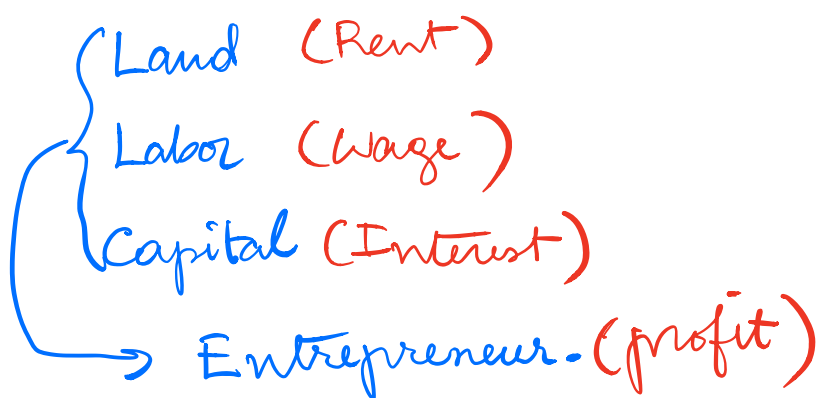


10 m \$.

- Gold, Bond, Stocks (Invest)
- Travel.
- Gamble.
- Land & House.



Example: Coffee shop owner.

Revenue = 200,000 \$/month.

Costs of Labor, Land, Capital = 150,000 \$/mon.

⇒ Profit = 50,000 \$/month → accounting profit.

If the owner could have worked in a company and earn 30,000 \$

⇒ Economic Profit =  $50,000 - 30,000$   
= 20,000 \$/month.

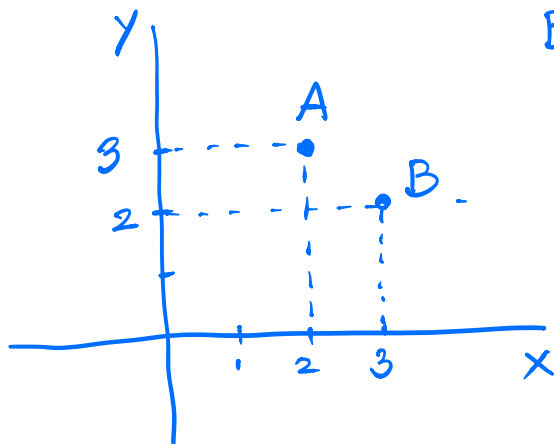
If the salary at the company = 50,000 \$ as well.

$$\Rightarrow \text{Economic Profit} = 50,000 - 50,000 = 0 \$$$

$\Rightarrow$  the owner receives Normal Profit.

If Economic Profit  $> 0 \Rightarrow$  Excess Profit.

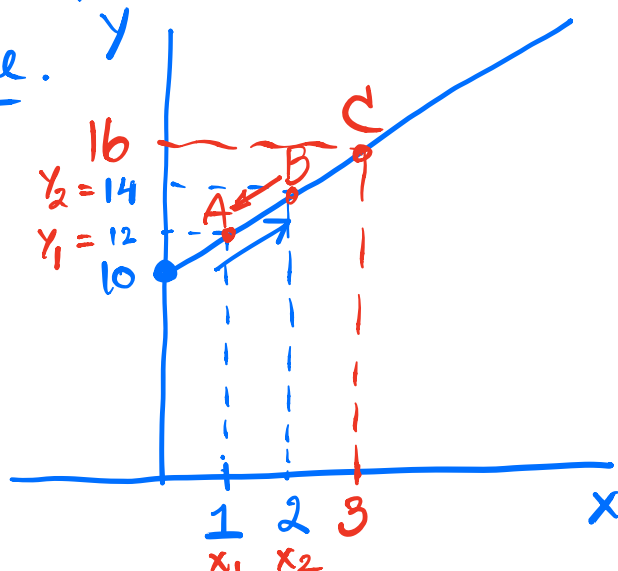
Graph.



$$A = (2, 3)$$

$$B = (3, 2)$$

Line.



X	Y
0	10
1	12
2	14

a line tells us the relationship between

$x$  &  $y$ . - that as

$x$  increases  
 $y$  also increases.

we can write the function to describe this relationship  
as  $Y = 10 + 2x$ . — a straight line.  
— linear function.

Slope = rate of change of  $Y$  per unit change of  $X$ .

