Intertemporal Consumption

Topics to be Discussed

- Consumption decision between times, a simple model with money market
- Determination of interest rates
- Consumption decision with investment
- Present Discounted Value

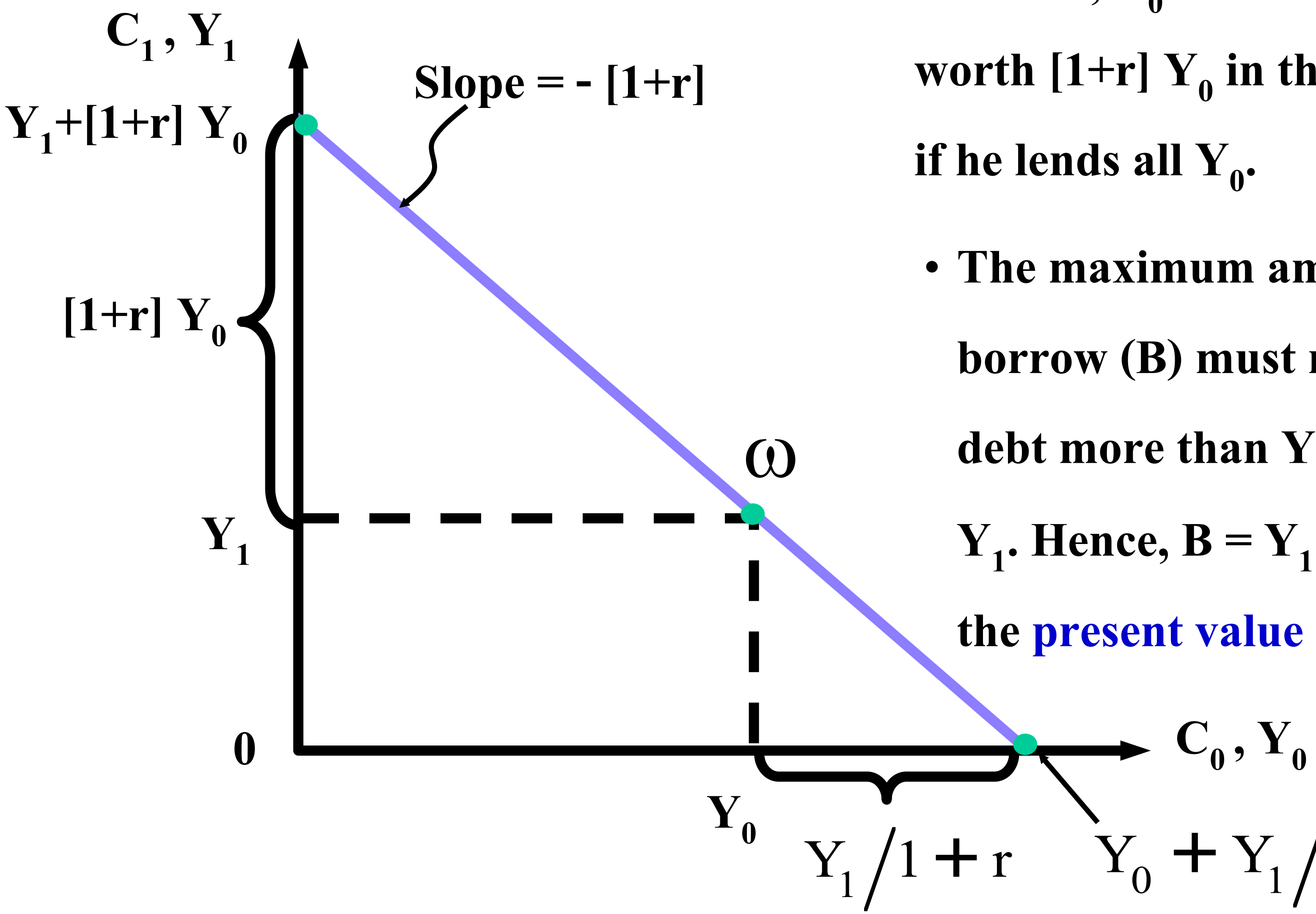
BASIC CONCEPT

- Consumers need to choose consumption over different periods of time.
- Consumers may choose to save some of the present consumption for the future.
- Or borrow some from the future in order to consume more today.

