1.)

```
. dfuller x, trend lags(1) regress

Augmented Dickey-Fuller test for unit root         Number of obs   =       498

                                 ------------ Interpolated Dickey-Fuller ------------
                    Test         1% Critical       5% Critical      10% Critical
                 Statistic           Value             Value            Value
------------------------------------------------------------------------------
 Z(t)              0.601            -3.980            -3.420           -3.130
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 0.9970

------------------------------------------------------------------------------
D.x |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
x            |
        L1.  |   .0001061   .0001764     0.60   0.548    -.0002405    .0004526
        LD.  |     .46018   .0349881    13.15   0.000     .3914361    .5289239
     _trend  |   4.166909    1.14105     3.65   0.000     1.924999    6.408818
      _cons  |   2128.626   140.0551    15.20   0.000     1853.449    2403.803
------------------------------------------------------------------------------

. dfuller y, trend lags(1) regress

Augmented Dickey-Fuller test for unit root         Number of obs   =       498

                                 ------------ Interpolated Dickey-Fuller ------------
                    Test         1% Critical       5% Critical      10% Critical
                 Statistic           Value             Value            Value
------------------------------------------------------------------------------
 Z(t)              1.000            -3.980            -3.420           -3.130
------------------------------------------------------------------------------
MacKinnon approximate p-value for Z(t) = 1.0000

------------------------------------------------------------------------------
D.y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
y            |
        L1.  |   .0001178   .0001179     1.00   0.318    -.0001137    .0003494
        LD.  |   .6997015   .0248993    28.10   0.000     .6507799    .7486231
     _trend  |   2.897751   1.159296     2.50   0.013      .619992    5.175511
      _cons  |   1811.233   147.0426    12.32   0.000     1522.327    2100.139
------------------------------------------------------------------------------
```

From MacKinnon approximate p-value, both y and x are non stationary

2.)

```
. vecrank y x, trend(t) lag(1/1) max

                       Johansen tests for cointegration
Trend: trend                                      Number of obs =     499
Sample:  2 - 500                                           Lags =       1
-------------------------------------------------------------------------
                                                        5%
maximum                                      trace   critical
  rank    parms       LL       eigenvalue  statistic   value
    0       4    -7387.4577        .       790.3357    18.17
    1       7    -6992.3441     0.79477      0.1085*    3.74
    2       8    -6992.2899     0.00022
-------------------------------------------------------------------------
                                                        5%
maximum                                       max    critical
  rank    parms       LL       eigenvalue  statistic   value
    0       4    -7387.4577        .       790.2272    16.87
    1       7    -6992.3441     0.79477      0.1085     3.74
    2       8    -6992.2899     0.00022
-------------------------------------------------------------------------
```

rank=1 > reject <

rank=1 > reject <

```
. vecrank y x, trend(rt) lag(1/1) max

                       Johansen tests for cointegration
Trend: rtrend                                     Number of obs =     499
Sample:  2 - 500                                           Lags =       1
-------------------------------------------------------------------------
                                                        5%
maximum                                      trace   critical
  rank    parms       LL       eigenvalue  statistic   value
    0       2    -8050.4781        .      2116.3764    25.32
    1       6    -7075.2453     0.97993    165.9107    12.25
    2       8    -6992.2899     0.28286
-------------------------------------------------------------------------
                                                        5%
maximum                                       max    critical
  rank    parms       LL       eigenvalue  statistic   value
    0       2    -8050.4781        .      1950.4657    18.96
    1       6    -7075.2453     0.97993    165.9107    12.52
    2       8    -6992.2899     0.28286
-------------------------------------------------------------------------
```

```
. vecrank y x, trend(c) lag(1/1) max

                       Johansen tests for cointegration
Trend: constant                                   Number of obs =     499
Sample:  2 - 500                                           Lags =       1
-------------------------------------------------------------------------
                                                        5%
maximum                                      trace   critical
  rank    parms       LL       eigenvalue  statistic   value
    0       2    -8050.4781        .      2086.8946    15.41
    1       5    -7083.8611     0.97923    153.6607     3.76
    2       6    -7007.0308     0.26504
-------------------------------------------------------------------------
                                                        5%
maximum                                       max    critical
  rank    parms       LL       eigenvalue  statistic   value
    0       2    -8050.4781        .      1933.2339    14.07
    1       5    -7083.8611     0.97923    153.6607     3.76
    2       6    -7007.0308     0.26504
-------------------------------------------------------------------------
```

```
. vecrank y x, trend(rc) lag(1/1) max

                       Johansen tests for cointegration
Trend: rconstant                                  Number of obs =     499
Sample:  2 - 500                                           Lags =       1
-------------------------------------------------------------------------
                                                        5%
maximum                                      trace   critical
  rank    parms       LL       eigenvalue  statistic   value
    0       0    -8811.3759        .      3607.1930    19.96
    1       4    -7092.1887     0.99898    168.8185     9.42
    2       6    -7007.7794     0.28703
-------------------------------------------------------------------------
                                                        5%
maximum                                       max    critical
  rank    parms       LL       eigenvalue  statistic   value
    0       0    -8811.3759        .      3438.3745    15.67
    1       4    -7092.1887     0.99898    168.8185     9.24
    2       6    -7007.7794     0.28703
-------------------------------------------------------------------------
```

```
. vecrank y x, trend(n) lag(1/1) max
```