Ex. from A to B.  $\Delta x = x_2 - x_1 = 1$   
 $\Delta y = y_2 - y_1 = 14 - 12 = 2$

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{2}{1} = 2.$$

From A to C  $\Delta x = 2$  } slope =  $\frac{4}{2} = 2$   
 $\Delta y = 4$

Linear  $\Rightarrow$  Slope is constant

—  $X$  can change by any amount and we will have  $Y$  change in such a way that we have same slope.

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} \text{ from A to B.}$$

From B to A,  $x$  decreases.

$$\Delta x = x_1 - x_2 = 1 - 2 = -1$$

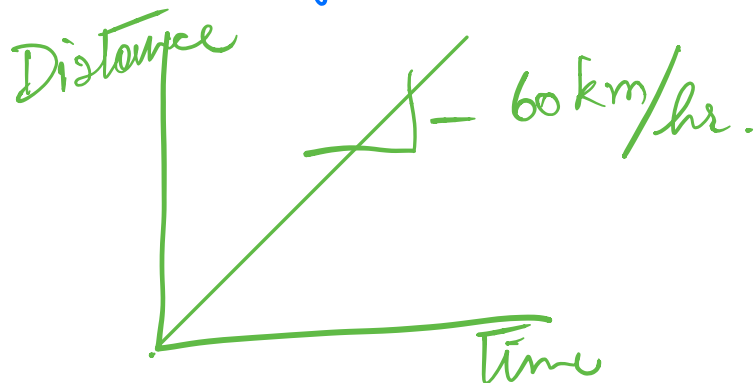
$$\Delta y = y_1 - y_2 = 12 - 14 = -2$$

$$\text{Slope} = \frac{\Delta y}{\Delta x} = \frac{-2}{-1} = 2$$

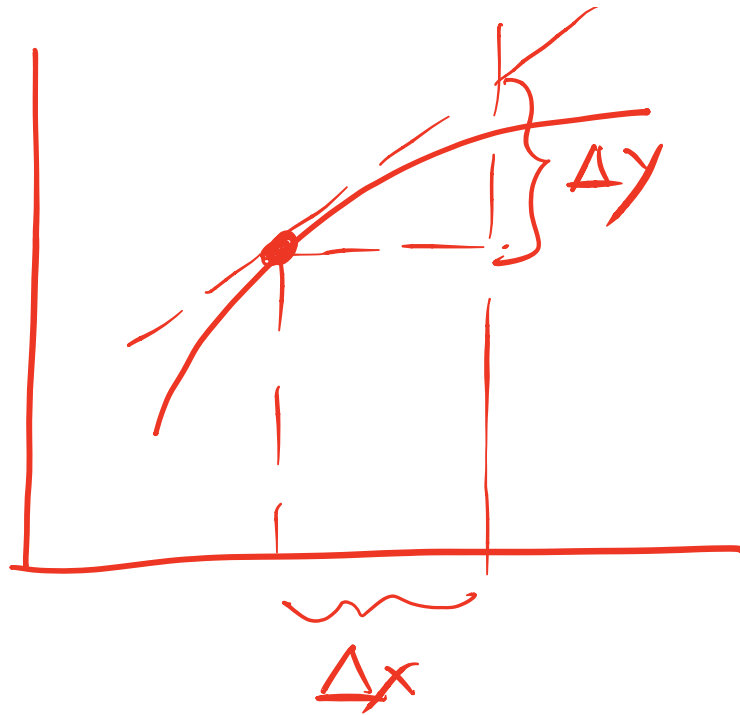
$$= \frac{y_1 - y_2}{x_1 - x_2} = \frac{-(y_2 - y_1)}{-(x_2 - x_1)} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y = 10 + 2x$$