Simple model: Assumptions

- C_0 and C_1 are consumption where 0 denotes this period, and 1 denotes next period.
- They are both normal goods
- Y_0 and Y_1 are endowments.
- The economy has a money market for lending and borrowing at an interest rate = r per period.
- Assume: no risk.
- No debt or bequest will be left after period 1.
- A consumer chooses the consumption bundle that maximises utility given the endowment and interest rate.

Budget line for 2 periods



- From ω, Y_0 has a future value or worth [1+r] Y_0 in the next period if he lends all Y_0.
  - The maximum amount he can borrow (B) must not generate debt more than Y_1 or [1+r] B = Y_1. Hence, B = Y_1 /[1+r] which is the present value of Y_1.

Budget line for 2 periods

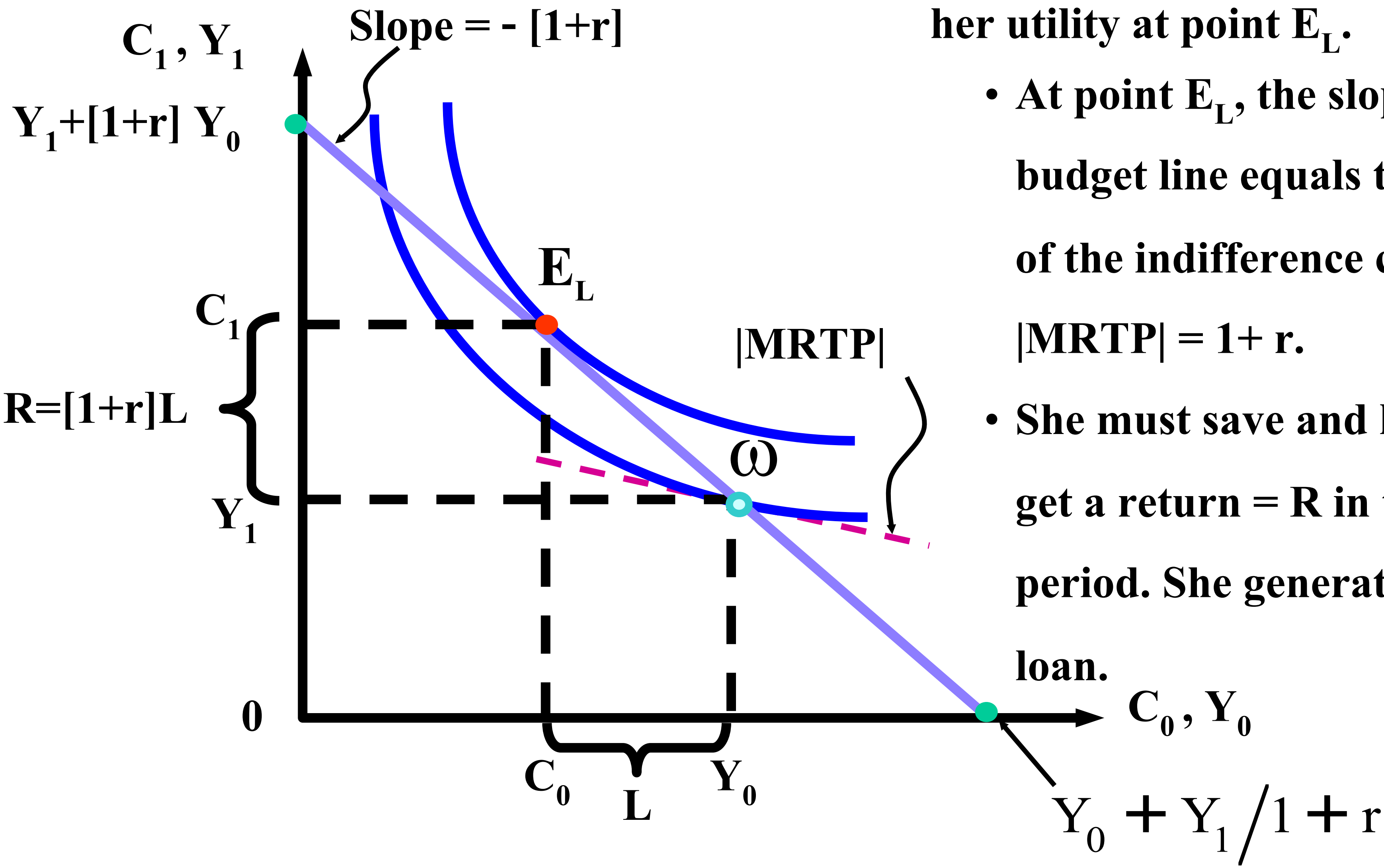
- Since the total consumption must be equal to the total endowment; hence,

