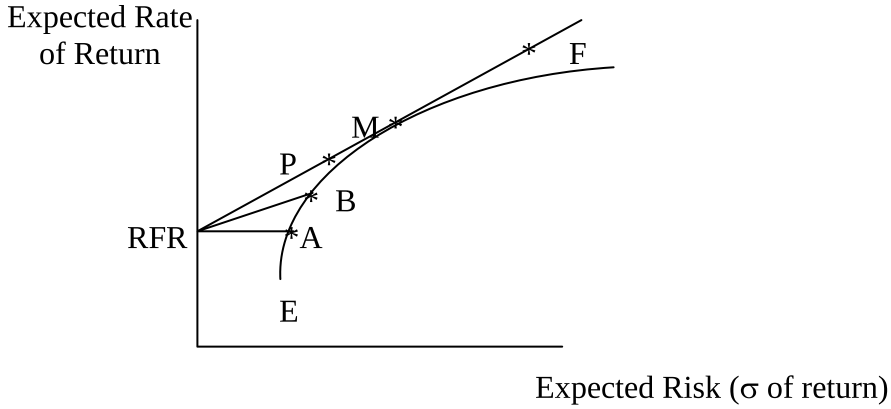
SOUTH-WESTERN
CENGAGE Learning

CHAPTER 8

AN INTRODUCTION TO ASSET PRICING MODELS

Answers to Questions

1.



The existence of a risk-free asset excludes the E-A segment of the efficient frontier because any point below A is dominated by the RFR. In fact, the entire efficient frontier below M is dominated by points on the RFR-M Line (combinations obtained by investing a part of the portfolio in the risk-free asset and the remainder in M), e.g., the point P dominates the previously efficient B because it has lower risk for the same level of return. As shown, M is at the point where the ray from RFR is tangent to the efficient frontier. The new efficient frontier thus becomes RFR-M-F.

2. Standard deviation would be expected to decrease with an increase in stocks in the portfolio because an increase in number will increase the probability of having more low and inversely correlated stocks. There will be a major decline from 4 to 10 stocks, a continued decline from 10 to 20 but at a slower rate. Finally, from 50 to 100 stocks, there is a further decline but at a very slow rate because almost all unsystematic risk is eliminated by about 18 stocks.

3. In a capital asset pricing model (CAPM) world the relevant risk variable is the security's systematic risk - its covariance of return with all other risky assets in the market. This risk cannot be eliminated. The unsystematic risk is not relevant because it can be eliminated through diversification - for instance, when you hold a large number of securities, the poor management capability, etc., of some companies will be offset by the above average capability of others.

4. Similarities: they both measure the relationship between risk and expected return.
Differences: First, the CML measures risk by the standard deviation (i.e., total risk) of the investment while the SML explicitly considers only the systematic component of an investment's volatility. Second, as a consequence of the first point, the CML can only be applied to portfolio holdings that are already fully diversified, whereas the SML can be

applied to any individual asset or collection of assets.

On the CML the lowest risk portfolio is 100 percent invested in the risk-free asset with a standard deviation of zero. The SML—focusing on the word “security” in its name—deals primarily with security or asset risk. Security risk is measured by the asset’s systematic risk, or beta. Beta can be negative (if the asset’s returns and market returns are negatively correlated) so the SML extends to the left of the vertical (expected return) axis.

5. Any three of the following are criticisms of beta as used in CAPM.

1. Theory does not measure up to practice. In theory, a security with a zero beta should give a return exactly equal to the risk-free rate. But actual results do not come out that way, implying that the market values something besides a beta measure of risk.

2. Beta is a fickle short-term performer. Some short-term studies have shown risk and return to be negatively related. For example, Black, Jensen and Scholes found that from April 1957 through December 1965, securities with higher risk produced lower returns than less risky securities. This result suggests that (1) in some short periods, investors may be penalized for taking on more risk, (2) in the long run, investors are not rewarded enough for high risk and are overcompensated for buying securities with low risk, and (3) in all periods, some unsystematic risk is being valued by the market.

3. Estimated betas are unstable. Major changes in a company affecting the character of the stock or some unforeseen event not reflected in past returns may decisively affect the security’s future returns.
4. Beta is easily rolled over. Richard Roll has demonstrated that by changing the market index against which betas are measured, one can obtain quite different measures of the risk level of individual stocks and portfolios. As a result, one would make different predictions about the expected returns, and by changing indexes, one could change the risk-adjusted performance ranking of a manager.