$$\frac{dy}{dx} = 2 - \text{slope when } \Delta x \rightarrow 0.$$



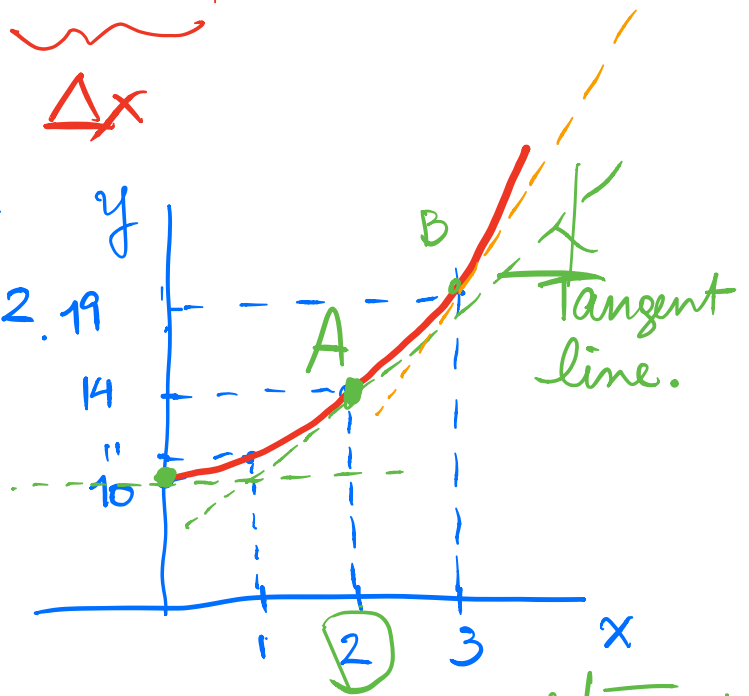




Nonlinear line.

x	y	$\frac{dy}{dx} = 2x$
0	10	0
1	11	2
2	14	4
3	19	6

$$y = 10 + x^2$$



Slope at a point  $x = 2$ .

= slope of tangent line touching the graph at  $x = 2$ .

$$\frac{dy}{dx} = \frac{d}{dx} (10 + x^2) = 2x$$

$$x=2 \quad \left| \quad \frac{dy}{dx} = 2x = 2(2) = 4.$$

- as  $x$  increases, the slope is increasing.

- when  $x=2, y=14$ . (slope = 4).

$\Delta x = 0.1$  —  $x$  increases from 2 to 2.1

$$\Delta y \approx \frac{dy}{dx} \cdot \Delta x$$

$$= 4 \times (0.1) = 0.4.$$

what is the real change in  $y$ ?

$$x = 2.1, y = 10 + (2.1)^2 = 14.41$$

$$\therefore \Delta y = 14.41 - 14 = 0.41$$

---

H.W.  $y = 10 + \sqrt{x}$ .

x	y	dy/dx
0		
1		
2		
3		

Find  $\frac{dy}{dx}$ .

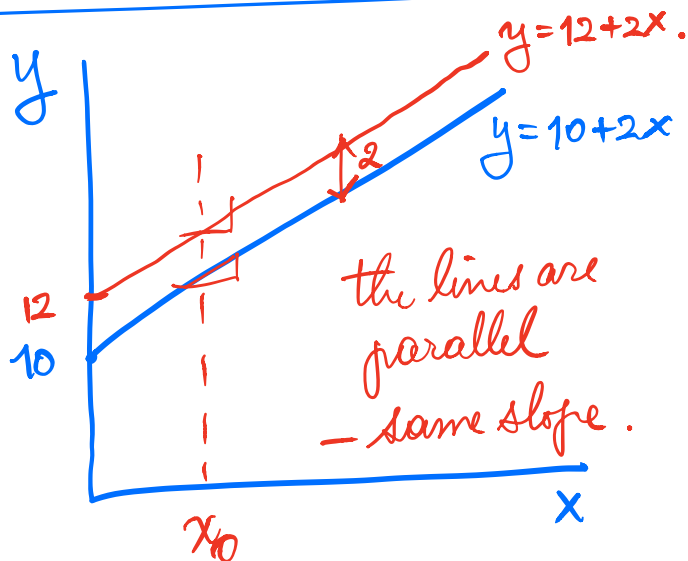
Approximate  $\Delta y$   
when  $x = 2$ ,  $\Delta x = 0.1$   
and  $\Delta x = -0.2$ .

Compare the actual  $\Delta y$   
to find the errors.