$$Y_0 + \frac{Y_1}{1+r} = C_0 + \frac{C_1}{1+r}$$

- The present value of the endowment must equal to the present value of the consumption

Saving and Lending Equilibrium

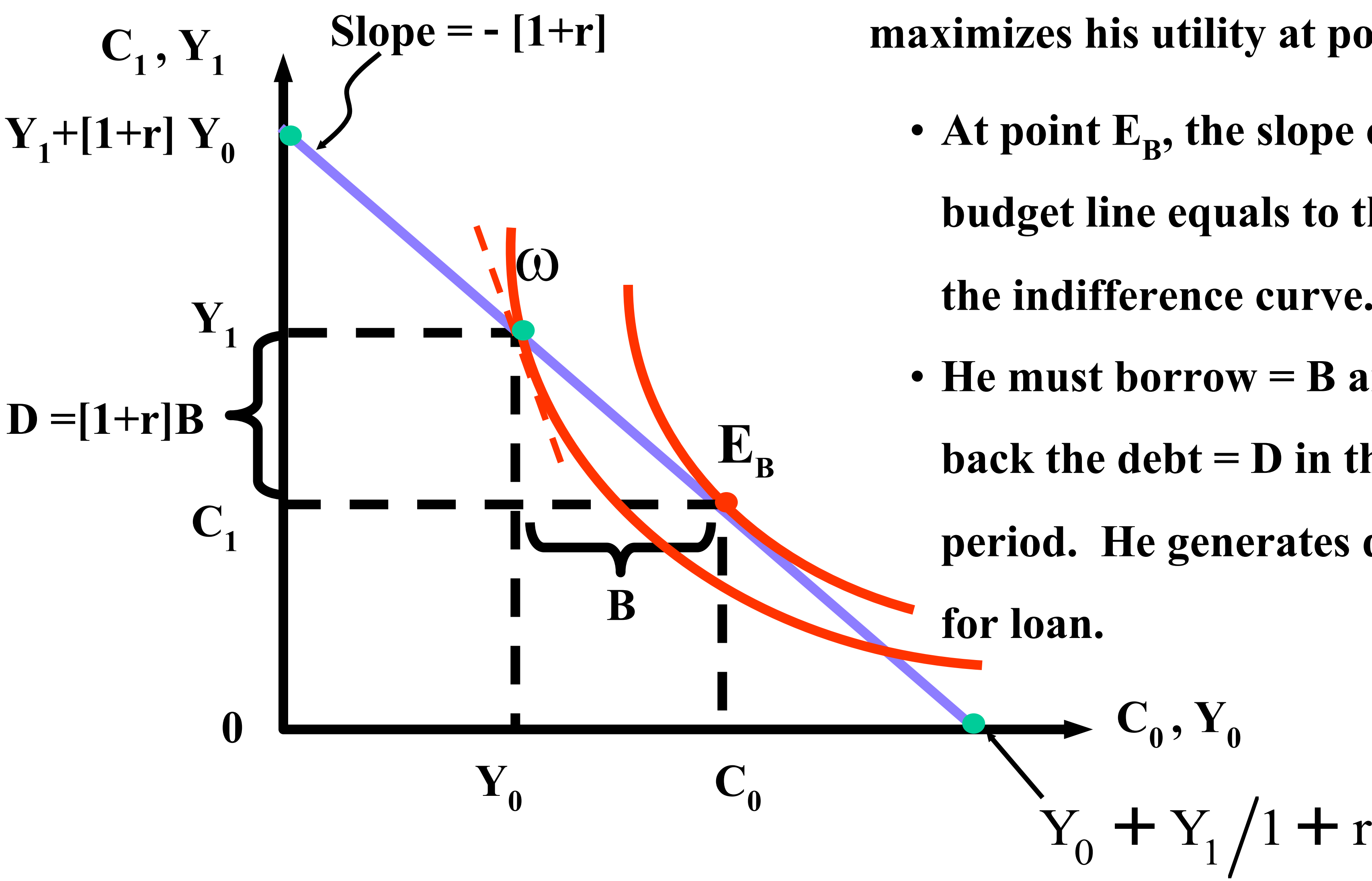
Relatively flat ICs compared to 1+r



- From ω, this consumer maximizes her utility at point E_L.
  - At point E_L, the slope of the budget line equals to the slope of the indifference curve or |MRTP| = 1+ r.
  - She must save and lend = L to get a return = R in the next period. She generates supply of loan.

