

## Assignment Solution MLE & GMM

### Part I

The study on bankruptcy firm employs the following regression model.

$$z_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i}$$

The log-likelihood function of this model is as follows:

$$\ln L = \begin{cases} \ln \Phi(z_i) & \text{if } y_i = 1 \\ \ln \Phi(-z_i) & \text{if } y_i = 0 \end{cases}$$

where:  $y_i$  is discrete binary variable, values either 0 or 1.

$x_{ki}$  are independent variables,  $k=1, 2, 3$ .

$\Phi(\cdot)$  = Cumulative normal density function.

From the given data set (Assign02\_MLE):

1. Estimate the above models using MLE with Newton-Raphson algorithm and BHHH.

---

```

program ml_probit
  args lnf theta
  quietly replace `lnf' = ln(normal(`theta')) if $ML_y1==1
  quietly replace `lnf' = ln(normal(-`theta')) if $ML_y1==0
end

```

---

```
. ml model lf ml_probit (y=x*)
```

```
. ml maximize
```

```

initial:      log likelihood = -346.57359
alternative:  log likelihood = -299.06229
rescale:     log likelihood = -299.06229
Iteration 0:  log likelihood = -299.06229
Iteration 1:  log likelihood = -137.13306
Iteration 2:  log likelihood = -126.54535
Iteration 3:  log likelihood = -126.24974
Iteration 4:  log likelihood = -126.24956
Iteration 5:  log likelihood = -126.24956

```

```

Log likelihood = -126.24956
Number of obs   =      500
Wald chi2(3)    =      138.82
Prob > chi2     =      0.0000

```

y	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
x1	.0176771	.0015548	11.37	0.000	.0146297 .0207244
x2	-.0269468	.0038815	-6.94	0.000	-.0345544 -.0193393
x3	.0178847	.003506	5.10	0.000	.0110131 .0247563
_cons	1.012678	.1126462	8.99	0.000	.7918951 1.23346

```
. ml model lf ml_probit (y=x*), tech(bhhh)
```

```
. ml maximize
```

```

initial:      log likelihood = -346.57359
alternative:  log likelihood = -299.06229
rescale:     log likelihood = -299.06229
Iteration 0:  log likelihood = -299.06229
Iteration 1:  log likelihood = -140.49244
...
Iteration 54: log likelihood = -126.24956

```

```
Number of obs   =      500
```

Log likelihood = -126.24956      Wald chi 2(3) = 134.45  
 Prob > chi 2 = 0.0000

y	Coef.	OPG Std. Err.	z	P> z	[95% Conf. Interval]	
x1	.0176751	.0016539	10.69	0.000	.0144335	.0209167
x2	-.0269433	.0037917	-7.11	0.000	-.0343749	-.0195116
x3	.0178852	.0031771	5.63	0.000	.0116583	.0241121
_cons	1.012449	.1477093	6.85	0.000	.7229442	1.301954

## 2. Perform hypothesis testing whether $\beta_1 = \beta_2 = \beta_3 = 0$ using LR-test and Wald test.

```
. ml model lf ml_probit (y=x*)
```

```
. ml maximize
```

```
initial:      log likelihood = -346.57359
alternative:  log likelihood = -299.06229
rescale:     log likelihood = -299.06229
Iteration 0:  log likelihood = -299.06229
Iteration 1:  log likelihood = -137.13306
...
Iteration 5:  log likelihood = -126.24956
```

Log likelihood = -126.24956      Number of obs = 500  
 Wald chi 2(3) = 138.82  
 Prob > chi 2 = 0.0000

y	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
x1	.0176771	.0015548	11.37	0.000	.0146297	.0207244
x2	-.0269468	.0038815	-6.94	0.000	-.0345544	-.0193393
x3	.0178847	.003506	5.10	0.000	.0110131	.0247563
_cons	1.012678	.1126462	8.99	0.000	.7918951	1.23346

### Wald Test

```
. test (x1=0) (x2=0) (x3=0)
```

```
( 1) [eq1]x1 = 0
( 2) [eq1]x2 = 0
( 3) [eq1]x3 = 0
```

chi 2( 3) = 138.82  
 Prob > chi 2 = 0.0000

### LR Test

```
. est store ur
```

```
. ml model lf ml_probit (y=)
```

```
. ml maximize
```

```
initial:      log likelihood = -346.57359
alternative:  log likelihood = -299.06229
rescale:     log likelihood = -299.06229
Iteration 0:  log likelihood = -299.06229
Iteration 1:  log likelihood = -298.34586
Iteration 2:  log likelihood = -298.3458
Iteration 3:  log likelihood = -298.3458
```

Log likelihood = -298.3458      Number of obs = 500  
 Wald chi 2(0) = .  
 Prob > chi 2 = .

y	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
_cons	.5709995	.0595003	9.60	0.000	.454381	.687618

```
. est store res
```

```
. lrtest ur res
```

Likelihood-ratio test      LR chi 2(3) = 344.19  
 (Assumption: res nested in ur)      Prob > chi 2 = 0.0000

```
. probit y x*
```

```
Iteration 0:   log likelihood =  -298.3458
```

```
Iteration 5:   log likelihood = -126.24956
```

```
Probit regression
```

```
Number of obs   =      500
LR chi2(3)      =     344.19
Prob > chi2     =      0.0000
Pseudo R2      =      0.5768
```

```
Log likelihood = -126.24956
```

y	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
x1	.0176771	.0015548	11.37	0.000	.0146297 .0207244
x2	-.0269468	.0038815	-6.94	0.000	-.0345544 -.0193393
x3	.0178847	.003506	5.10	0.000	.0110131 .0247563
_cons	1.012678	.1126462	8.99	0.000	.7918951 1.23346

Note: 0 failures and 5 successes completely determined.

## Part II

From the given data set (assign02gmm):

Estimate the interest rate structure models applying all 9 models using GMM.