

Quiz 2

1. Let $A = \{0, 1, 2\}$ be the domain of variable x . Let the set of real number $\mathbb{R}$ be the domain of variable y . Consider the predicate

$$P(y) : \quad \forall x \in A, (x < y) \vee (x + y > 10).$$

- (a) Write the negation of $P(y)$ (without using negation symbol “ $\sim$ ” in the final answer).
 (b) Determine the **truth set** of the predicate $P(y)$. Explain your answer.

Answer:

- (a) Write the negation of $P(y)$ (without using negation symbol “ $\sim$ ” in the final answer).

$$\sim P(y) : \quad \exists x \in A, (x \geq y) \wedge (x + y \leq 10).$$

- (b) Determine the **truth set** of the above statement. Explain your answer.

Consider each value of $x \in \{0, 1, 2\}$.

- (i) For $x = 0$, we have $(0 < y) \vee (y > 10)$,
 which is true for $y > 0$ or $y > 10$. That is, $y \in (0, \infty)$.
 (ii) For $x = 1$, we have $(1 < y) \vee (1 + y > 10)$,
 which is true for $y > 1$ or $y > 9$. That is, $y \in (1, \infty)$.
 (iii) For $x = 2$, we have $(2 < y) \vee (2 + y > 10)$,
 which is true for $y > 2$ or $y > 8$. That is, $y \in (2, \infty)$.

The given statement has the universal quantifier $\forall$, so we need this to be true for all values of $x \in A = \{0, 1, 2\}$ and the value of y has to be an element in the intersection of all cases from (i)-(iii), i.e.

$$y \in (0, \infty) \cap (1, \infty) \cap (2, \infty) = (2, \infty).$$

That is, the truth set for $P(y)$ is the interval $(2, \infty)$.