6. Under CAPM, the only risk that investors should be compensated for bearing is the risk that cannot be diversified away (systematic risk). Because systematic risk (measured by beta) is equal to one for both portfolios, an investor would expect the same return for Portfolio H and Portfolio L.

Since both portfolios are fully diversified, it doesn’t matter if the specified risk for each individual security is high or low. The specific risk has been diversified away for both portfolios.

7.

7(a). The concepts are explained as follows:
The Foundation’s portfolio currently holds a number of securities from two asset classes. Each of the individual securities has its own risk (and return) characteristics, described as *specific risk*. By including a sufficiently large number of holdings, the specific risk of the

individual holdings offset each other, diversifying away much of the overall specific risk and leaving mostly nondiversifiable or market-related risk.

Systematic risk is market-related risk that cannot be diversified away. Because systematic risk cannot be diversified away, investors are rewarded for assuming this risk.

The *variance* of an individual security is the sum of the probability-weighted average of the squared differences between the security's expected return and its possible returns. The *standard deviation* is the square root of the variance. Both variance and standard deviation measure total risk, including both systematic and specific risk. Assuming the rates of return are normally distributed, the likelihood for a range of rates may be expressed using standard deviations. For example, 68 percent of returns may be expressed using standard deviations. Thus, 68 percent of returns can be expected to fall within + or -1 standard deviation of the mean, and 95 percent within 2 standard deviations of the mean.

Covariance measures the extent to which two securities tend to move, or not move, together. The level of covariance is heavily influenced by the degree of correlation between the securities (the correlation coefficient) as well as by each security's standard deviation. As long as the correlation coefficient is less than 1, the portfolio standard deviation is less than the weighted average of the individual securities' standard deviations. The lower the correlation, the lower the covariance and the greater the diversification benefits (negative correlations provide more diversification benefits than positive correlations).
The capital asset pricing model (CAPM) asserts that investors will hold only fully diversified portfolios. Hence, total risk as measured by the standard deviation is not relevant because it includes specific risk (which can be diversified away).

Under the CAPM, *beta* measures the systematic risk of an individual security or portfolio. Beta is the slope of the characteristic line that relates a security's returns to the returns of the market portfolio. By definition, the market itself has a beta of 1.0. The beta of a portfolio is the weighted average of the betas of each security contained in the portfolio. Portfolios with betas greater than 1.0 have systematic risk higher than that of the market; portfolios with betas less than 1.0 have lower systematic risk. By adding securities with betas that are higher (lower), the systematic risk (beta) of the portfolio can be increased (decreased) as desired.

7(b). Without performing the calculations, one can see that the portfolio return would increase because: (1) Real estate has an expected return equal to that of stocks. (2) Its expected return is higher than the return on bonds.

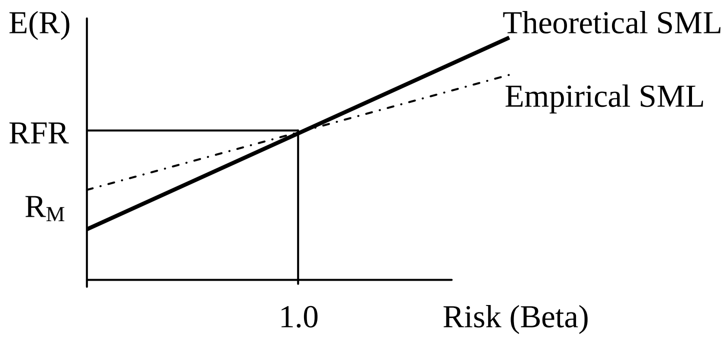
The addition of real estate would result in a reduction of risk because: (1) The standard deviation of real estate is less than that of both stocks and bonds. (2) The correlation of real estate with both stocks and bonds is negative.

The addition of an asset class that is not perfectly correlated with existing assets will reduce variance. The fact that real estate has a negative correlation with the existing asset classes will reduce risk even more.

7(c). Capital market theory holds that efficient markets prevent mispricing of assets and that expected return is proportionate to the level of risk taken. In this instance, real estate is expected to provide the same return as stocks and a higher return than bonds. Yet, it is expected to provide this return at a lower level of risk than both bonds and stocks. If these expectations were realistic, investors would sell the other asset classes and buy real estate, pushing down its return until it was proportionate to the level of risk.

