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EE 325 Section 1 (Aj.Wanwiphang) Homework Assignment 1

Due date: 31 January 2020 before 11pm

*** Please submit this assignment on Moodle. For those who work on paper, please scan or submit the pictures of your work.***

1. Find the answers following questions (please also show your calculation)

a. $\sum_{i=1}^{5}(a+bx_i) = (a+bx_1)+(a+bx_2)+(a+bx_3)+(a+bx_4)+(a+bx_5)$

$= 5a+bx_1+bx_2+bx_3+bx_4+bx_5$

$= 5a+b(x_1+x_2+x_3+x_4+x_5)$

b. $\sum_{y=0}^{5} f(x+y) = f(x+0)+f(x+1)+f(x+2)+f(x+3)+f(x+4)+f(x+5)$

c. $\sum_{i=1}^{10} i^2 = 1^2+2^2+3^2+4^2+5^2+6^2+7^2+8^2+9^2+10^2$

$= 1+4+9+16+25+36+49+64+81+100$

$= 385$

d. $\sum_{x=1}^{2}\sum_{y=2}^{3}(2x+y) = [2(1)+2]+[2(2)+3]$

$= (2+2)+(4+3)$

$= 4+7 = 11$

2. Given X is discrete random variable. The probability distribution function (PDF) of this variable is shown in the table

X	-2	-1	0	1	2	3	4
$f(x)$	0.5b	b	2.25b	2b	1.5b	0.5b	0.25b

** when b is constant number

a. Find the value of b

$f(x) = P(X=x)$: $0.5b+b+2.25b+2b+1.5b+0.5b+0.25b = 1$

$8b = 1$

$b = \frac{1}{8} = 0.125$

b. Find the answer for $P(X \le 2)$

$P(X \le 2) = 1 - P(X > 2)$

$= 1 - [P(X=3)+P(X=4)]$

$= 1 - [0.5(0.125)+0.25(0.125)]$

$= 1 - (0.0625+0.03125)$

$= 1 - 0.09375$

$= 0.90625$

c. Find the answer for $P(-2 \le X \le 3)$

$P(-2 \le X \le 3) = 1 - P(X=4)$

$= 1 - 0.25(0.125)$

$= 1 - 0.03125 = 0.96875$

d. Find the answer for $P(X \ge 1)$

$P(X \ge 1) = P(X=1)+P(X=2)+P(X=3)+P(X=4)$

$= 2(0.125)+1.5(0.125)+0.5(0.125)+0.25(0.125)$

$= 0.25+0.1875+0.0625+0.03125$

$= 0.53125$

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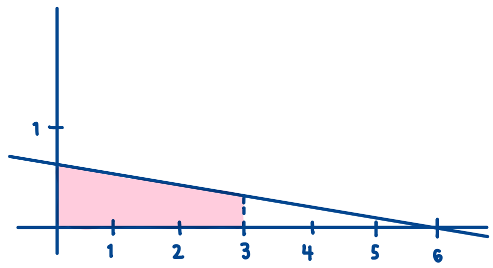
3. Given X is continuous random variable. The probability distribution function (PDF) of this variable is

$$f(x) = -\frac{1}{9}x + \frac{6}{9}, 0 \le x \le 3$$

a. Plot graph for $f(x)$



b. Find the answer for $P(1 \le X \le 3)$

$$P(1 \le X \le 3) = \int_1^3 f(x)\,dx$$

$$= \int_1^3 -\frac{1}{9}x + \frac{6}{9}\,dx$$

$$= -\frac{x^2}{18} + \frac{6x}{9} \Big|_1^3$$

$$= \left[-\frac{3^2}{18} + \frac{6(3)}{9}\right] - \left[-\frac{1}{18} + \frac{6(1)}{9}\right]$$

$$= \left(-\frac{9}{18} + \frac{18}{9}\right) - \left(-\frac{1}{18} + \frac{6}{9}\right)$$

$$= -\frac{9}{18} + \frac{18}{9} + \frac{1}{18} - \frac{6}{9}$$

$$= -\frac{8}{18} + \frac{12}{9}$$

$$= -\frac{8}{18} + \frac{24}{18}$$

$$= \frac{16}{18} = \frac{8}{9}$$

c. Find the answer for $P(X \ge 2)$

$$P(x \ge 2) = \int_2^3 f(x)\,dx$$

$$= \int_2^3 -\frac{1}{9}x + \frac{6}{9}\,dx$$

$$= -\frac{x^2}{18} + \frac{6}{9}x \Big|_2^3$$

$$= \left[-\frac{3^2}{18} + \frac{6(3)}{9}\right] - \left[\frac{-2^2}{18} + \frac{6(2)}{9}\right]$$

