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Group Homework 1

Semester 2/2022 EE320 Introductory mathematical economics

Due date: Feb 3rd 2022 (before midnight /B.E. moodle).

Note: Late homework will not be accepted. Use the format of filename as required; this will cost you two points if you don't follow the instruction.

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1. Consider the market for good x. Suppose that the market demand equation is given by

$$P_x = 8 - bQ_x^d + cP_y; \quad b > 0 \; and \; c > 0$$

and the equation for market supply is given by

$$P_x = 20 + dQ_x^S; \quad d > 0$$

where P_x is price of good X, P_y is price of good Y, Q_x^d is quantity demanded for good X and Q_x^S is quantity supplied for good X. Answer the following questions.

a) What is(are) the endogenous variable(s) in the model? What is(are) the exogenous variable(s) in the model?

b) Specify condition(s) under which the market equilibrium for good x is guaranteed to exist. What restrictions do we need to place on the values of P_y so that the market equilibrium exists?

c) Solve for the equilibrium price (P_x^*) and equilibrium quantity (Q_x^*) of the market for good x.

d) Calculate the magnitude of the response of equilibrium quantity to the change in exogenous variable(s).

2. (an old midterm exam question) Consider a market with *10 identical consumers*. Each of the consumer's demand function is given by:

$$P = 10 + k_1 P_x + k_2 Y - Q_j^d,$$

where P is the unit price of the product sold in this market, Q_j^d is the amount of quantity demanded by the j-th consumer, P_x is the price of product x, and Y is the level of income. Assume further that the industry is controlled by two producers, each of whom has the following supply function:

$$P = 5 + k_3 W + Q_1^s, \qquad \text{and}$$

$$P = 20 + k_4 T + 2Q_2^s,$$

where Q_1^s and Q_2^s are the amount of quantity supplied by the first and second producer, respectively. W is the price of gasoline and T is the level of technology. All the parameters are STRICTLY positive.

Use the information given to answer the following questions:

a. Is "product x" considered as a *substitute* product or a *complementary* product?
b. Derive the market demand equation.

Now, I supplement two pieces of information to be used in the remaining parts of this question. That is, I assume that

(i) $0 < (20 + k_4 T) < (5 + k_3 W)$ and (ii) $10 + k_1 P_x + k_2 Y > 5 + k_3 W$

c. What does the first condition mean in terms of the relative cost advantages between the two firms? Given your interpretation, derive the market supply equation.
d. Given (i) and (ii), state the condition under which the market ceases to have only *single* firm that stays active in the business?

Continue with the information given above, but now consider a specific case where the value of coefficients and exogenous variables are given in the following table:

Coefficients	k_1	k_2	k_3	k_4
Value	2	2	3	1

Variables	Y	P_x	w	T
Value	5	10	10	5

e. Solve for the equilibrium price *and* quantity.

f. Suppose that the government provides a subsidy of $5 for each unit of output that the consumers have purchased. Calculate the benefit that consumers and each of the two producers receive under the subsidy program.

3. Consider a simple macroeconomics model given below

$C = 0.3Y_d - k_1 r;$ $\qquad k_1 > 0.$

$I = 0.5Y - k_2 r;$ $\qquad k_2 > 0.$

$Y_d = Y - T$

$G = G_0$

$T = T_0$

$M^d = k_3 Y - k_4 r;$ $\qquad k_3 > 0 \;\; and \;\; k_4 > 0.$

$M^s = M_0$

where $Y = GDP, \; C = consumption, \;\; I = investment, \; G = government\ purchase, \; T = tax, \; r = interest\ rate, \; M^d = money\ demand, \ and\ M^s = money\ supply$

a. Write the system of linear equations in matrix form with two variables included, namely Y and r.
b. Solve for the equilibrium solution (Y^*, r^*)
c. Has fiscal policy, for example, the change in government purchase, become *more effective* if both k_1 and k_2 are getting bigger? Show your numerical result, and explain your result with some economic intuitions. (Hint: how do we measure the effectiveness of a policy?)

4. Let the demand function be $P = 14 - 3Q$ and the supply function be $P = 4 + 2Q$. Suppose that the government imposes tax by \$t per unit of output. This tax is assumed to impose on consumer. Answer the following questions.

   a. Find the equilibrium under pre-tax situation. That is, when "t" is set to equal to zero.
   b. State the condition that links between consumer's and producer's price
   c. Find the equilibrium after tax. (Hint: your solution should be written in terms of "t".)
   d. Calculate the consumers' and producers' burden. Which group is paying more for the tax under the equilibrium?
   e. Find the expression of the revenue that the government can collect from the market under the equilibrium.
   f. If the government were to collect tax so that total revenue is maximized, what is the appropriate level of unit tax, "t", that it should impose to the market?