Appraised values differ from transaction prices, reducing the accuracy of return and volatility measures for real estate. Capital market theory was developed and applied to the stock market, which is a very liquid market with relatively small transaction costs. In contrast to the stock market, real estate markets are very thin and lack liquidity.

8.



In the empirical line, low risk securities did better than expected, while high risk securities did not do as well as predicted.

9. The "market" portfolio contains all risky assets available. If a risky asset, be it an obscure bond or rare stamp, was not included in the market portfolio, then there would be no demand for this asset and, consequently, its price would fall. Notably, the price decline would continue to the point where the return would make the asset desirable such that it would be part of the "market" portfolio. The weights for all risky assets are equal to their relative market value. The typical proxy for the market portfolio are stock market indexes.

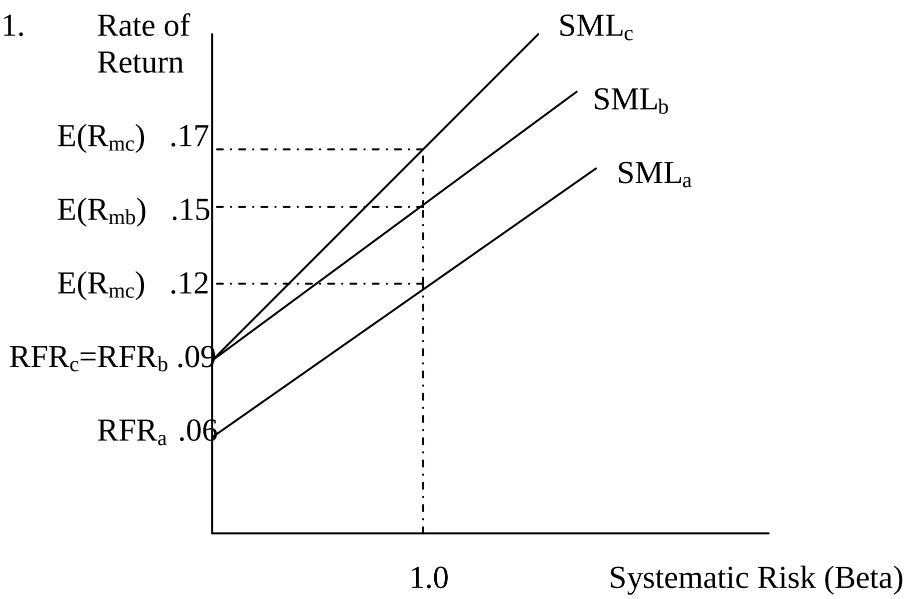
According to Roll, a mistakenly specified proxy for the market portfolio can have two effects. First, the beta computed for alternative portfolios would be wrong because the market portfolio is inappropriate. Second, the SML derived would be wrong because it goes from the RFR through the improperly specified market portfolio. In general, when comparing the performance of a portfolio manager to the "benchmark" portfolio, these errors will tend to **overestimate** the performance of portfolio managers because the proxy market portfolio employed is probably not as efficient as the true market portfolio, so the slope of the SML will be underestimated.

10. Studies of the efficient markets hypothesis suggest that additional factors affecting estimates of expected returns include firm size, the price-earnings ratio, and financial leverage. These variables have been shown to have predictive ability with respect to security returns.

CHAPTER 8

Answers to Problems

1.



In (b), a change in risk-free rate, with other things being equal, would result in a new SML_b, which would intercept with the vertical axis at the new risk-free rate (.09) and would be parallel in the original SML_a.

In (c), this indicates that not only did the risk-free rate change from .06 to .09, but the market risk premium per unit of risk [$E(R_m) - R_f$] also changed from .06 (.12 - .06) to .08 (.17 - .09). Therefore, the new SML_C will have an intercept at .09 and a different slope so it will no longer be parallel to SML_a.

2. $E(R_i) = RFR + \beta_i(R_M - RFR)$

$= .10 + \beta_i(.14 - .10)$

$= .10 + .04\beta_i$

2a.