$$= \left(-\frac{9}{18} + \frac{18}{9}\right) - \left(-\frac{4}{18} + \frac{12}{9}\right)$$

$$= -\frac{9}{18} + \frac{18}{9} + \frac{4}{18} - \frac{12}{9}$$

$$= -\frac{5}{18} + \frac{6}{9}$$

$$= -\frac{5}{18} + \frac{12}{18}$$

$$= \frac{7}{18}$$

d. Find the expected value of X

$$E(x) = \int_0^3 x f(x)\,dx$$

$$= \int_0^3 x\left(-\frac{1}{9}x + \frac{6}{9}\right)dx$$

$$= \int_0^3 -\frac{1}{9}x^2 + \frac{6}{9}x\,dx$$

$$= -\frac{1}{27}x^3 + \frac{6}{18}x^2 \Big|_0^3$$

$$= \left[-\frac{3^3}{27} + \frac{6(3)^2}{18}\right] - \left[-\frac{0^3}{27} + \frac{6(0)^2}{18}\right]$$

$$= -1 + 3 - 0$$

$$= 2$$

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4. Let random variable X be the outcome of throwing one dice and random variable Y be the outcome of tossing one coin. Coin has two sided that has valued 1 and 0.

   a. Construct the joint probability distribution function (PDF) table of X and Y

Y\X	1	2	3	4	5	6
0	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$
1	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

   b. Find the marginal probability distribution function (PDF) of X

$f(X=1) = \frac{1}{6}$ | $f(X=4) = \frac{1}{6}$

$f(X=2) = \frac{1}{6}$ | $f(X=5) = \frac{1}{6}$

$f(X=3) = \frac{1}{6}$ | $f(X=6) = \frac{1}{6}$

   c. Find the marginal probability distribution function (PDF) of Y

$f(Y=0) = \frac{1}{2}$

$f(Y=1) = \frac{1}{2}$

   d. Find the conditional probability distribution function (PDF) of X given Y is equal to 1

$$f(X|Y=1) = \frac{f(x,y)}{f_Y(1)} = \frac{f(x,1)}{1/2}$$

X	1	2	3	4	5	6
f(x|y=1)	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

   e. Find the expected value of X given Y is equal to 1

$$E(X|Y=1) = \sum x_i P(X=x_i|Y=1) = \frac{\sum x_i P(X=x_i, Y=1)}{P(Y=1)}$$

$$= \frac{[(1\cdot\frac{1}{12})+(2\cdot\frac{1}{12})+(3\cdot\frac{1}{12})+(4\cdot\frac{1}{12})+(5\cdot\frac{1}{12})+(6\cdot\frac{1}{12})]}{1/2} = \frac{7}{2} = 3.5$$

   f. Find the variance of X given Y is equal to 1

$$var(X|Y=1) = \sum (x-E(X|Y=1))^2 \cdot P(X|Y=1)$$

$$= \{[(1-\tfrac{7}{2})^2\cdot\tfrac{1}{6}]+[(2-\tfrac{7}{2})^2\cdot\tfrac{1}{6}]+[(3-\tfrac{7}{2})^2\cdot\tfrac{1}{6}]+[(4-\tfrac{7}{2})^2\cdot\tfrac{1}{6}]+[(5-\tfrac{7}{2})^2\cdot\tfrac{1}{6}]+[(6-\tfrac{7}{2})^2\cdot\tfrac{1}{6}]\}$$

$$= [(\tfrac{25}{4}\cdot\tfrac{1}{6})+(\tfrac{9}{4}\cdot\tfrac{1}{6})+(\tfrac{1}{4}\cdot\tfrac{1}{6})+(\tfrac{1}{4}\cdot\tfrac{1}{6})+(\tfrac{9}{4}\cdot\tfrac{1}{6})+(\tfrac{25}{4}\cdot\tfrac{1}{6})]$$

$$= \frac{25}{24}+\frac{9}{24}+\frac{1}{24}+\frac{1}{24}+\frac{9}{24}+\frac{25}{24}$$

$$= \frac{70}{24} = 2.917$$

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5. If X_1, X_2, X_3 is a random sample from a population with mean u and variance σ^2. X_1, X_2, X_3 are not independent

