B.E. International Program
Faculty of Economics, Thammasat University

EE 320 Introductory Mathematical Economics
Semester 1/2019

Assignment 2 - Solution

Due 26th September 2019

Question 1: *IS-LM model (Matrix Algebra)*

Consider the following IS-LM model:

Commodity market:

$Y = C + I + G_0, \quad (G_0 > 0)$

$C = a + bY_d, \quad (0 < b < 1)$

$Y_d = Y - T,$

$T = T_0 + tY, \quad (T_0 > 0, 0 < t < 1)$

$I = I_0 - kr, \quad (I_0 > 0, k > 0)$

Money market:

$M_s = M_0$

$M_D = mY - hr, \quad (m > 0, h > 0)$

a. (2 points) Write a matrix form of the IS-LM equations with Y and r as the endogenous variables.

Ans.

$$\begin{bmatrix} 1-b(1-t) & k \\ m & -h \end{bmatrix} \begin{bmatrix} Y \\ r \end{bmatrix} = \begin{bmatrix} a - bT_0 + I_0 + G_0 \\ M_0 \end{bmatrix}$$

b. (2 points) State the condition for the existence of the equilibrium national income and interest rate.

Ans.

$$\begin{vmatrix} 1-b(1-t) & k \\ m & -h \end{vmatrix} \neq 0$$

$\Rightarrow \quad -h[1-b(1-t)] - km \neq 0,$

Or $\quad -h + hb - hbt - km \neq 0$

Or $\quad h[1-b(1-t)] + km \neq 0$

c. (4 points) Solve for the equilibrium level of national income and interest rate by using Cramer's rule.

Ans.

$$Y^* = \frac{\begin{vmatrix} a-bT_0+I_0+G_0 & k \\ M_0 & -h \end{vmatrix}}{\begin{vmatrix} 1-b(1-t) & k \\ m & -h \end{vmatrix}} = \frac{h(a-bT_0+I_0+G_0)+kM_0}{h[1-b(1-t)]+km}$$

$$r^* = \frac{\begin{vmatrix} 1-b(1-t) & a-bT_0+I_0+G_0 \\ m & M_0 \end{vmatrix}}{\begin{vmatrix} 1-b(1-t) & k \\ m & -h \end{vmatrix}} = \frac{m(a-bT_0+I_0+G_0)-[1-b(1-t)]M_0}{h[1-b(1-t)]+km}$$

d. (2 points) Determine the rate of change of equilibrium national income with respect to the government expenditure (*G*), assuming that everything else remains constant.

Ans.

$$\frac{dY^*}{dG_0} = \frac{h}{h[1-b(1-t)]+km} > 0$$

Question 2: *Market equilibrium (Matrix Algebra)*

Given the following supply and demand functions:

$Q_d = 100 - 3P$

$$Q_s = 80 + 2P$$

a. (2 points) Write the equilibrium condition for this market, and translate the system of equations into matrix notation.

Eq'm condition: $Q_d = Q_s$.

System of equations:

$$\begin{aligned} Q_d - Q_s &= 0 \\ Q_d + 3P &= 100 \\ Q_s - 2P &= 80 \end{aligned}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 1 & 0 & 3 \\ 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} Q_D \\ Q_S \\ P \end{bmatrix} = \begin{bmatrix} 0 \\ 100 \\ 80 \end{bmatrix}$$

b. (2 points) Use <u>matrix inversion</u> to solve for the equilibrium quantity and equilibrium price.

$$A^{-1} = \frac{1}{5}\begin{bmatrix} 3 & 2 & 3 \\ -2 & 2 & 3 \\ -1 & 1 & -1 \end{bmatrix}$$

$$\Rightarrow \rightarrow \begin{bmatrix} Q_D \\ Q_S \\ P \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 3 & 2 & 3 \\ -2 & 2 & 3 \\ -1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 0 \\ 100 \\ 80 \end{bmatrix} = \begin{bmatrix} 88 \\ 88 \\ 4 \end{bmatrix}$$

c. (6 points) Suppose that the government subsidizes the consumption of this good by giving the consumer \$5 per unit of the goods consumed. Write the new equilibrium condition for this market in the matrix form, and use <u>Cramer's rule</u> to solve for (i) the equilibrium price paid by the consumer, (ii) the price received by the producer, and (iii) the amount of money the government needs for this subsidization.

(i) $P_d^* = P_s^* - 5 = \$7 - \$5 = \$2$

(ii) $P_s^* = \$7$

(iii) $Q^* = 94;\ S^* = 94x\$5 = \$470$