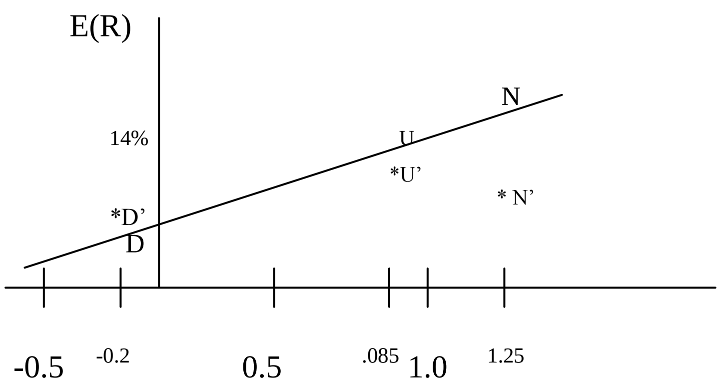
Stock	Beta	(Required Return) $E(R_i) = .10 + .04\beta_i$
U	85	.10 + .04(.85) = .10 + .034 = .134
N	1.25	.10 + .04(1.25)= .10 + .05 = .150
D	-.20	.10 + .04(-.20) = .10 - .008 = .092

2b.

Stock	Current Price	Expected Price	Expected Dividend	Estimated Return
U	22	24	0.75	$\frac{24-22+0.75}{22} = .1250$
N	48	51	2.00	$\frac{51-48+2.00}{48} = .1042$
D	37	40	1.25	$\frac{40-37+1.25}{37} = .1149$

Stock	Beta	Required	Estimated	Evaluation
U	.85	.134	.1250	Overvalued
N	1.25	.150	.1042	Overvalued
D	-.20	.092	.1149	Undervalued

If you believe the appropriateness of these estimated returns, you would buy stocks D and sell stocks U and N.



3.

3a. Q: 4.8%/10.5% = 0.4571
R: 7%/14% = 0.5000
S: 1.6%.5% = 0.3200
T: 8.7%/18.5% = 0.4703
U: 3.2%/7.5% = 0.4267

3b. The CML slope, $[E(R_{MKT}) - RFR]/ \sigma_{MKT}$, is the ratio of risk premium per unit of risk. Portfolio R has the highest ratio, 0.5000, of these five portfolios so it is most likely the market portfolio. Thus, the slope of the CML is 0.5; its intercept is 3%, the risk-free rate.

3c. The CML equation, based on the above analysis, is $E(R_{portfolio}) = 3\% + (0.50)\,\sigma_{portfolio}$. If the desired standard deviation is 7.0% the expected portfolio return is 6.5%: $E(R_{portfolio}) = 3\% + (0.50)\,(7\%) = 6.5\%$. The answer is no, it is not possible to earn an expected return of 7% with a portfolio whose standard deviation is 7%.

3d. Using the CML equation, we set the expected portfolio return equal to 7% and solve for the standard deviation:
$E(R_{portfolio}) = 7\% = 3\% + (0.50)\,\sigma_{portfolio} \Rightarrow 4\% = (0.50)\,\sigma_{portfolio} \Rightarrow \sigma = 4\%/0.50 = 8\%$.
Thus, 8% is the standard deviation consistent with an expected return of 7%.
To find the portfolio weights with result in a risk of 8% and expected return of 7%, recall that the covariance between the risk-free asset and the market portfolio is zero. Thus, the portfolio standard deviation calculation simplifies to: $\sigma_{portfolio} = w_{MKT}\,(\sigma_{MKT})$ and the weight of the risk-free asset is $1 - w_{MKT}$.
Doing this, we have $\sigma_{portfolio} = 8\% = w_{MKT}\,(14.0\%)$, so $w_{MKT} = 8\%/14.0\% = 0.5714$ and $w_{risk\text{-}free\ asset} = 1 - 0.5714 = 0.4286$. As a check, the weighted average expected return should equal 7%:
0.5714 (10%) + 0.4286(3%) = 7.0% which it does. Remember to use the expected return of the market portfolio, 10%, in this calculation.

3e. To find the portfolio weights with result in a risk of 18.2%, recall that the covariance between the risk-free asset and the market portfolio is zero. Thus, the portfolio standard deviation calculation simplifies to: $\sigma_{portfolio} = w_{MKT}\,(\sigma_{MKT})$ and the weight of the risk-free asset is $1 - w_{MKT}$.
Doing this, we have $\sigma_{portfolio} = 18.2\% = w_{MKT}\,(14.0\%)$, so $w_{MKT} = 18.2\%/14.0\% = 1.30$; $w_{risk\text{-}free\ asset} = 1 - (1.3) = -0.30$. This portfolio is a borrowing portfolio; 30% of the funds will be borrowed (we will use margin) and 130% of the initial funds are invested in the market portfolio.
The expected return will be the weighted average of the risk-free and market portfolio returns:
1.30 (10%) + (-0.30) (3%) = 12.1% .
We can also use the CML equation to find the expected return:
$E(R_{portfolio}) = 3\% + (0.50)\,\sigma_{portfolio} = 3\% + (0.50)(18.2\%) = 12.1\%$. Thus, both methods agree, as they should, on the expected portfolio return.