Johansen tests for cointegration

Trend: none — Number of obs = 499
Sample: 2 - 500 — Lags = 1

maximum rank	parms	LL	eigenvalue	trace statistic	5% critical value
0	0	-8811.3759	.	3204.1193	12.53
1	3	-7211.7592	0.99836	4.8860	3.84
2	4	-7209.3163	0.00974		

maximum rank	parms	LL	eigenvalue	max statistic	5% critical value
0	0	-8811.3759	.	3199.2333	11.44
1	3	-7211.7592	0.99836	4.8860	3.84
2	4	-7209.3163	0.00974		

3.)

```
. vecrank y x, trend(t) lag(1) max
```

Johansen tests for cointegration

Trend: trend — Number of obs = 499
Sample: 2 - 500 — Lags = 1

maximum rank	parms	LL	eigenvalue	trace statistic	5% critical value
0	4	-7387.4577	.	790.3357	18.17
1	7	-6992.3441	0.79477	0.1085*	3.74
2	8	-6992.2899	0.00022		

maximum rank	parms	LL	eigenvalue	max statistic	5% critical value
0	4	-7387.4577	.	790.2272	16.87
1	7	-6992.3441	0.79477	0.1085	3.74
2	8	-6992.2899	0.00022		

```
. vecrank y x, trend(t) lag(2) max

                         Johansen tests for cointegration
Trend: trend                                            Number of obs =      498
Sample:   3 - 500                                                Lags =        2
-------------------------------------------------------------------------------
                                                         5%
maximum                                      trace    critical
  rank    parms       LL       eigenvalue  statistic    value
    0       8     -6795.5337          .     187.5771    18.17
    1      11     -6703.3826    0.30932       3.2749*    3.74
    2      12     -6701.7451    0.00655
-------------------------------------------------------------------------------
                                                         5%
maximum                                       max     critical
  rank    parms       LL       eigenvalue  statistic    value
    0       8     -6795.5337          .     184.3022    16.87
    1      11     -6703.3826    0.30932       3.2749     3.74
    2      12     -6701.7451    0.00655
-------------------------------------------------------------------------------

. vecrank y x, trend(t) lag(3) max

                         Johansen tests for cointegration
Trend: trend                                            Number of obs =      497
Sample:   4 - 500                                                Lags =        3
-------------------------------------------------------------------------------
                                                         5%
maximum                                      trace    critical
  rank    parms       LL       eigenvalue  statistic    value
    0      12      -6756.658          .     140.0434    18.17
    1      15      -6688.113    0.24106       2.9534*    3.74
    2      16     -6686.6363    0.00592
-------------------------------------------------------------------------------
                                                         5%
maximum                                       max     critical
  rank    parms       LL       eigenvalue  statistic    value
    0      12      -6756.658          .     137.0900    16.87
    1      15      -6688.113    0.24106       2.9534     3.74
    2      16     -6686.6363    0.00592
-------------------------------------------------------------------------------
```

4.) The most appropriated set up is the model with trend with maximum rank 1.

5.)

```
. vec y x, lags(1/1)

Vector error-correction model

Sample:  2 - 500                                        Number of obs     =       499
                                                        AIC               =   28.41227
Log likelihood = -7083.861                              HQIC              =   28.42883
Det(Sigma_ml)  =  7.34e+09                              SBIC              =   28.45448

Equation           Parms      RMSE     R-sq      chi2     P>chi2
D_y                    2   331.479   0.9988   399343.6   0.0000
D_x                    2   430.726   0.9953   105050.2   0.0000

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
D_y          |
        _ce1 |
         L1. |  -1.303404   .0095397  -136.63   0.000    -1.322102   -1.284707
             |
       _cons |  -107.843    69.40373    -1.55   0.120    -243.8718    28.18583
-------------+----------------------------------------------------------------
D_x          |
        _ce1 |
         L1. |  -.8364345   .0123959   -67.48   0.000    -.8607301   -.812139
             |
       _cons |  168.0502    90.1839     1.86   0.062    -8.706964    344.8074
------------------------------------------------------------------------------

Cointegrating equations

Equation           Parms      chi2     P>chi2
_ce1                   1   1.71e+10   0.0000

Identification:  beta is exactly identified

                 Johansen normalization restriction imposed

------------------------------------------------------------------------------
        beta |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
_ce1         |
           y |          1          .        .       .            .           .
           x |  -1.500508   .0000115  -1.3e+05   0.000    -1.500531   -1.500486
       _cons |  -1678.204          .        .       .            .           .
------------------------------------------------------------------------------
```