### Shift of graph.

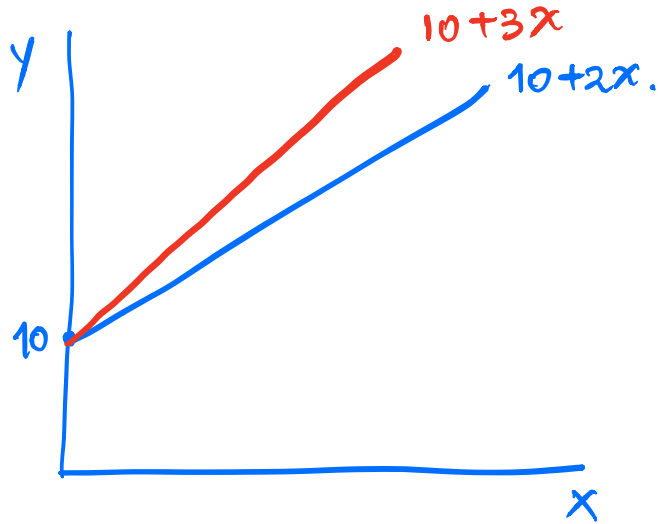
$y = 10 + 2x$ .  
y-intercept  
slope

$y = 12 + 2x$ .

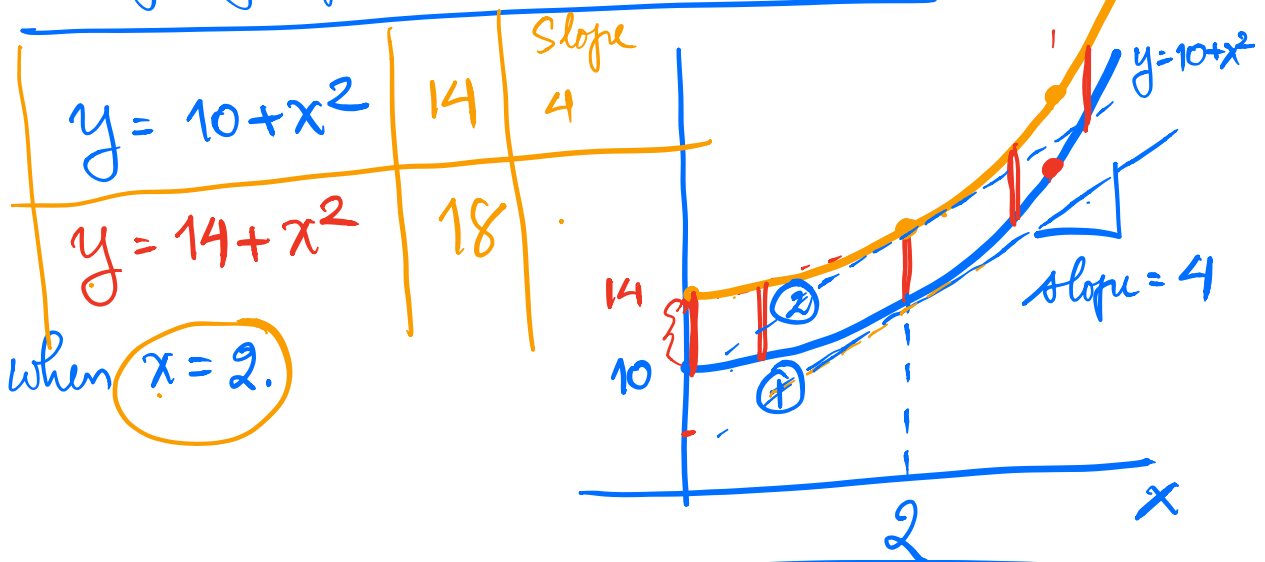


$$y = 10 + 2x$$

$$y = 10 + 3x$$



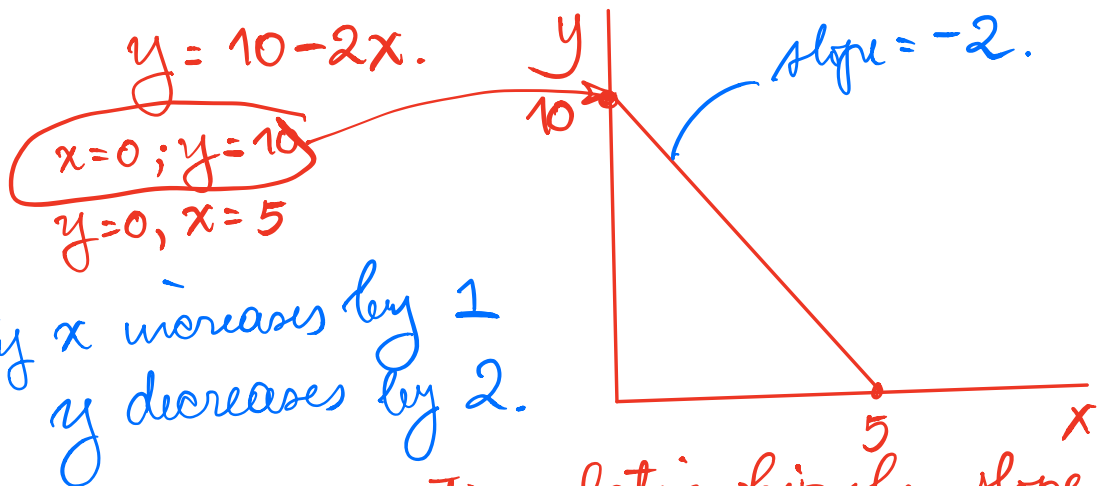
Shift of graph of Nonlinear Line.



