Assignment 2: Due date: February 17, 2022 before 2.00 pm

Question 1 (30 Points)

Score.....................

At this moment, all of your assets are invested in asset A with the following return and risk characteristics:

$$E(r_A) = 10\%$$

$$\sigma_A = 10\%$$

Another asset (call it"B") becomes avaiable; the characteristics of B are as follows;

$$E(r_B) = 20\%$$

$$\sigma_B = 25\%$$

. Furthermore, the correlation of A's and B's return patterns is -1.

Questions: $corr(r_A, r_B) = \frac{cov(r_A, r_B)}{SD(r_A)SD(r_B)} = -1 \iff cov(r_A, r_B) = -SD(r_A)SD(r_B)$

(1) Using MATLAB to write down the syntax (.m file) for determining the optimal weight (w) of asset A and B in order to achieve the lowest variance, or, in other words, determining the optimal weight for the minimum-variance portfolio.

(2) Find out the Expected return and its variance of the min-variance portfolio using the MATLAB.

(3) By reallocating your portfolio to include some of asset B, how much additional return could you expect to receive if you wanted to maintain your portfolio's risk at $\sigma_p = 10\%$. (Hint: Solve for W_B, not for the W_A).

Note: You must submit both the.m file and your answer in the next page's supplied space.

(1) Let w_i^*, $i = A, B$ be the optimal weight of assets i, $i = A, B$ such that the portfolio has the lowest variance

$$w^* = \begin{bmatrix} w_A^* \\ w_B^* \end{bmatrix} = \begin{bmatrix} 0.7143 \\ 0.2857 \end{bmatrix} \quad \#$$

(2) Let $\bar{R}_{mv}$ and σ_{mv}^2 be the expected return and variance of the minimum-variance portfolio respectively.

$$\bar{R}_{mv} = \underbrace{1.1286}_{\text{Gross return.}} - 1 = 0.1286 = 12.86\ \%$$

$$\sigma_{mv}^2 = 4.2483e^{-19} \approx 0.00\ \% \quad \#$$

— All calculations are done in MATLAB .

Gross Return Vector

```
c =

    1.1000    1.2000
```

Find Covariance and Variance of R_a and R_b

```
cov_RaRb =

   -0.0250

var_Ra =

    0.0100

var_Rb =

    0.0625
```

Variance-Covariance Matrix

```
H =

    0.0100   -0.0250
   -0.0250    0.0625

H_inv =

   1.0e+18 *

    1.2010    0.4804
    0.4804    0.1922
```

Others

```
e =

     1     1
```

```
alpha =

   2.6565e+18


sigma =

   2.9981e+18


delta =

   2.3539e+18
```

Minimum-Variance Portfolio

```
R_bar =

    1.1286


variance =

   4.2483e-19
```

(2)

```
std =

   6.5179e-10


weight =

    0.7143    0.2857
```

(1)

(3) The investor chooses w_b^* such that

$$w_b^* = \arg\max_{w_b} \; (1-w_b)(1.1) + w_b(1.2) \Rightarrow 1.1 + 0.1 w_b$$

$$\text{s.t.} \quad (1-w_b)^2(0.01) + w_b^2(0.0625) + 2(1-w_b)w_b(-1)(0.1)(0.25) = 0.01$$

Lagrangian :

$$L(w_b, \lambda) = (1-w_b)(1.1) + w_b(1.2) + \lambda\left[0.01 - (1-w_b)^2(0.01) - w_b^2(0.0625) + 2(1-w_b)w_b(0.1)(0.25)\right]$$

F.O.C : $\dfrac{\partial L}{\partial w_b} = 0 \Rightarrow -1.1 + 1.2 + \lambda^*\left[0.02(1-w_b^*) - 0.125 w_b^* + 0.05\left(1 - 2w_b^*\right)\right] = 0$ — (1)

$\dfrac{\partial L}{\partial \lambda} = 0 \Rightarrow (1-w_b^*)^2(0.01) + (w_b^*)^2(0.0625) - 2(1-w_b^*)w_b^*(0.1)(0.25) = 0.01$ — (2)

From (2) : $\sqrt{\left(0.1(1-w_b^*) - 0.25 w_b^*\right)^2} = 0.1$

$\left|0.1(1-w_b^*) - 0.25 w_b^*\right| = 0.1$ — (3)

Solve (3) : $0.1(1-w_b^*) - 0.25 w_b^* = 0.1$ or $0.1(1-w_b^*) - 0.25 w_b^* = -0.1$

$0.1 - 0.35 w_b^* = 0.1$ or $0.1 - 0.35 w_b^* = -0.1$

$w_b^* = 0$ or $w_b^* = 0.5714$

Since we require $w_b^* > 0$, thus $w_b^* = 0.5714$.

So, $\bar{R}_p^* = \max \; (1-w_b)(1.1) + w_b(1.2) = (1-w_b^*)(1.1) + w_b^*(1.2) = 1.15714$

Initially the investor invests in asset A alone, and take the gross return equal to 1.1 with the risk 10%.

Including assets B by restricting the risk at the same level 10%, the investor now can have the gross return equal to $\bar{R}_p^* = 1.15714$.

Hence, the investor expects the additional return $= \bar{R}_p^* - 1.1 = 1.151714 - 1.1 = 0.051714 = 5.1714\%$ from including the more risky asset — assets B — into the portfolio given the same level of risk = 10%. #