$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \cdots + \beta_k X_{ki} + u_i$ — Unrestricted Model $\rightarrow RSS_{UR}$

$\boxed{Y_i = \beta_1 + u_i}$ — Restricted Model $\rightarrow RSS_R$

$H_0: \beta_2 = \beta_3 = \cdots = \beta_k = 0$

$$\frac{ESS/(k-1)}{RSS/(n-k)} \qquad \frac{RSS_R - RSS_{UR}/(k-1)}{RSS_{UR}/(n-k)} \sim F_{(k-1,n-k)}$$

Residual

$u_i = Y_i - \beta_1$

$$\boxed{\sum \hat{u}_i^2 = \sum (Y_i - \bar{Y})^2 = RSS_R}$$

$$\sum (Y_i - \bar{Y})^2 = TSS$$

$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \cdots + \beta_k X_{ki} + u_i \quad \forall i = 1, 2, \ldots, n$

$$\begin{array}{ll}
i=1 & Y_1 = \beta_1 + \beta_2 X_{21} + \beta_3 X_{31} + \cdots + \beta_k X_{k1} + u_1 \\
i=2 & Y_2 = \beta_1 + \beta_2 X_{22} + \beta_3 X_{32} + \cdots + \beta_k X_{k2} + u_2 \\
\vdots & \vdots \\
i=n & Y_n = \beta_1 + \beta_2 X_{2n} + \beta_3 X_{3n} + \cdots + \beta_k X_{kn} + u_n
\end{array}$$

$$\underset{n\times 1}{\begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix}} = \underset{n\times k}{\begin{bmatrix} 1 & X_{21} & X_{31} & \cdots & X_{k1} \\ 1 & X_{22} & X_{32} & \cdots & X_{k2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & X_{2n} & X_{3n} & \cdots & X_{kn} \end{bmatrix}} \cdot \underset{k\times 1}{\begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \vdots \\ \beta_k \end{bmatrix}} + \underset{n\times 1}{\begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_n \end{bmatrix}}$$

$$\mathbb{Y} = \mathbb{X}\mathbb{B} + \mathbb{U}$$

$$RSS = \sum \hat{u}_i^2$$

$$\boxed{\hat{\mathbb{U}} = Y - X\hat{\beta}}$$

$$\underline{\underline{\frac{\partial \sum \hat{u}_i^2}{\partial \hat{\beta}} = 0}}$$

$$\frac{\partial \hat{u}'\hat{u}}{\partial \hat{\beta}} = \underset{1\times n}{[\hat{u}_1\ \hat{u}_2 \cdots \hat{u}_n]} \underset{n\times 1}{\begin{bmatrix} \hat{u}_1 \\ \hat{u}_2 \\ \vdots \\ \hat{u}_n \end{bmatrix}} = \sum \hat{u}_i^2$$

$$\hat{u}'\hat{u} = (Y - X\hat{\beta})'(Y - X\hat{\beta})$$

$$= Y'Y + \hat{\beta}'X'X\hat{\beta} - 2\hat{\beta}'X'Y$$

$$\frac{\partial \hat{u}'\hat{u}}{\partial \hat{\beta}} = 0 + \cancel{2}X'X\hat{\beta} - \cancel{2}X'Y = 0$$

$$X'X\hat{\beta} - X'Y = 0$$

$$X'X\hat{\beta} = X'Y$$

$$\cancel{(X'X)^{-1}}\cancel{X'X}\,\hat{\beta} = (X'X)^{-1}X'Y$$

$$\boxed{\hat{\beta} = (X'X)^{-1}X'Y}$$

$$\underset{n\times 1}{u}\cdot\underset{1\times n}{u'} = \begin{bmatrix} \cancel{u_1} \\ \cancel{u_2} \\ \vdots \\ \cancel{u_n} \end{bmatrix} \begin{bmatrix} \cancel{u_1} & \cancel{u_2} & \cdots & u_n \end{bmatrix}$$

$$= \underset{n\times n}{\begin{bmatrix} u_1u_1 & u_1u_2 & \cdots & u_1u_n \\ u_2u_1 & u_2u_2 & \cdots & u_2u_n \\ \vdots & \vdots & \ddots & \vdots \\ u_nu_1 & u_nu_2 & \cdots & u_nu_n \end{bmatrix}}$$

$$\sum (u_i - \bar{u})^2$$

$$E\left[(u_i - \underset{0}{\underset{\|}{E(u_i)}})^2\right]$$

$$E\left[u_i^2\right] = Var(u_i)$$

$$= \sigma^2$$

$$\underset{\underset{n\times n}{\Sigma}}{\underset{\|}{E(uu')}} = \begin{bmatrix} E(u_1^2) & E(u_1u_2) & \cdots & E(u_1u_n) \\ E(u_2u_1) & E(u_2^2) & \cdots & E(u_2u_n) \\ \vdots & \vdots & \ddots & \vdots \\ E(u_nu_1) & E(u_nu_2) & \cdots & E(u_n^2) \end{bmatrix}$$

$A = A$

$a = \alpha$

$B = B$

$b = \beta$

$s = \sigma$

$S = \Sigma$

$$= \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_2^2 & \cdots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \cdots & \sigma_n^2 \end{bmatrix} \Rightarrow E(uu')_{OLS} = \begin{bmatrix} \sigma^2 & 0 & \cdots & 0 \\ 0 & \sigma^2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ & & & \sigma^2 \\ 0 & 0 & \cdots & \end{bmatrix}$$

$$= \sigma^2 \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

$$\boxed{\Sigma_{OLS} = \sigma^2 I}$$

$$\hat{\beta} = (X'\Sigma^{-1}X)^{-1}X'\Sigma^{-1}Y$$

$$= (X'(\sigma^2 I)^{-1}X)^{-1}X'(\sigma^2 I)^{-1}Y$$

$$\cancel{\sigma^2} \cdot \frac{1}{\cancel{\sigma^2}}(X'X)^{-1}X'Y$$

$$= (X'X)^{-1}X'Y$$