Slope  $> 0 \Rightarrow$  when  $x$  increases,  $y$  also increases  
 (decreases) (decreases)

i.e.  $x$  &  $y$  have positive relation.

Slope can be negative.



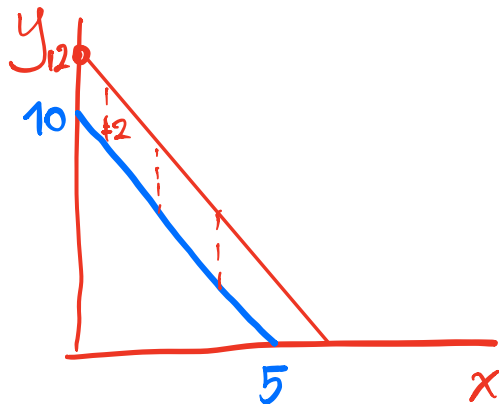
$y$   $x$  increases by 1  
 $y$  decreases by 2.

$x$  &  $y$  have negative relationship when slope  $< 0$ .

Shift of graph with negative slope.

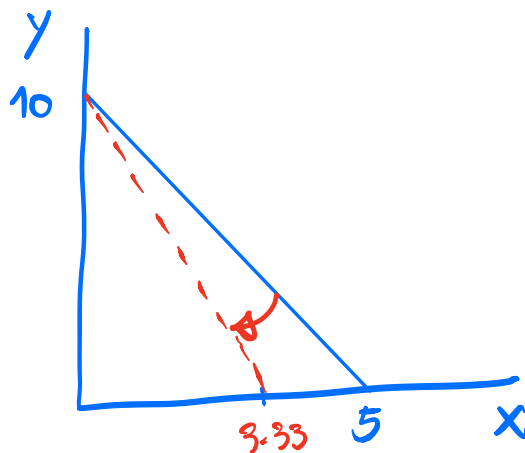
$y = 10 - 2x$

$y = 12 - 2x$



$y = 10 - 2x$

$y = 10 - 3x$

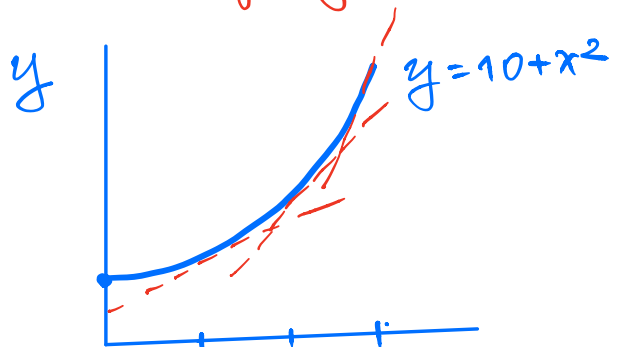


$$y = 10 + x^2$$

$x$	$y$	$\frac{dy}{dx} = 2x$ <i>slope</i>	$\frac{d^2y}{dx^2} = 2$
0	10	0	2
1	11	2	2
2	14	4	2
3	19	6	2

$$\frac{d^2y}{dx^2} = \frac{d}{dx} \left( \frac{dy}{dx} \right) = \frac{d}{dx} 2x = 2$$

= slope of slope.



Here slope of slope =  $2 > 0$   
 It means as  $x$  increases the slope is increasing

$$\text{slope} = \frac{dy}{dx}$$



H.W. Find 2<sup>nd</sup> order derivative of  $y = 10 + \sqrt{x}$   
 and plot the graph of  $y$  and  $\frac{dy}{dx}$ .  
 Is the slope of slope a constant.