Chapter 8 Integration and its application

8.1) Definition and Basic integration formula

- An **antiderivative** of a function *f* is a function *F* such that $F'(x) = f(x)$

  In differential notation, $dF = f(x)dx$

- Integration states that

  $\int f(x)dx = F(x) + C$ if only $F'(x) = f(x)$

- Basic Integration

  Properties:

1.	$\int k\,dx = kx + C$ $\quad$ k is a constant
2.	$\int x^n\,dx = \dfrac{x^{n+1}}{n+1} + C$ $\quad$ $n \neq -1$
3.	$\int x^{-1}\,dx = \int \dfrac{1}{x}dx = \int \dfrac{dx}{x} = \ln x + C$ $\quad$ for $x > 0$
4.	$\int e^x\,dx = e^x + C$
5.	$\int kf(x)\,dx = k\int f(x)\,dx$ $\quad$ k is a constant
6.	$\int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

Example: Find $\int \left(2\sqrt[5]{x^4} - 7x^3 + 10e^x - 1\right)dx$

$$\int 2\sqrt[5]{x^4}\,dx - \int 7x^3\,dx + \int 10e^x\,dx - \int 1\,dx$$

$$\int 2x^{4/5}\,dx \quad \Downarrow \quad \Downarrow \quad \Downarrow$$

$$\Downarrow$$

$$\frac{2\cdot x^{4/5+1}}{\frac{4}{5}+1} \qquad \frac{7x^{3+1}}{3+1} \qquad 10e^x \qquad x \qquad +C$$

Constant term

$$\frac{10}{9}\cdot x^{\frac{9}{5}} - \frac{7x^4}{4} + 10e^x - x + C.$$

B/S-C.

Example: If y is a function of x such that dy/dx = 8x – 4 and y(2) = 5, find y.

$$\frac{dy}{dx} = 8x - 4$$

$$dy = (8x - 4) . dx$$

$$\int dy = \int (8x-4) dx$$

$$y = \frac{8x^2}{2} - 4x + C$$

$$= 4x^2 - 4x + C$$

$y(2) = 5$

$$\Rightarrow 5 = 4(2)^2 - 4(2) + C$$

$$= 16 - 8 + C$$

$$= 8 + C \Rightarrow C = -3$$

$$y = 4x^2 - 4x - 3 \quad \#$$

Example: In the manufacture of a product, fixed costs per week are \$4000. (Fixed costs are costs, such as rent and insurance, that remain constant at all levels of production during a given time period.) If the marginal-cost function is

$$\frac{dc}{dq} = 0.000001(0.002q^2 - 25q) + 0.2$$

where c is the total cost (in dollars) of producing q pounds of product per week, find the cost of producing 10,000 lb in 1 week.

$\int \frac{dc}{dq} \Rightarrow \int MC \Rightarrow$ Cost f^n

$\int dc = \int (0.0000001(0.002 \cdot q^2 - 25q) + 0.2) \cdot dq$

$Cost = 0.000001\left(0.002 \cdot \frac{q^3}{3} - \frac{25q^2}{2}\right) + 0.2q$

+ Constant term.

$c(q) = 0.00001\left(0.002\frac{q^3}{3} - \frac{25q^2}{2}\right) + 0.2q$

$+ 4{,}000$

$q = 10{,}000$

$C(10{,}000)$

$MR = 10 - X$

$TR = 10X - \frac{X^2}{2} + C$

$C = 0$; b/c $X = 0 \rightarrow TR = 0$

Integration by substitution method

Example: a. $\int (x+1)^{20}\,dx$ b. $\int 3x^2\left(x^3+7\right)^3 dx$

8.2) Definite integral

- If f is continuous on the interval $[a, b]$ and F is any antiderivative of f on $[a, b]$, then