$Y_t = 1678.204 + 1.5X_t$

```
. vec y x, lags(2/2)

Vector error-correction model

Sample:  3 - 500                        Number of obs     =       498
                                        AIC               =   27.18032
Log likelihood = -6758.899              HQIC              =   27.21018
Det(Sigma_ml)  =  2.11e+09              SBIC              =   27.25641

Equation           Parms      RMSE     R-sq      chi2      P>chi2
------------------------------------------------------------------
D_y                    4   213.415   0.9995   964110.3    0.0000
D_x                    4   263.876   0.9982   280728.2    0.0000
------------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
D_y          |
        _ce1 |
         L1. |   -.427199   .0483639    -8.83   0.000    -.5219905   -.3324075
             |
           y |
         LD. |   .4022605   .0224005    17.96   0.000     .3583563    .4461646
             |
           x |
         LD. |   .4814843   .0598769     8.04   0.000     .3641276    .5988409
             |
       _cons |   171.4546   44.03061     3.89   0.000     85.15624    257.7531
-------------+----------------------------------------------------------------
D_x          |
        _ce1 |
         L1. |   .2903827   .0597995     4.86   0.000     .1731779    .4075875
             |
           y |
         LD. |   .5734396   .0276971    20.70   0.000     .5191544    .6277249
             |
           x |
         LD. |   .3676105   .0740347     4.97   0.000     .2225051    .5127159
             |
       _cons |    252.237   54.44157     4.63   0.000     145.5335    358.9405
------------------------------------------------------------------------------

Cointegrating equations

Equation           Parms      chi2     P>chi2
------------------------------------------------
_ce1                   1  8.09e+08     0.0000
------------------------------------------------

Identification:  beta is exactly identified

                 Johansen normalization restriction imposed

------------------------------------------------------------------------------
        beta |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
_ce1         |
           y |          1          .        .       .            .           .
           x |  -1.500176   .0000527 -2.8e+04   0.000    -1.50028   -1.500073
       _cons |  -577.9622          .        .       .            .           .
------------------------------------------------------------------------------
```

$Y_t = 577.96 + 1.5 X_t$

```
. vec y x, lags(3/3)

Vector error-correction model

Sample:  4 - 500                                  Number of obs     =       497
                                                  AIC               =   27.1245
Log likelihood = -6727.439                        HQIC              =  27.16771
Det(Sigma_ml)  =  1.96e+09                        SBIC              =  27.23459

Equation           Parms      RMSE     R-sq      chi2     P>chi2
------------------------------------------------------------------
D_y                    6   211.673   0.9995  980003.4     0.0000
D_x                    6   253.098   0.9984  305162.2     0.0000
------------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
D_y          |
        _ce1 |
         L1. |  -.3193909    .058155    -5.49   0.000    -.4333726   -.2054092
             |
           y |
         LD. |   .3553209   .0559202     6.35   0.000     .2457194    .4649225
        L2D. |   .0174858   .0312998     0.56   0.576    -.0438607    .0788323
             |
           x |
         LD. |   .6042668   .0763396     7.92   0.000     .4546439    .7538896
        L2D. |   .0521968   .0634683     0.82   0.411    -.0721988    .1765925
             |
       _cons |   181.5838   45.38519     4.00   0.000     92.63049    270.5372
-------------+----------------------------------------------------------------
D_x          |
        _ce1 |
         L1. |   .4434064   .0695359     6.38   0.000     .3071185    .5796943
             |
           y |
         LD. |   .4713427   .0668638     7.05   0.000     .3402922    .6023932
        L2D. |    .012763   .0374252     0.34   0.733     -.060589    .0861151
             |
           x |
         LD. |   .5220109   .0912792     5.72   0.000     .3431069     .700915
        L2D. |   .0911602   .0758891     1.20   0.230    -.0575797    .2399001
             |
       _cons |   130.797   54.26707     2.41   0.016     24.43548    237.1585
------------------------------------------------------------------------------

Cointegrating equations

Equation           Parms      chi2     P>chi2
------------------------------------------------------------------
_ce1                   1  6.64e+08     0.0000
------------------------------------------------------------------

Identification:  beta is exactly identified

                 Johansen normalization restriction imposed

------------------------------------------------------------------------------
        beta |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
_ce1         |
           y |          1          .        .       .            .           .
           x |  -1.500049   .0000582 -2.6e+04   0.000    -1.500163   -1.499935
       _cons |  -92.13439          .        .       .            .           .
------------------------------------------------------------------------------
```

Cointegration eqn:
$y_t = 92.13 + 1.5 x_t$

Long 3 has the lowest SBIC

Speed of adj. (x) $= \frac{1}{1-.44} \approx 2.27$

Speed of adj. (y) $= \frac{1}{|-.32|} \approx 3$