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where P_x is price of good X, P_y is price of good Y, Q_x^d is quantity demanded for good X and Q_x^S is quantity supplied for good X. Answer the following questions.

a) What is(are) the endogenous variable(s) in the model? What is(are) the exogenous variable(s) in the model?

Ans. Endogenous variables : Q^d, Q^s, P_x

Exogenous variables : P_y

b) Specify condition(s) under which the market equilibrium for good x is guaranteed to exist. What restrictions do we need to place on the values of P_y so that the market equilibrium exists?

Ans. $P_x = 8 - bQ_x^d + cP_y$ — ①

$P_x = 20 + dQ_x^S$ — ②

① = ②

$8 - bQ_x^d + cP_y = 20 + dQ_x^S$

$cP_y = 20 + dQ_x^S + bQ_x^d - 8$

$P_y = \dfrac{12 + dQ_x^S + bQ_x^d}{c}$

∴ Restriction we need to place is $c > 0$

c) Solve for the equilibrium price (P_x^*) and equilibrium quantity (Q_x^*) of the market for good x.

$Q_x^* = Q_x^d = Q_x^S$

① = ②

$8 - bQ_x^* + cP_y = 20 + dQ_x^*$

$8 - 20 + cP_y = dQ_x^* + bQ_x^*$

$-12 + cP_y = (d+b)Q_x^*$

$Q_x^* = \dfrac{12 + cP_y}{(d+b)}$

Plug in Q_x^* in equation 2

$P_x^* = 20 + d\left(\dfrac{12 + cP_y}{d+b}\right)$

$P_x^* = \dfrac{20b + 8d + cdP_y}{d+b}$ ※

d) Calculate the magnitude of the response of equilibrium quantity to the change in exogenous variable(s).

magnitute → $Q_x^* = \frac{-12 + cp_y}{(d+b)}$

$$\frac{\partial Q_x^*}{\partial p_y} = \frac{c}{d+b}$$ #

2. (an old midterm exam question) Consider a market with *10 identical consumers*. Each of the consumer's demand function is given by:

$$P = 10 + k_1 P_x + k_2 Y - Q_j^d,$$

where P is the unit price of the product sold in this market, Q_j^d is the amount of quantity demanded by the j-th consumer, P_x is the price of product x, and Y is the level of income. Assume further that the industry is controlled by two producers, each of whom has the following supply function:

$$P = 5 + k_3 W + Q_1^s, \qquad \text{and}$$

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where Q_1^s and Q_2^s are the amount of quantity supplied by the first and second producer, respectively. W is the price of gasoline and T is the level of technology. All the parameters are STRICTLY positive.

Use the information given to answer the following questions:

a. Is "product x" considered as a *substitute* product or a *complementary* product?

$$P = 10 + K_1 P_x + K_2 Y - Q_j^d$$

$$\rightarrow Q_j^d = 10 + K_1 P_x + K_2 Y - P$$

10 identical consumers $= 10(Q_j) = 10(10 + K_1 P_x + K_2 Y - P)$
$= 100 + 10K_1 P_x + 10K_2 Y - 10P$

①: $P = 5 + K_3 W + Q_1^s$

$Q_1^s = 5 + K_3 W - P$

$Q_j^d = 10 + K_1 P_x + K_2 P_y - P$

$P \uparrow \rightarrow Q_j^d \downarrow$

②: $P = 20 + K_4 T + 2Q_2^s$

$Q_2^s = \dfrac{20 + K_4 T - P}{2}$

$Q_j^d = 10 + K_1 P_x + K_2 Y - P$

$P \uparrow \rightarrow Q_j^d \downarrow$

∴ Product X is a complementary product since the prices from Producer ① & ② increases make Q_j^d decreases.

b. Derive the market demand equation.

Now, I supplement two pieces of information to be used in the remaining parts of this question. That is, I assume that

(i) $0 < (20 + k_4 T) < (5 + k_3 W)$ and (ii) $10 + k_1 P_x + k_2 Y > 5 + k_3 W$

$$Q_j^d = 10 + k_1 P_x + k_2 Y - P$$

$$\rightarrow Q^d = 10(Q_i)$$
$$= 10(10 + K_1 P_x + K_2 Y - P)$$
$$= 100 + 10K_1 P_x + 10K_2 Y - 10P \quad \#$$

$$Q^d = \begin{cases} 10(10 + K_1 P_x + K_2 Y - P), & P > 10 + K_1 P_x + K_2 Y \\ 0, & P \leq 10 + K_1 P_x + K_2 Y \end{cases} \quad \#$$

c. What does the first condition mean in terms of the relative cost advantages between the two firms? Given your interpretation, derive the market supply equation.

