Lecture note 1: EE320
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Reference:
Ch. 2/3/4: CW

Topics:

1) Math review on relation and function.
2) Some applications for economic model
3) Math review on solving for solution of system of equations.
4) Some applications of solving for solution of system of equations: static equilibrium analysis models

1) Math review: *relation and function*

1.1) Relation

Relation is a set-to-set mapping, defining the relationship between two sets of variables.

$$R = R : A \dashrightarrow B = A \times B = \{(x,y) | \, x \in A \text{ and } y \in B\}$$

We read: "relation R" is a cartesian product between A and B, such that the outcome from cartesian products result in a set of ordered-pairs (x,y) in which x belongs to A and y belongs to B.

For example, A is set value of price and B is set value of the amount of goods.

A = { 0 , 1, 2 , 3, … ,10}
B = { 0 , 1, 2 , 3, … ,10 }

A complete set of cartesian product from A to B is 10 x 10 elements of ordered pairs.

Relation from A to B could be specifically defined for only a *subset of the complete set* of cartesian product.

R = { (0,10) , (1,9), (2,8), (3,7), (4,6), (5,5), (6,4), (7,3), (8,2), (9,1), (10,0) }.

Graphically illustrated, mapping from A to B can be also represented by ordered pairs plotted on x-y coordinate.

Alternatively, a more common approach to characterize/represent relation is to use equation.

The relation above can be represented by $R = \{(x,y) \in R \times R | \, y = 10 - x \}$.

In term of economic interpretation, this relation captures the relationship that postulates demand of consumer. Why is it so?

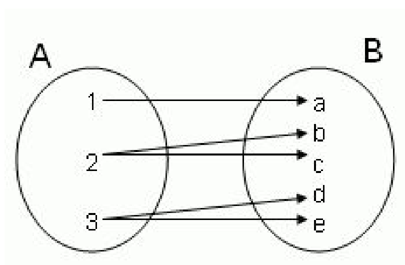
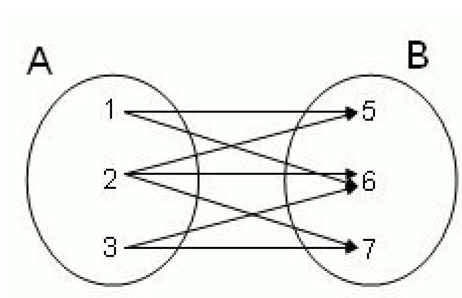
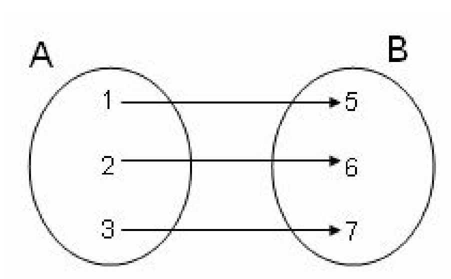
Mathematically, there are three types of relation.

a. one-to-many: a point in domain set is mapped into a number of points in anti-domain (range) set.
b. many-to-one: many points in the domain set are mapped into a single value in the anti-domain (range) set.
c. one-to-one (injective): each point in the domain set is mapped to different value in the anti-domain set.

Another special type of relation that mathematicians also define is an “onto (Surjective)” relation. An onto relation is a relation in which every single value in anti-domain has at least a single corresponding mapped from the domain set. To understand this easier, remember that an onto relation is the relation that set of range are all used up.

- Bijective relation = one-to-one + onto

Example: what types of relation are they?

R1		R4	
R2		R5	
R3		R6	

1.2) Function

Sometimes, we are only interested in special types of relation.

Function is the only two types of relation: *many-to-one and one-to-one*.

Some important operations on function.

- Inverse: reverse the ordered pairs of a given function

Example: $f(x) = x^2$, $g(x) = 2x + 1$ find inverse of "f" and "g".

- Composite: range of function has become domain of another function.

Example: $f(x) = x^2$, $g(x) = 2x + 1$. Find $f \circ g$, $g \circ f$, $g \circ g^{-1}$

- Algebraic: *addition, subtraction, multiplication, division*

Example: f(x) = 2x-1 and g(x) = 5. Find f+g, g/f and f*g. Determine domain and range of these functions.

1.3) Two major classes of function: Linear vs non-linear function.

1.3.1) Linear function

Functional form: $Y = mX + b$.

"m" is the slope of linear function = change in Y over change in X = "rise" *over* "run".

b is the value of y-intercept. That is, the value of Y when X = 0.

Properties of linear function:

(i) the value of "m" tells us about the shape of linear function.
$m > 0$ == upward sloping
$m < 0$ == downward sloping
$m = 0$ == Horizontal line that is parallel to x-axis.

(ii) the large value of "m" measured in absolute term, the stepper the line has.
So, the extreme case for the value of slope is when "m" is infinity. That is, we have vertical line parallel to y-axis.

(iii) Two lines are parallel to each other when $m1 = m2$. (two lines that never cross on x-y axis.)

(iv) Two lines are perpendicular/orthogonal when $m1*m2 = -1$.

<u>What do we do with linear equation?</u>

Things you would have to normally do: knowing how to plot linear equation or deriving linear equation from given sets of information.

<u>Example:</u> plotting linear equations
L1: $y = -3x + 4$;
L2: $-3x + y = 7$;
L3: $y = (\frac{1}{3})x - 5$;
L4: $y = 5$.
L5: $x = 7$.