4. With a risk premium of 5% and risk-free rate of 4.5%, the security market line is:
E(return) = 4.5% + (5%)β. Information about the level of diversification of the portfolios is not given, nor is information about the market portfolio. But a portfolio's beta is the weighted average of the betas of the securities held in the portfolio so the SML can be used to evaluate managers Y and Z.

4a. Expected return (Y) = 4.5% + (5%)β = 4.5% + (5%)(1.20) = 10.50%.
Expected return (Z) = 4.5% + (5%)β = 4.5% + (5%)(0.80) = 8.50%.

4b. Alpha is the difference between the actual return and the expected return based on portfolio risk:
Alpha of manager Y = actual return – expected return = 10.20% - 10.50% = -0.30%
Alpha of manager Z = actual return – expected return = 8.80% - 8.50% = 0.30%

4c. A positive alpha means the portfolio outperformed the market on a risk-adjusted basis; it would plot above the SML. A negative alpha means the opposite, that the portfolio underperformed the market on a risk-adjusted basis; it would plot below the SML.

In this case, manager Z outperformed the market portfolio on a risk-adjusted basis by 30 basis points (0.30%). Manager Y underperformed, returning 30 basis points less (-0.30%) than expected based upon the risk of Y's portfolio.

5(a).

$$B_i = \frac{COV_{i,m}}{\sigma_m^2} \text{ and } r_{i,m} = \frac{COV_{i,m}}{(\sigma_i)(\sigma_m)}$$

then $COV_{i,m} = (r_{i,m})(\sigma_i)(\sigma_m)$

For Intel:

$COV_{i,m} = (.72)(.1210)(.0550) = .00479$

$$\text{Beta} = \frac{.00479}{(.055)^2} = \frac{.00479}{.003025} = 1.583$$

For Ford:

$COV_{i,m} = (.33)(.1460)(.0550) = .00265$

$$\text{Beta} = \frac{.00265}{.003025} = .876$$

For Anheuser Busch:

$COV_{i,m} = (.55)(.0760)(.0550) = .00230$

$$\text{Beta} = \frac{.00230}{.003025} = .760$$

For Merck:

$COV_{i,m} = (.60)(.1020)(.0550) = .00337$

$$\text{Beta} = \frac{.00337}{.003025} = 1.114$$

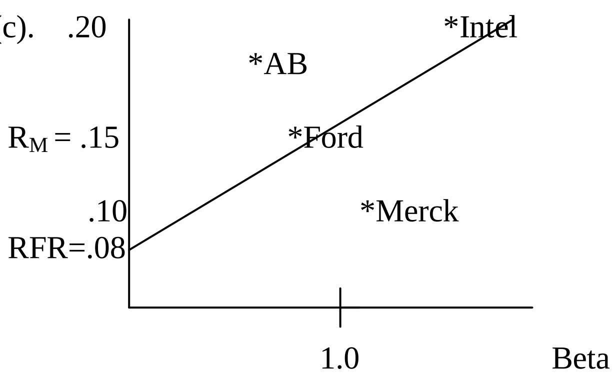
5(b). $E(R_i) = RFR + B_i(R_M - RFR)$

$= .08 + B_i(.15 - .08)$

$= .08 + .07B_i$

Stock	Beta	$E(R_i) = .08 + .07B_i$
Intel	1.583	0.1908
Ford	.876	0.1413
Anheuser Busch	.760	0.1332
Merck	1.114	0.1580

5(c).



Intel, Ford, and Anheuser all have estimated return (given in part c) exceeding their expected returns (computed in part b); they are undervalued and are potential "buy" candidates. Merck is overvalued as its estimated return (10%) is less than the return required by the SML (15.8%); it is a potential candidate for selling.

6.