$$Cov(X_1, X_2) = Cov(X_1, X_3) = Cov(X_2, X_3) = \frac{1}{4}\sigma^2$$

$\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{3}(X_1 + X_2 + X_3)$

Find $E(\bar{X})$ and $\text{var}(\bar{X})$

$$E(\bar{x}) = E\left(\frac{1}{N}\sum_{i=1}^{3} x_i\right)$$
$$= \frac{1}{N} E(x_1 + x_2 + x_3)$$
$$= \frac{1}{N}\left(E(x_1) + E(x_2) + E(x_3)\right)$$
$$= \frac{1}{3}(\mu_x + \mu_x + \mu_x)$$
$$= \frac{1}{3}(3\mu_x)$$
$$= \mu_x$$

$$\text{var}(\bar{x}) = \text{var}\left(\frac{1}{N}\sum_{i=1}^{3} x_i\right)$$
$$= \frac{1}{N^2}\text{var}(x_1 + x_2 + x_3)$$
$$= \frac{1}{N^2}[\text{var}(x_1) + \text{var}(x_2) + \text{var}(x_3) + 2\text{cov}(x_1 x_2) + 2\text{cov}(x_1 x_3) + 2\text{cov}(x_2 x_3)]$$
$$= \frac{1}{3^2}\left(\frac{1}{4}\sigma_x^2 + \frac{1}{4}\sigma_x^2 + \frac{1}{4}\sigma_x^2 + \frac{1}{2}\sigma_x^2 + \frac{1}{2}\sigma_x^2 + \frac{1}{2}\sigma_x^2\right)$$
$$= \frac{1}{9}\left(\frac{9}{4}\sigma_x^2\right)$$
$$= \frac{1}{4}\sigma_x^2$$
$$= 0.25\,\sigma_x^2$$

6. Given X_1, X_2, X_3, X_4 are independent identically distributed random variables from population with mean u and variance σ^2. $\bar{X}$ is estimator used to estimate mean value. $\bar{X} = \frac{1}{4}(X_1 + X_2 + X_3 + X_4)$

a. Find $E(\bar{X})$ and $\text{var}(\bar{X})$ in term of μ and σ

$$E(\bar{x}) = E\left(\frac{1}{N}\sum_{i=1}^{4} x_i\right)$$
$$= \frac{1}{N} E(x_1 + x_2 + x_3 + x_4)$$
$$= \frac{1}{N}\left(E(x_1) + E(x_2) + E(x_3) + E(x_4)\right)$$
$$= \frac{1}{4}(\mu_x + \mu_x + \mu_x + \mu_x)$$
$$= \frac{1}{4}(4\mu_x)$$
$$= \mu_x$$

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$$var(\bar{x}) = var\left(\frac{1}{N}\sum_{i=1}^{4} x_i\right)$$

$$= \frac{1}{N^2} var(x_1 + x_2 + x_3 + x_4)$$

$$= \frac{1}{4^2} (var(x_1) + var(x_2) + var(x_3) + var(x_4))$$

$$= \frac{1}{16} (\sigma_x^2 + \sigma_x^2 + \sigma_x^2 + \sigma_x^2)$$

$$= \frac{1}{16} (4\sigma_x^2)$$

$$= \frac{1}{4} \sigma_x^2$$

b. Given $\tilde{X} = \frac{1}{8}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \frac{1}{2}X_4$ is another estimator of μ. Show that $\tilde{X}$ is an unbiased estimator of u

$$\tilde{x} = \frac{1}{8}x_1 + \frac{1}{4}x_2 + \frac{1}{8}x_3 + \frac{1}{2}x_4$$

$$= \frac{1}{4}(0.5x_1 + x_2 + 0.5x_3 + 2x_4)$$

$$E(\tilde{x}) = E\left(\frac{1}{N}\sum_{i=1}^{4} x_i\right)$$

$$= \frac{1}{4} E(0.5x_1 + x_2 + 0.5x_3 + 2x_4)$$

$$= \frac{1}{4}[E(0.5x_1) + E(x_2) + E(0.5x_3) + E(2x_4)]$$

$$= \frac{1}{4}[0.5E(x_1) + E(x_2) + 0.5E(x_3) + 2E(x_4)]$$

$$= \frac{1}{4}(0.5\mu_x + \mu_x + 0.5\mu_x + 2\mu_x)$$

$$= \frac{1}{4}(4\mu_x)$$

$$= \mu_x$$

So, $\tilde{x}$ is an unbiased estimator of μ

c. Between $\bar{X}$ and $\tilde{X}$, which one is the better estimator for μ? Why?

Between $\bar{x}$ and $\tilde{x}$, $\bar{x}$ is the better estimator for μ than $\tilde{x}$ because $\bar{x}$ has smaller value of variance than $\tilde{x}$ which is $\bar{x} = 0.25\,\sigma_x^2 < \tilde{x} = 0.34\,\sigma_x^2$