Example: A(1,0) lies on L1. Find the equation for L1 if slope is equal to 5. (This is point-slope case!)

Example: A line goes through the following two points: A(2,4) and B(7,6). Find the equation for the line that goes through these two points. (This is two-point formula case.)

Step 1:

Step 2:

Application: a simple linear economic model for Break-even analysis

A representative firm is operating in a perfectly competitive market. Assume that market price is equal to $7. Suppose that firm faces a constant value of AVC (average variable cost) equal to $3. Additionally, firm must pay for the fixed cost of the operation equal to $100. (Suppose the following notations: Q is quantity of output, P is price per unit.)

a. Find the level of output that results in break-even situation of this firm.
b. Would the answer as in “a” change if fixed cost is change to $150 and $200, respectively. Can we drawn any implications from this question?

Application: Understanding **“**Ceteris Paribus”

One usually sees in textbooks/examples that individual demand function takes the form of $Q_x = a - bP_x$. Economically, such form is actually being simplified from a true underlying structural relationship that is given by $Q^D{}_x = 100 - 0.2 * P_x + 0.2 * P_y - 0.01 * P_z + 0.002 * Income$

a. Calculate quantity demanded for “x” if Px= 10, Py = 10, Pz = 100 and Income = 1,000.
b. Suppose all everything is fixed to the numeric values given in “a”, except Px that is allowed to be varied. Plot demand for x on p-q diagram, where P is vertical axis and Q is horizontal axis.
c. How would you prove the pairwise relationship between goods x, good y and good z. (no need to think about the relationship between good y and good z.)
d. Is X a normal goods?

Application: Elasticity of supply

Suppose that the individual supply is equal to $P = 3 + Q$.

a. Calculate the value of elasticity of supply when $P = 1$.
b. Show in general case for $P = a + bQ$ that elasticity of supply is always greater than one if $a > 0$, equal to 1 if $a=0$, and less than one if $a < 0$.

Application: Restricting domain set for demand and the construction of market demand.

A study has shown that there are three groups of iphone user, namely, *crazy, love-it, and just-live-with-it.* Demand for iphone of each group can be given by:

crazy: $Qc = 100 - p$;
Love-it: $p = 50 - Ql$;
Just-live-with-it: $Qj = 20 - p$;

a. Find the domain set of price that justifies demand equation for each group of iphone user.
b. At what price, do all the three types of iphone users stay active in the market?
c. Find the market demand for iphone. Be precise about what is needed to make your equation justified.

1.3.2) Non-linear function

We define non-linear function as any function that is not linear. (Sound weird? But, this is how mathematicians define things in mathematics.)

Now, we will discuss about some types of non-linear function that will be commonly used in economics analysis.

a. Polynomial function

Functional form: $y = a_0 + a_1 x + a_2 x^2 + ... + a_n x^n$

When n = 1, polynomial function becomes just linear function.

When n = 2, polynomial function becomes known as the *quadratic function*.

- Quadratic function is the function that has the so-called parabola shape. That's, the function that produces either a U-shape and an inverted U-shape curve, depending on the value of coefficient associated with the second-order term in the function.
- Suppose that the function take the following form: $y = a + bx + cx^2$.
- If c > 0 ⇒ U-shape. If c < 0 ⇒ inverted U-shape.
- A U-shape parabola would have a single turning point which is global minimum point, and vice versa for the case of inverted U-shape parabola.

The extreme point x* would be equal to $-b/2a$. How about extreme value?

<u>Example</u>: (Revenue function in the quadratic form) Suppose that demand function is given by: P = 200 - Q. P is the price per unit, and Q is the amount of goods purchased.

a. Find the revenue function.
b. Plot graph for the revenue function, and locate some important points.
c. What is the level of revenue-maximizing output?
d. Suppose further that cost function can be given by $C(Q) = 200Q + 100$, find break-even output.
e. What is the level of profit-maximizing output/price?

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b. Exponential function

$y = a^x$

Exponential function with base equal to “a”.

Property of power function associated to value of a.

- a > 1: upward sloping
- 0 < a < 1: downward sloping
- a < 0 = oscailte pattern

Example: $y = 3^{2x}$, $y = (1/4)^x$ $y = 4^{-x}$

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c. Logarithm function

An inverse operation of the exponential function.

$y = a^x \Rightarrow y = log_a x$

When the value of “a” is equal to the exponent number, we call the inverse of such exponential function as natural log function.

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2) **Solving for the system of equations.**

<u>Linear case</u>

We would now only focus on *2-by-2 case.* That is, we only have two variables.

This is a simple case that we can solve for the solution of simultaneous equation by hands. The method is called the *substitution method.*

<u>Example:</u> Types of solution

System 1: $2x + 3y = 7$ and $x - y = 5$.

System 2: $y = -2x + 6$ and $4x + 2y = 12$.

System 3: $x + 2y = 14$ and $3x + 6y = 8$

Application: Market equilibrium model (general idea)

Suppose that market demand and supply can be given by the following two equations:
Qd = 10 - 3P and P = 2 + Qs.

a. Graph demand and supply equation.
b. Find market equilibrium for quantity of output and price.
c. Suppose now we generalize that Qd = 30 - 5*income - 3P. At what level of income, does this generalized form coincide with the form of demand equation given above?
d. Solve for the market equilibrium by using the specification of demand given in "c". What happen to equilibrium output/price when income is $5 higher than the level associated to that in "c". Is the product normal/inferior goods?