Year	Chelle (R_1)	General Index (R_M)	$R_1 - E(R_1)$	$R_M - E(R_M)$	$(R_1 - E(R_1))$ x $R_M - E(R_M)$
1	37	15	27.33	6	163.98
2	9	13	-.67	4	-2.68
3	-11	14	-20.67	5	-103.35
4	8	-9	-1.67	-18	30.06
5	11	12	1.33	3	3.99
6	4	9	-5.67	0	0.00
	Σ = 58	Σ = 54			Σ = 92.00

$E(R_1) = 9.67$ $E(_M) = 9$

$$\text{Var}_1 = \frac{1211.33}{5} = 242.267 \qquad \text{Var}_M = \frac{410}{5} = 82.00$$

$$\sigma_1 = \sqrt{242.267} = 15.5649 \qquad \sigma_M = \sqrt{82} = 9.0554$$

$$\text{COV}_{1,M} = \frac{92.00}{5} = 18.40$$

6(a). The correlation coefficient can be computed as follows:

$$r_{1,M} = \frac{\text{COV}_{1,M}}{\sigma_1 \sigma_M} = \frac{18.40}{(15.5649)(9.90554)} = \frac{18.40}{140.9464} = .13$$

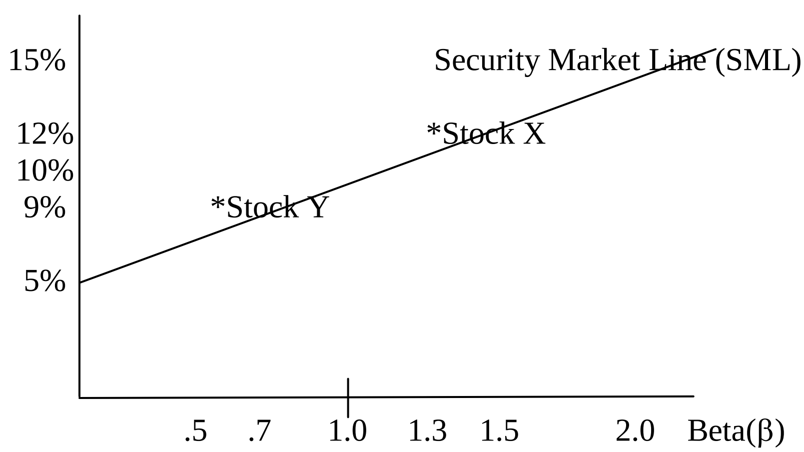
6(b). The standard deviations are: 15.5649% for Chelle Computer and 9.0554% for index, respectively.

6(c). Beta for Chelle Computer is computed as follows:

$$\text{Beta}_1 = \frac{\text{COV}_{1,M}}{\text{Var}_M} = \frac{18.40}{82.00} = .2244$$

7.

7(a). The security market line (SML) shows the required return for a given level of systematic risk. The SML is described by a line drawn from the risk-free rate: expected return is 5 percent, where beta equals 0 through the market return; expected return is 10 percent, where beta equal 1.0. Using the SML:
Stock X expected return = 5% + 1.3% (10% - 5%) =11.50%
Stock Y expected return = 5% + 0.7%(10% - 5%) = 8.50%



7(b). The expected risk-return relationship of individual securities may deviate from that suggested by the SML, and that difference is the asset's alpha. Alpha is the difference between the expected (estimated) rate of return for a stock and its required rate of return based on its systematic risk Alpha is computed as

ALPHA (α) $= E(r_i) - [r_f + \beta(E(r_M) - r_f)]$

where

$E(r_i)$ = expected return on Security i
r_f = risk-free rate
β_i = beta for Security i
$E(r_M)$ = expected return on the market

Calculation of alphas:

Stock X: = 12% - [5% + 1.3% (10% - 5%)] = 12% - 11.5% = 0.5%
Stock Y: = 9% - [5% + 0.7%(10% - 5%)] = 9% - 8.5% = 0.5%

In this instance, the alphas are equal and both are positive, so one does not dominate the other.

7(c). By increasing the risk-free rate from 5 percent to 7 percent and leaving all other factors unchanged, the slope of the SML [$E(R_{MKT})$ - RFR] flattens and now becomes (10% - 7%) or 3%. Using the formula for alpha, the alpha of Stock X increases to 1.1 percent and the alpha of Stock Y falls to -0.1 percent. In this situation, the expected return (12.0 percent) of Stock X exceeds its required return (10.9 percent) based on the CAPM. Therefore, Stock X's alpha (1.1 percent) is positive. For Stock Y, its expected return (9.0 percent) is below its required return (9.1 percent) based on the CAPM. Therefore, Stock Y's alpha (-0.1 percent) is negative. Stock X is preferable to Stock Y under these circumstances.
Calculations of revised alphas:

