SUBJECT: .... NO: .... DATE: ..../..../....

Summary : Determinantal Tests for Relative Constrained Extremum

I. Max (Min) $z = f(x,y)$ s.t. $g(x,y) = c$

$\Rightarrow L(x,y,\lambda) = f(x,y) + \lambda[c - g(x,y)]$

	Maximum	Minimum
FONC :	$L_x = L_y = L_\lambda = 0$	
SOSC :	$\lvert\bar{H}\rvert > 0$	$\lvert\bar{H}\rvert < 0$

II Max (Min) $z = f(x_1, x_2, \ldots, x_n)$ s.t. $g(x_1, \ldots, x_n) = c$

$\Rightarrow L(x_1, x_2, \ldots, x_n, \lambda) = f(x_1, x_2, \ldots, x_n) + \lambda[c - g(x_1, x_2, \ldots, x_n)]$

	Maximum	Minimum
FONC:	$L_1 = L_2 = \ldots = L_n = L_\lambda = 0$	
SOSC:	$\lvert\bar{H}_2\rvert > 0;\ \lvert\bar{H}_3\rvert < 0;\ \ldots;\ (-1)^n \lvert\bar{H}_n\rvert > 0$	$\lvert\bar{H}_2\rvert, \lvert\bar{H}_3\rvert, \ldots, \lvert\bar{H}_n\rvert < 0$

III Max (Min) $z = f(x_1, x_2, \ldots, x_n)$ s.t. $g^j(x_1, \ldots, x_n) = c_j$, $j = 1, \ldots, k$ $(k < n)$.

$$\Rightarrow L = f(x_1, x_2, \ldots, x_n) + \sum_{j=1}^{k} \lambda_j \left[c_j - g^j(x_1, \ldots, x_n) \right]$$

	Maximum	Minimum
FONC :	$L_1 = L_2 = \ldots = L_n = L_{\lambda_1} = L_{\lambda_2} = \ldots = L_{\lambda_n} = 0$	
SOSC :	$\lvert\bar{H}_{k+1}\rvert, \lvert\bar{H}_{k+2}\rvert, \ldots, \lvert\bar{H}_n\rvert$ alternates in sign; where sign of $\lvert\bar{H}_{k+1}\rvert$ is sign of $\boxed{(-1)^{k+1}}$	sign of all $\lvert\bar{H}_{k+1}\rvert, \ldots, \lvert\bar{H}_n\rvert$ is sign of $\boxed{(-1)^{k}}$

Note : The subscript number of the bordered Hessian is the dimension of the Hessian inside the $\lvert\bar{H}\rvert$. Ex.

$$\lvert\bar{H}_2\rvert = \begin{vmatrix} 0 & g_1 & g_2 \\ g_1 & L_{11} & L_{12} \\ g_2 & L_{21} & L_{22} \end{vmatrix} \ (2 \times 2) ; \quad \lvert\bar{H}_3\rvert = \begin{vmatrix} 0 & g_1 & g_2 & g_3 \\ g_1 & L_{11} & L_{12} & L_{13} \\ g_2 & L_{21} & L_{22} & L_{23} \\ g_3 & L_{31} & L_{32} & L_{33} \end{vmatrix} \ (3 \times 3) ; \ \ldots$$