- Producer ② has lower reservation than ①
  has higher cost advantages among two producer

$$Q_1^s = P - (5 + K_3 W) \; ; \; Q_1^s > 0 \; ; \; P > (5 + K_3 W)$$
$$Q_1^s = 0 \; ; \; P \leq (5 + K_3 W)$$

$$Q_2^s = P - (20 + K_4 T)\tfrac{1}{2} \; ; \; Q_2^s > 0 \; ; \; P > (20 + K_4 T)$$
$$Q_2^s = 0 \; ; \; P \leq (20 + K_4 T)$$

$$Q_s \begin{cases} 0 + 0, & 0 \leq P \leq (20 + K_4 T) \\ P - (20 + K_4 T)\tfrac{1}{2}, & (20 + K_4 T) < P \leq (5 + K_3 W) \\ Q_1^s + Q_2^s, & P > (5 + K_3 W) \end{cases}$$

d. Given (i) and (ii), state the condition under which the market ceases to have only *single* firm that stays active in the business?

Continue with the information given above, but now consider a specific case where the value of coefficients and exogenous variables are given in the following table:

Coefficients	k_1	k_2	k_3	k_4
Value	2	2	3	1

Variables	Y	P_x	w	T
Value	5	10	10	5

$$Q^d = \begin{cases} 10(10 + k_1 P_x + k_2 y - P), & P > 10 + k_1 P_x + k_2 y \\ 0, & P \leq 10 + k_1 P_x + k_2 y \end{cases}$$

$$Q^s = \begin{cases} 0 + 0, & 0 \leq P \leq (20 + k_4 T) \\ P - (20 k_4 T) \times \frac{1}{2}, & (20 + k_4 T) < P \leq (5 + k_3 w) \\ Q_1^s + Q_2^s, & P > (5 + k_3 w) \end{cases}$$

e. Solve for the equilibrium price *and* quantity.

Equilibrium ; $Q^d = Q^s$

$$100 + 10k_1 P_x + 10k_2 y - 10P = (P - 5 - k_3 w) + \frac{(P - 20 - k_4 T)}{2}$$

$$\times 2; \quad 200 + 20k_1 P_x + 20k_2 y - 20P = 2P - 10 - 2k_3 w + P - 20 - k_4 T$$

$$210 + 20k_1 P_x + 20k_2 y = 23P - 2k_3 w - 20 - k_4 T$$

$$P = \frac{210 + 20kP_x + 20k_2 y + 20k_3 w + 20 + k_4 T}{23}$$

$$P = \frac{210 + 20(20) + 20(10) + 2(30) + 20 + 5}{23}$$

$$= \frac{895}{23} \approx 38.91 \qquad P^* = 38.91$$

sub in supply equation

$$Q^* = (P-5-k_3W) + \frac{(P-20-k_4T)}{2}$$

$$Q^* = \frac{2P-10-2k_3W+P-20-k_4T}{2}$$

$P = 38.91$; $Q = 13.365$

sub in demand $Q^* = 100 + 10k_1P_x + 10k_2y - 10P$

$= 100 + 200 + 100 - 389.1$

$= 10.9$

$(P-5-k_3w) + (P-20-k_4T)$

$3P - 10 - 2k_3W - 20 - k_4T$

$3(38.91) - 10 - 2(3.10) - 20 - 5 = Q^*$

6.2

$Q^* = 75.53$ #

f. Suppose that the government provides a subsidy of $5 for each unit of output that the consumers have purchased. Calculate the benefit that consumers and each of the two producers receive under the subsidy program.

$$p_1^s - 5 = p^d \rightarrow 5 + k_3 w + Q_1^s - 5 = 10(10 + K_1 Y + K_2 Y - Q_j^d)$$

$$p_2^s - 5 = p^d \rightarrow 20 + K_4 T + 2Q_2^s - 5 = 10(10 + K_1 P_x + K_2 Y - Q_j^d)$$

with subsidy

$$k_3 w + Q_1^s = 100 + 10k_1 P_x + 10 k_2 Y - 10Q^d$$

$$15 + k_4 T + 2Q_2^s = 100 + 10k_1 P_x + 10 k_2 Y - 10Q^d$$

Producer 1

$$11Q = 370$$

$$Q_1^* = 33.63$$

$$p_1^{d*} = (10 + k_1 P_x + K_2 Y - Q^d) \times 10$$

$$= (10 + 20 + 10 - 33.63) \times 10$$

$$= \$63.7$$

$$p_1^{s*} = p^d + 5$$

$$= \$68.7$$

	Before subsidy	after subsidy
Q_1	31.51	33.63
p_1^d	42.17	63.7
p_1^s	42.17	68.7
Subsidy	0	168.15

