

Quiz 6

1. Let

$$f(x, y, z) = \frac{\sqrt{y^2z + \ln(x^2 + z)}}{e^{xy}}.$$

Find $\frac{\partial f}{\partial x}$.**Solution**

By the quotient rule,

$$\begin{aligned}\frac{\partial f}{\partial x} &= \frac{1}{e^{2xy}} \left[e^{xy} \frac{\partial}{\partial x} [y^2z + \ln(x^2 + z)]^{1/2} + [y^2z + \ln(x^2 + z)]^{1/2} \frac{\partial}{\partial x} e^{xy} \right] \\ &= \frac{1}{e^{2xy}} \left[e^{xy} \frac{1}{2} [y^2z + \ln(x^2 + z)]^{-1/2} \frac{\partial}{\partial x} [y^2z + \ln(x^2 + z)] + [y^2z + \ln(x^2 + z)]^{1/2} e^{xy} \frac{\partial}{\partial x} (xy) \right] \\ &= \frac{1}{e^{2xy}} \left[e^{xy} \frac{1}{2} [y^2z + \ln(x^2 + z)]^{-1/2} \frac{1}{x^2 + z} \frac{\partial}{\partial x} (x^2 + z) + [y^2z + \ln(x^2 + z)]^{1/2} e^{xy} [y] \right] \\ &= \frac{1}{e^{2xy}} \left[e^{xy} \frac{1}{2} [y^2z + \ln(x^2 + z)]^{-1/2} \frac{2x}{x^2 + z} + y [y^2z + \ln(x^2 + z)]^{1/2} e^{xy} \right]\end{aligned}$$

Note: This can be done through the product rule as well. ■