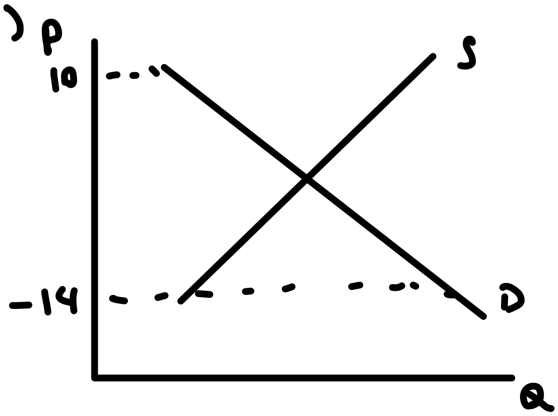
<u>EE320 Placement test</u>

1. Attempt all.
2. Submit your work (in .pdf) on the Moodle. The required format of your filename is studentID_PT
3. You will get **TWO** bonus points if you submit this placement test by the deadline.
4. **This placement test is due on Friday 14th, at 11 AM. Late submission will not be accepted.**

1. Suppose that market demand is given by $P = 10 - Q^2$ and the market supply is given by $Q = a + P$, where P is the unit price, Q is the quantity of output, and a is the coefficient in the supply equation.
   1.1) Graph the market demand and market supply curve in a P-Q diagram. Set the value of a equal to – 14.
   1.2) Solve for the market equilibrium quantity (Q*) and price (P*) when a = -14. Show your work.
   1.3) If "a" increases to -12, what would happen to the market equilibrium quantity and price? State the qualitative predictions without redoing the algebra.

1.1)



1.2) $P = Q$

$= 10 - Q^2 = -14 + P$

1.3) if a increase quantity will increase and shif of supply curve to the left make a new equilibrium

2. Suppose that the revenue function is given by $R(Q) = \ln(Q^2+1) + 3\left(\frac{Q}{Q+1}\right), Q \geq 0$. Use the derivative technique and calculate the marginal revenue function. Is the revenue function an increasing or decreasing function?

$$\frac{dR}{dQ} = \frac{1}{Q^2+1} 2Q + 3\left(\frac{Q+1-Q}{(Q+1)^2}\right)$$

$$= \frac{2Q}{Q^2+1} + \frac{3}{(Q+1)^2}$$

This derivative is always positive when $Q \geq 0$

so the revenue function an increasing function

3. Suppose that the profit function is given by $\pi(Q) = -\frac{1}{3}Q^3 - Q^2 + 8Q - 1$ where Q is the level of output. Use the calculus and solve for the level of profit-maximizing output. Confirm your answer with the second derivative.

$$\frac{d\pi}{dQ} = -Q^2 - 2Q + 8 = 0$$

$$-(-Q^2 - 2Q + 8) = 0$$

$$Q^2 + 2Q - 8 = 0$$

$$(Q+4)(Q-2) = 0$$

$$Q = -4, 2$$

$$\frac{d^2\pi}{dQ^2} = -2Q - 2$$

$$Q = -4; -2(-4) - 2 = 6$$

$$Q = 2 \quad -2(2) - 2 = -6$$

Q = 2 is a maximum value

4. Suppose that $A = \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$, calculate the following object. Show your work.

4.1 A + B

undefined

4.2 A*B

$$\begin{bmatrix} 8\times1+9\times4 & 8\times2+9\times5 & 8\times3+9\times6 \\ 10\times1+9\times4 & 10\times2+9\times5 & 10\times3+9\times6 \end{bmatrix} = \begin{bmatrix} 44 & 61 & 78 \\ 54 & 75 & 96 \end{bmatrix}$$

4.3 det(A)

$11\times8 - 9\times10 = -2$

4.4 det(B)

undefined

4.5 det(C)

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \begin{matrix} 1 & 2 \\ 4 & 5 \\ 7 & 8 \end{matrix}$$

$(1\times5\times9 + 2\times6\times7 + 3\times4\times8) - (7\times5\times3 + 8\times6\times1 + 9\times4\times2)$

$= 0$

5. Suppose that $U(x, y) = x^a y^b + \ln\left(\frac{x}{x+y}\right)$. Use the partial derivative technique, calculate $\frac{\partial U}{\partial x}$ and $\frac{\partial U}{\partial y}$.

$$\frac{\partial U}{\partial x} = a(x^{a-1})y^b + \left(\frac{x+y}{x}\right)\left(\frac{x+y-x}{(x+y)^2}\right)$$

$$\frac{\partial U}{\partial y} = x^a y(y^{b-1}) + \left(\frac{x+y}{x}\right)\left(\frac{x+y-x}{x+y}\right)$$