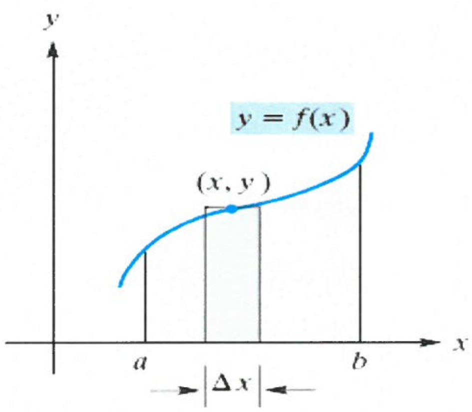
$$\int_a^b f(x)dx = F(b) - F(a)$$

- Property of definite integral

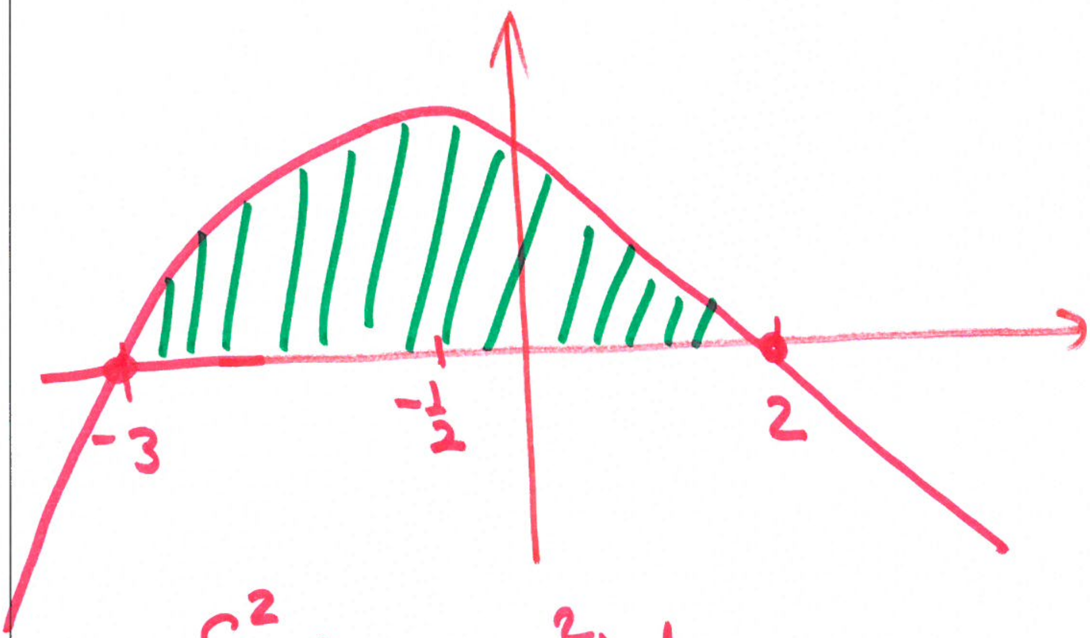
$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx\,; \quad c \in [a, b]$$

- Definite integral as the area under curve
  - Suppose we are interested in finding the area under f(x) bounded by (over the interval) [a,b].
  - Geometrically.
    - The width of the vertical element is Δx. The height is the y-value of the curve.
  - The area is defined as

$$\sum f(x)\Delta x \to \int_a^b f(x)dx = area$$



Example: Find the area of the region bounded by the curve $y = 6 - x - x^2$ for $x \in [-3,2]$



$$\int_{-3}^{2} (6-x-x^2)\,dx$$

$$F(x) = \int (6-x-x^2)\,dx = 6x - \frac{x^2}{2} - \frac{x^3}{3} + C$$

$$= F(2) - F(-3)$$

$$= \left(12 - 2 - \frac{8}{3} + C\right) - \left(-18 - \frac{9}{2} + \frac{\cancel{27}^{\,9}}{\cancel{3}} + C\right)$$

$$= \left(10 - \frac{8}{3} \cancel{+C}\right) \cancel{+C} + 18 + \frac{9}{2} - 9 \cancel{-C}$$

$$= 28 - \frac{8}{3} + \frac{9}{2} - 9$$

Example: Find the area of the region under the curve $y = x^3$ for $x \in [-2,2]$