312	-2.31	0.022	-.0009874	-.0000789
	_cons	.3213781	.100009	3.21	0.001	.1249041	.5178521

Comments: This regression is not the same as the previous one. It uses "single male" as the base group. (The previous one use male & single as 2 base groups)

- δ_0 measures the expected diff. in wage of married male as compared with single males, holding other factors constant.
- δ_1 measures the expected diff. in wage of married female as compared with single males, holding others factors constant.
- δ_2 → Same rationale



Case 2 We can use dummy variables to represent multiple categories of a variable Consider the relationship between law school rankings and starting salaries

$$\log(salary) = \beta_0 + \delta_0 top10 + \delta_1 r11\_25 + \delta_3 r26\_40 + \delta_4 r41\_60 + \beta_1 LSAT + \beta_2 GPA + \beta_3 \log(libvol) + \beta_4 \log(cost) + u.$$

where $top10$, $r11\_25$, $r26\_40$, $r41\_60$ would be equal to 1 when the variable $rank$ falls into the appropriate range.

** Rank below 60 would be the base case.

* In many cases the "range of value" serve as a better explanatory variable than the "value" itself.

eg. age may explain the model better if split into generations. young 0-15 gen z 16-29 etc.

```
. regress lsalary top10 r11_25 r26_40 r41_60 LSAT GPA llibvol lcost

      Source |       SS           df       MS      Number of obs   =       136
-------------+----------------------------------   F(  8,     127) =    120.15
       Model |  9.16538532         8  1.14567316   Prob > F        =    0.0000
    Residual |   1.2109665       127  .009535169   R-squared       =    0.8833
-------------+----------------------------------   Adj R-squared   =    0.8759
       Total |  10.3763518       135  .076861865   Root MSE        =    .09765

------------------------------------------------------------------------------
     lsalary |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       top10 |   .5393428    .053542    10.07   0.000     .4333927    .6452928
      r11_25 |   .4716199   .0390921    12.06   0.000     .3942637     .548976
      r26_40 |   .2790977   .0346972     8.04   0.000     .2104383    .3477571
      r41_60 |    .182382   .0283098     6.44   0.000      .126362     .238402
        LSAT |   .0060482   .0034919     1.73   0.086    -.0008616     .012958
         GPA |   .1305893   .0818678     1.60   0.113    -.0314122    .2925908
     llibvol |   .0725522   .0289213     2.51   0.013     .0153221    .1297824
       lcost |   .0249169   .0283224     0.88   0.381     -.031128    .0809619
       _cons |   8.363103   .4457314    18.76   0.000     7.481081    9.245125
------------------------------------------------------------------------------
```

the baseline is ranking 61th and worse.

Comments:

1) δ_0 measures the difference in expected log(salary) of a law-school graduate from a top 10 university compared to expected log(salary) of those who graduated from the school ranked 61th and worse.

2) δ_1 → use the same rationale.

rank	top 10	11-25	26-40
1	1	0	0
2	1	0	0
3	1	0	0
⋮	⋮	0	⋮
10	1	0	0
⋮	0	1	0
25	0	1	0
26	0	⋮	1
⋮	0	0	1
40	0	0	1
⋮	0	⋮	1
	⋮	⋮	0
			0

etc.