



EE 320 Introductory Mathematical Economics

Semester 1/2015

Practice Problem 8

(Unconstrained Optimization Problems – Multiple-Independent Variables)

Question 1: Suppose that there are two firms in the industry, and they are competing in quantities. The amount of the commodity sold by firm i is q_i , $i = 1, 2$. The market demand function is given by $P = 50 - 3q$, where $q = q_1 + q_2$. The cost functions for each firm is given by $TC_i = 25 + 5q_i$, $i = 1, 2$.

- Find the profit-maximizing quantity for each firm, and determine each firm's profit level.
- Suppose that both firms merge. Compute the new profit-maximizing quantity and the new profit of the merged firm. Do firms have incentive to merge, and why?

Question 2: Given the production function

$$Q = f(K, L) = 8K^{1/2}L^{1/4}$$

Suppose that the price per unit of Q is $\$P$, and the per unit input prices for K and L are $\$r$ and $\$w$, respectively. (P , w , and r are positive constants.)

- Solve for the values K and L that maximizes the profit. Verify that the second-order sufficient condition is met.

- b. Find the comparative statics derivatives $\frac{\partial K^*}{\partial r}$ and $\frac{\partial L^*}{\partial P}$, evaluate the signs, and interpret their economic meanings.

Question 3: Each of two firms A and B produces its own brand of a commodity, such as mineral water, in amounts denoted by x and y , and these are sold at prices p and q unit, respectively. Each firm determines its own price and produces exactly as much as is demanded. The demands for the two brands are given by

$$x = 29 - 5p + 4q$$

$$y = 16 + 4p - 6q$$

Firm A has total costs $5 + x$, whereas firm B has total costs $3 + 2y$.

(Assume that the functions to be maximized have maxima, and at positive prices.)

- Initially, the two firms collude in order to maximize their combined profits, as one monopolist would. Find the prices (p, q) , the production levels (x, y) , and the profits of firms A and B.
- Then, an antitrust authority prohibits collusion, so each producer maximizes its own profit, taking the other's price as given.
 - If q is fixed, how will A choose p ? (Find p as a function $p = p_A(q)$.)
 - If p is fixed, how will B choose q ? (Find q as a function $q = q_B(p)$.)
- Under the assumptions in part b), what constant equilibrium prices are possible? What are the production levels and profits in this case?

Question 4 A firm produces two different kinds A and B of a commodity. The daily cost of producing Q_1 units of A and Q_2 units of B is:

$$C(Q_1, Q_2) = 0.1(Q_1^2 + Q_1Q_2 + Q_2^2) .$$

Suppose that the firm sells all its output at a price per unit $P_1 = 120$ for A and $P_2 = 90$ for B.

- Find the daily production levels that maximize profit.

- b. What prices (P_1) per unit of A would imply that the optimal daily production level for A is 400 units?

Question 5:

Let the total cost function depend on goods x, y and z ;

$$TC = 1,000 + 3x^2 + 2y^2 + 2z^2 - 2xy - 40z - 20x$$

Determine the level of x , y and z which minimize total cost and determine the minimum total cost.

Question 6:

A monopolist produces two products, A, and B. The joint-cost function is $C = 5000 + 5q_A + 3q_B$, where c is the total cost of producing q_A units of A and q_B units of B. The demand functions for these products are given by $p_A = 205 - 2q_A - q_B$ and $p_B = 153 - q_A - q_B$, where p_A and p_B are the prices of A and B, respectively. Consider the following problem.

- Determine the profit-maximizing level output for both products.
- How much should the monopolist set the price of the two products?

Question 7:

A manufacturer produces products A and B for which the average costs of production are constant at 3 and 5 (dollars per unit), respectively. The quantities q_A , q_B of A and B that can be sold each week are given by the joint-demand functions $q_A = 10 - p_A + p_B$ and $q_B = 12 + p_A - 3p_B$, p_A and p_B are the prices (in dollars per unit) of A and B, respectively. Determine the prices of A and B at which the manufacturer can maximize profit.

Question 8:

Consider a market with 2 firms. Each firm sells an identical product, facing the same market demand equation given by $p = 10 - Q$. For the first firm, denoted by firm 1, the cost function is given by $C = c_1 Q_1$. For the second firm, the cost function is given by $C = c_1 Q_2^2$. Consider the following problem.

- Determine the level of output that each firm will choose to produce under the Cournot equilibrium.
- State the requirement for c_1 in order to ensure that both firms stay active in the equilibrium.
- Do they share the same market size under the equilibrium? Explain your result with some economic intuitions.
- Determine the level of output that each firm will produce under the collusion.

Question 9:

The profit function of a firm is $\pi(x, y) = px + qy - \alpha x^2 - \beta y^2$, where p and q are the prices per unit and $\alpha x^2 + \beta y^2$ are the costs of producing x units of the first good and y units of the other. The constants are all positive.

- Find the values of x and y that maximize profits. Denote them by x^* and y^* . Verify that the second-order conditions are satisfied.
- Define $\pi(p, q) = \pi(x^*, y^*)$ as the optimal profit function. The function generates the level of maximum profit that firm attain under different combination of profit-maximizing output bundles. Verify that $\partial\pi(p,q)/\partial p = x^*$ and $\partial\pi(p,q)/\partial q = y^*$. Give these results economic interpretations.
- Show that $\pi(p,q)$ is convex in p and q . That is, you show that Hessian matrix of the optimal profit function is positive definiteness.

Question 10:

Define $f(x,y)$ for all (x,y) by $f(x, y) = e^{x+y} + e^{x-y} - \frac{3}{2}x - \frac{1}{2}y$

- a. Derive the Hessian matrix of $f(x, y)$.
- b. Show that $f(x,y)$ is monotonically convex function. That's, the function is convex for the entire domain sets defining $f(x,y)$.
- c. Find the global extrema of $f(x,y)$

Question 11:

Consider a function $f(x, y) = x^2 - y^2 - xy - x^3$

- a. Find and classify the stationary points of $f(x, y)$
- b. Find the domain set of $f(x, y)$ where $f(x, y)$ is concave, and find the largest value of $f(x, y)$ in that domain set.