Subsidy = $5 per unit × 33.63 unit

= $ 168.15

Producer

$42.17 - 63.7 \times 33.63$

$= -724.05$

consumer

$(68.7 - 42.17) \times 33.63$

$= 892.2$

Producer 2

$$15 + 5 + 2Q = 100 + 200 + 100 - 10Q$$

$$20 + 2Q = 400 - 10Q$$

$$12Q = 380$$

$$Q = 31.667$$

P^d

③

a. Write the system of linear equations in matrix form with two variables included, namely Y and r.

$$IS: \; Y = C + I + G$$

$$Y = 0.3Y_d - K_1 r + 0.5Y - K_2 r + G_0$$

$$Y = 0.3(Y - t) - K_1 r + 0.5Y - K_2 r + G_0$$

$$Y = 0.3Y - 0.3t - K_1 r + 0.5Y - K_2 r + G_0$$

$$Y - 0.3Y - 0.5Y = -0.3t - K_1 r + 0.5Y - K_2 r + G_0$$

$$0.2Y = -0.3t - K_1 r + 0.5Y - K_2 r + G_0$$

$$Y = -1.5t + 5G_0 - 5r(K_1 + K_2) \; \#$$

b. Solve for the equilibrium solution (Y^*, r^*)

$$r \rightarrow \frac{M^s}{P} = L^d$$

$$M^s = M^d$$

$$M_0 = K_3 Y - K_4 r$$

$$r^* = \frac{K_3 Y - M_0}{K_4} \; \#$$

Equilibrium: $Y = -1.5T + 5G_0 - 5\left(\frac{K_3Y - M_0}{K_4}\right)(K_1 + K_2)$

$$Y = -1.5T + 5G_0 - 5K_1\left(\frac{K_3Y - M_0}{K_4}\right) - 5K_2\left(\frac{K_3Y - M_0}{K_4}\right)$$

$$Y = -1.5T + 5G_0 - \frac{5K_1K_3Y + 5K_1M_0}{K_4} - \frac{5K_2K_3Y + 5K_2M_0}{K_4}$$

$$Y = -1.5T + 5G_0 - \frac{5K_1K_3Y}{K_4} + \frac{5K_1M_0}{K_4} - \frac{5K_2K_3Y}{K_4} + \frac{5K_2M_0}{K_4}$$

$$Y + \frac{5K_1K_3Y}{K_4} + \frac{5K_2K_3Y}{K_4} = -1.5T + 5G_0 + \frac{5K_1M_0}{K_4} + \frac{5K_2M_0}{K_4}$$

$$Y\left(1 + \frac{5K_1K_3Y}{K_4} + \frac{5K_2K_3Y}{K_4}\right) = -1.5T + 5G_0 + \frac{5K_1M_0}{K_4} + \frac{5K_2M_0}{K_4}$$

$$Y^* = \frac{-1.5T + 5G_0 + (5K_1M_0/K_4) + (5K_2M_0/K_4)}{1 + (5K_1K_3/K_4) + (5K_2K_3/K_4)}$$

$$Y^* = \frac{(-1.5TK_4 + 5G_0K_4 + 5K_1M_0 + 5K_2M_0)/\cancel{K_4}}{(K_4 + 5K_1K_3 + 5K_2K_3)/\cancel{K_4}}$$

$$y^* = \frac{-1.5TK_4 + 5G_0K_4 + 5K_1M_0 + 5K_2M_0}{K_4 + 5K_1K_3 + 5K_2K_3}$$ #

c. Has fiscal policy, for example, the change in government purchase, become *more effective* if both k_1 and k_2 are getting bigger? Show your numerical result, and explain your result with some economic intuitions. (Hint: how do we measure the effectiveness of a policy?)

$$y^* = \frac{-1.5TK_4 + 5G_0K_4 + 5K_1M_0 + 5K_2M_0}{K_4 + 5K_1K_3 + 5K_2K_3}$$

$$\frac{\Delta y^*}{\Delta G_0} = \frac{5K_4}{K_4 + 5K_1K_3 + 5K_2K_3}$$

∴ No, fiscal policy has not become more effective since if k_1 and k_2 are getting bigger, y^* will fall down.

