Assignment 7 (Optional)

1. Find all solution(s) of the following systems of linear equations by using **Gaussian elimination method**.

(a)
$$\begin{aligned} x_1 + x_2 + x_3 + x_4 &= 6 \\ x_1 + 2x_2 + 3x_3 + 4x_4 &= 16 \\ 2x_1 + 3x_2 + 5x_3 + 6x_4 &= 25 \\ x_1 + x_2 + 2x_3 + 3x_4 &= 11 \end{aligned}$$

(b)
$$\begin{aligned} x + 2y + 3z &= 1 \\ 3x + 2y + z &= 1 \\ 7x + 2y - 3z &= 1 \end{aligned}$$

2. Consider the system of 3 equations and 3 unknowns x, y, z:

$$\begin{aligned} x + 2y + 3z &= 1 \\ x + 3y + 4z &= 3 \\ x + 4y + kz &= m, \end{aligned}$$

where m and k are some constants.

   (a) Suppose $k = 5$. Find the value of m that makes the above system of equations have infinitely many solutions.

   (b) Suppose $m = 4$. Find the value of m that makes the above system of equations have no solution.

3. Solve the following system of linear equations by using **Gaussian elimination method**.

$$\begin{aligned} x_1 + x_2 + x_4 + x_5 &= 6 \\ x_1 + 2x_2 + 3x_3 + 4x_4 + x_5 &= 17 \\ 2x_1 + 3x_2 + 5x_3 + 6x_5 &= 19 \\ x_2 + x_3 + 2x_4 + 3x_5 &= 10 \\ x_1 + 2x_2 + 2x_3 + x_4 &= 9 \end{aligned}$$

4. The Russian-born U.S. economist and Novel laureate Wassily Leontief (1906-1999) was interested in the following question:

   What output industries in an economy produce to satisfy the total demand of all products?

   Here, we will consider a very simple example of input-output analysis, an economy with only two industries, A and B. Assume that the consumer demand for their products is, respectively, 310 and 100, in millions of dollars per year.

   Let a and b be the output (in millions of dollars per year) of industries A and B, respectively. What amount of outputs a and b should the two industries generate to satisfy the demand? Ones may be tempted to say 1000 and 780, respectively. However, things are not quite as simple as that. We have to take into account the *interindustry demand* as well. E.g. suppose A produces electricity and B produces mechanical devices. Suppose also that

   - industry B needs \$0.3 worth of electrical power from A for each \$1 of output B produces;
   - industry A needs \$0.5 worth of mechanical devices from B for each \$1 of output A produces.

   Construct the corresponding system of linear equations to find the outputs a and b needed to satisfy both consumer and interindustry demand.