

Quiz 1

1. Find

$$\lim_{x \rightarrow -5} \frac{\sqrt{x+6} - 1 + \sin(|x| - 5)}{|x| - 5}.$$

Solution:

$$\lim_{x \rightarrow -5} \frac{\sqrt{x+6} - 1 + \sin(|x| - 5)}{|x| - 5} = \lim_{x \rightarrow -5} \frac{\sqrt{x+6} - 1}{|x| - 5} + \lim_{x \rightarrow -5} \frac{\sin(|x| - 5)}{|x| - 5}$$

(i) The first term: Since $|x| = \begin{cases} -x, & \text{for } x < 0 \\ x, & \text{for } x > 0 \end{cases}$ and since we consider x approaching negative number -5 ($x \rightarrow -5$), we use $|x| = -x$.

$$\begin{aligned} \lim_{x \rightarrow -5} \frac{\sqrt{x+6} - 1}{|x| - 5} &= \lim_{x \rightarrow -5} \frac{\sqrt{x+6} - 1}{-x - 5} \\ &= \lim_{x \rightarrow -5} \frac{\sqrt{x+6} - 1}{-x - 5} \cdot \frac{\sqrt{x+6} + 1}{\sqrt{x+6} + 1} \\ &= \lim_{x \rightarrow -5} \frac{(\sqrt{x+6})^2 - 1^2}{-(x+5)(\sqrt{x+6} + 1)} \\ &= \lim_{x \rightarrow -5} \frac{x+6-1}{-(x+5)(\sqrt{x+6} + 1)} = \lim_{x \rightarrow -5} \frac{x+5}{-(x+5)(\sqrt{x+6} + 1)} \\ &= \lim_{x \rightarrow -5} \frac{-1}{(\sqrt{x+6} + 1)} \\ &= -\frac{1}{2} \end{aligned}$$

(ii) The second term: Let $w = |x| - 5$ so that as $x \rightarrow -5$, we have $w \rightarrow 0$ and

$$\lim_{x \rightarrow -5} \frac{\sin(|x| - 5)}{|x| - 5} = \lim_{w \rightarrow 0} \frac{\sin(w)}{w} = 1.$$

Hence

$$\lim_{x \rightarrow -5} \frac{\sqrt{x+6} - 1 + \sin(|x| - 5)}{|x| - 5} = \lim_{x \rightarrow -5} \frac{\sqrt{x+6} - 1}{|x| - 5} + \lim_{x \rightarrow -5} \frac{\sin(|x| - 5)}{|x| - 5} = -\frac{1}{2} + 1 = \frac{1}{2}$$

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