a. Find the equilibrium under pre-tax situation. That is, when "t" is set to equal to zero. $t = 0$

4

$$\text{Demand} = \text{Supply}$$
$$14-3Q = 4+2Q$$
$$5Q = 10$$
$$Q = 2$$

∴ Pre tax equilibrium ; $Q^* = 2$ → plug in Q in Demand equation

$$14-3(Q) = 14-3(2)$$
$$P^* = 8$$

b. State the condition that links between consumer's and producer's price

Demand ; $P = 14-3(Q)$

$$Q^d = \frac{14-P}{3}$$

$$Q^d = \begin{cases} 0, & P \geq 14 \\ \frac{14-P}{3}, & 0 \leq P < 14 \end{cases}$$ #

Supply ; $P = 4+2Q^2$

$$Q^s = \frac{P-4}{2}$$

$$Q^s = \begin{cases} 0, & 0 \leq P < 4 \\ \frac{P-4}{2}, & P > 4 \end{cases}$$ #

∴ The reservation price for consumer is \$14 while the reservation price for producer is \$ 4. Thus, an equilibrium exists.

c. Find the equilibrium after tax. (Hint: your solution should be written in terms of "t".)

Tax on consumer; $P^d = P^s + t$

Equilibrium after tax; $4 + 2Q + t = 14 - 3Q$

$$5Q + t = 10$$

$$t = 10 - 5Q$$

and $Q^{*\ a/f\ tax} = \frac{10-t}{5}$

Q^* after tax : sub ① into demand function before tax,

$$14 - \left(\frac{10-t}{5} \times 3\right) = P^{d*}$$

$$14 - \frac{30+3t}{5} = P^{d*}$$

$$P^{d*\ a/f\ tax} = \frac{40+3t}{5} = 8 + \frac{3}{5}t \ \#$$

sub ① into supply function before tax to find P^{s*} after tax,

$$P^{s*\ af\ tax} = 4 + 2 \cdot \left(\frac{10-t}{5}\right)$$

$$= 4 + \frac{20-2t}{5} = 8 - \frac{2}{5}t$$

d. Calculate the consumers' and producers' burden. Which group is paying more for the tax under the equilibrium?

$$\text{Tax burden} = P^{d \text{ after tax}} + P^{s \text{ after tax}}$$

$$= \left(8 + \frac{3}{5}t\right) + \left(8 - \frac{2}{5}t\right)$$

$$\text{Producer percentage} = \frac{2}{5} \times 100 = 40\%$$

$$\text{Consumer percentage} = \frac{3}{5} \times 100 = 60\%$$

Therefore, consumers pay more for the tax under the equilibrium

e. Find the expression of the revenue that the government can collect from the market under the equilibrium.

tax = $t per unit

Output sold after tax = $\frac{20-t}{5}$ units

Tax revenue = ($ t per unit) (unit output sold)

$= t \times \left(\frac{10-t}{5}\right)$

$= \$ \frac{10t - t^2}{5}$ #

f. If the government were to collect tax so that total revenue is maximized, what is the appropriate level of unit tax, "t", that it should impose to the market?

$$\left.\begin{aligned} P^d &= a - bQ^d = 14 - 3Q^d \\ P^s &= c + dQ^s = 4 + 2Q^s \end{aligned}\right\}$$

t^* maximizes $T(t)$;

$$t^* = \frac{-\left(\frac{a-c}{b+d}\right)}{2\left(\frac{-1}{b+d}\right)}$$

$$= \frac{-\left(\frac{14-4}{-3+2}\right)}{2\left(\frac{-1}{-3+2}\right)}$$

$$= \frac{10}{2} = 5$$ #

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tax = \$t per unit

Output sold after tax $= \frac{10-t}{5}$ units

Tax revenue = (\$ t per unit) (unit output sold)

$$= t \times \left(\frac{10-t}{5}\right)$$

$$= \$ \frac{10t - t^2}{5} \quad \#$$

f. If the government were to collect tax so that total revenue is maximized, what is the appropriate level of unit tax, "t", that it should impose to the market?

$$\left.\begin{aligned} P^d &= a - bQ^d = 14 - 3Q^d \\ P^s &= c + dQ^s = 4 + 2Q^s \end{aligned}\right\}$$

t^* maximizes $T(t)$;

$$t^* = \frac{-\left(\frac{a-c}{b+d}\right)}{2\left(\frac{-1}{b+d}\right)}$$

$$= \frac{-\left(\frac{14-4}{-3+2}\right)}{2\left(\frac{-1}{-3+2}\right)}$$

$$= \frac{10}{2} = 5 \quad \#$$