Stock X = 12% - [7% + 1.3 (10% - 7%)]
= 12% - 10.90% = 1.1%

Stock Y = 9% - [7% + 0.7(10% - 7%)]
= 9% - 9.1% = -0.1%

8.
8(a). Security Market Line

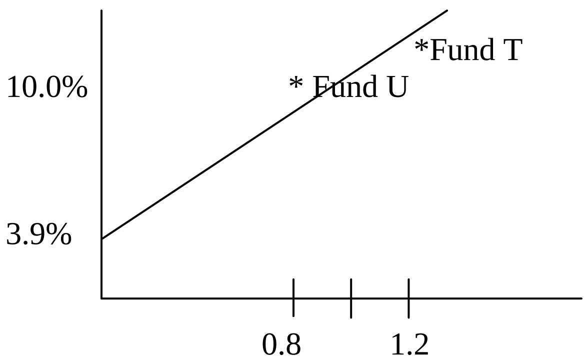
i. *Fair-value plot*. The following template shows, using the CAPM, the expected return, ER, of Fund T and Fund U on the SML. The points are consistent with the following equations:

ER on stock = Risk-free rate + Beta x (Market return – Risk-free rate)

ER for Fund T = 3.9% + 1.2(6.1%)
= 11.22%

ER for Fund U = 3.9% + 0.8(6.1%)
= 8.78%

ii. *Analyst estimate plot*. Using the analyst's estimates, Fund T plots below the SML and Fund U, above the SML.



8(b). Over vs. Undervalue

Fund T is overvalued (a potential "sell" candidate) because it should provide a 11.22% return according to the CAPM whereas the analyst has estimated only a 9.0% return.

Fund U **is undervalued** (a potential "buy" candidate) because it should provide a 8.8% return according to the CAPM whereas the analyst has estimated a 10% return.

9. Recall beta equals Cov(i, market)/Var(market). Using a spreadsheet, the following results are obtained:

	Proxy	True
Variance of market	205.2	109.3
Covariance (Sophie, market)	256.7	187.6

Computing betas for Sophie using the market proxy and the true market portfolio, we have:

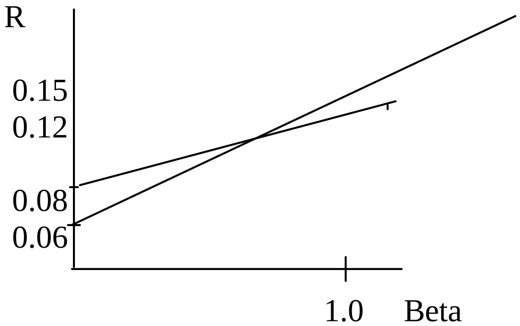
β_{proxy} = 256.7/205.2 = 1.251
β_{true} = 187.6/109.3 = 1.716.

According to the true market measure, Sophie's stock is more risky than when measured using the market proxy. Estimated returns, using the market proxy, will understate the expected returns for Sophie. If Sophie's returns approximate those given the true market, Sophie's stock will appear to consistently outperform compared to the market proxy.
For a given level of the risk-free rate the market proxy is not mean-variance efficient, as it is dominated by the true market portfolio. This is evident from comparing the returns and risks of Sophie, the market proxy, and the true market portfolio:

	Sophie	Market Proxy	True Market
Average Return	2.2	1.2	1.6
Standard deviation of returns	18.00	14.32	10.45

The market proxy has a lower average return and higher risk than the true market portfolio.

10.



10(b). $\beta = \text{Cov}_{i,m}/(\sigma_m)^2$

From a spreadsheet program, we find for Radar Tire and the Proxy,

$\text{Cov}_{i,m} = 187.4$

$\sigma_m^2 = 190.4$

Using the proxy:

$\beta_{\text{using proxy}} = 187.4/190.4 = .984$

Using the true index the covariance (Radar, true index) is 176.4 so

$\beta_{\text{using true}} = 176.4/168 = 1.05$

10(c). Using the proxy:

$E(R_R) = 0.08 + 0.984(0.12 - 0.08)$
$= 0.08 + 0.0394$
$= .1194$ or 11.94 percent

Using the true market:

$E(R_R) = 0.06 + 1.05(0.12 – 0.06)$
$= 0.06 + 0.063$
$= 0.123$ or 12.3 percent

Rader's performance of 11 percent would be inferior compared to either.