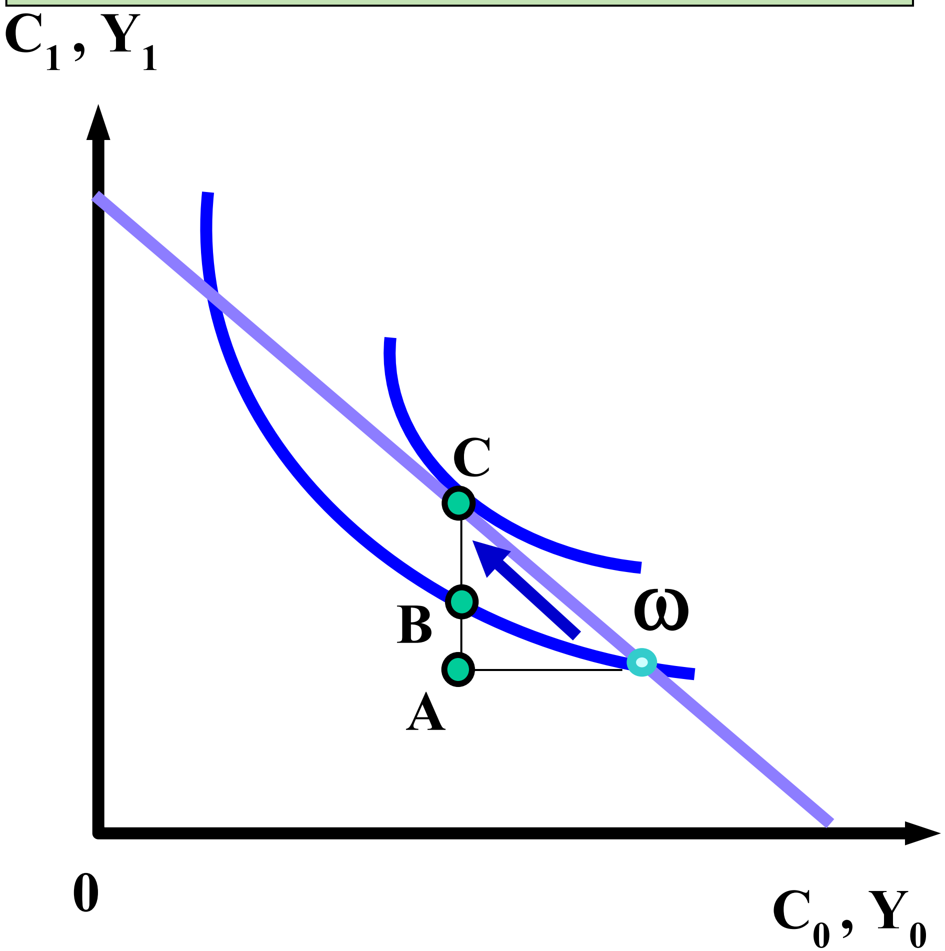
Borrowing Equilibrium

Relatively steep ICs compared to 1+r

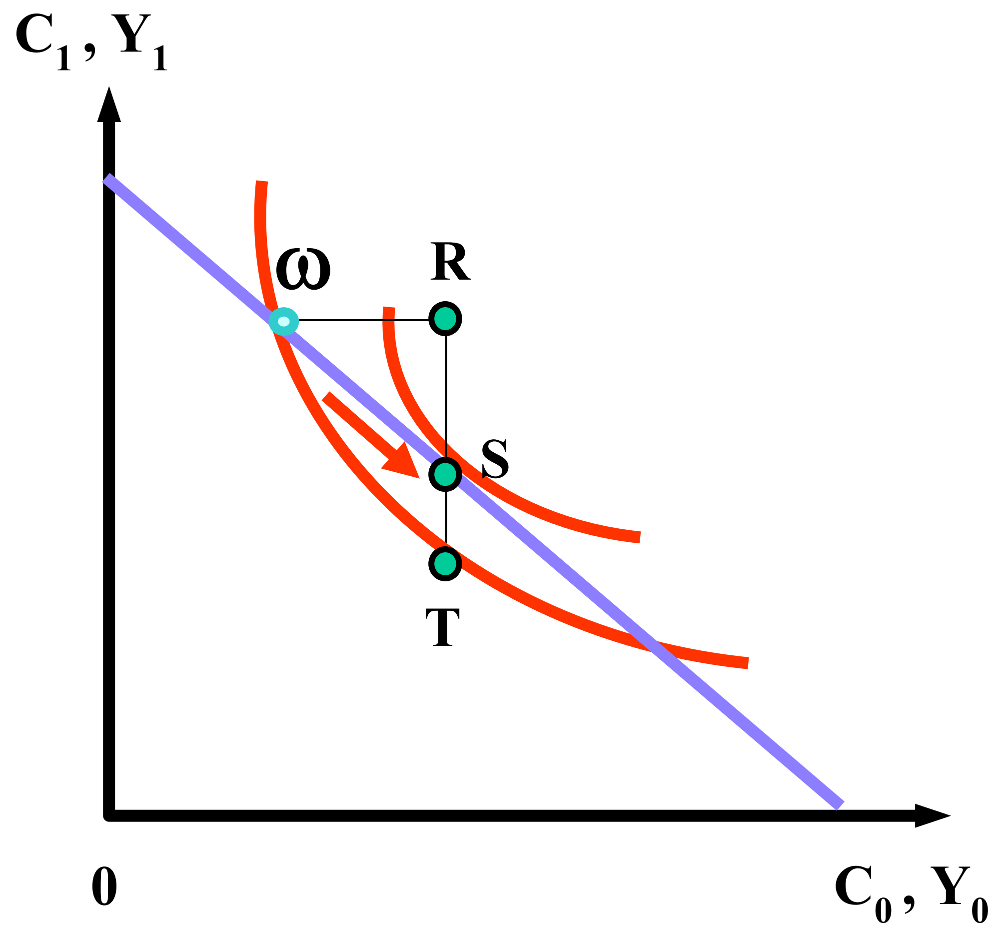


- From ω, another consumer maximizes his utility at point E_B.
  - At point E_B, the slope of the budget line equals to the slope of the indifference curve.
  - He must borrow = B and pay back the debt = D in the next period. He generates demand for loan.

- Willing to give up 1 unit of C_0 in exchange for AB
- but money market offers AC
- should lend money



- Willing to pay up to RT to get 1 more unit of C_0
- but money market asks only RS
- should borrow money



Equilibrium condition

- The consumer maximise utility when

$$|MRTP| = 1 + r$$

- Marginal Rate of Time Preferences: |MRTP| the rate at which consumer is willing to give up C_1 to get 1 additional unit of C_0.
- If $|MRTP| < 1 + r$, the consumer is preparing to accept C_1 less than what the money market offer --> decreases C_0 by lending.
- If $|MRTP| > 1 + r$, the consumer is willing to give up C_1 more than what the money market requires --> increases C_0 by borrowing.

Exercise

- If the loan interest rate is higher than the lending rate, how does the budget line look like?
- The oversea Chinese in the past who came to Thailand with nothing, gradually accumulated their wealth until some became very rich. Yet some of their descendants have turned their inheritance to debts. Use the theory of Intertemporal consumption to explain what might be the difference between the two groups of people?

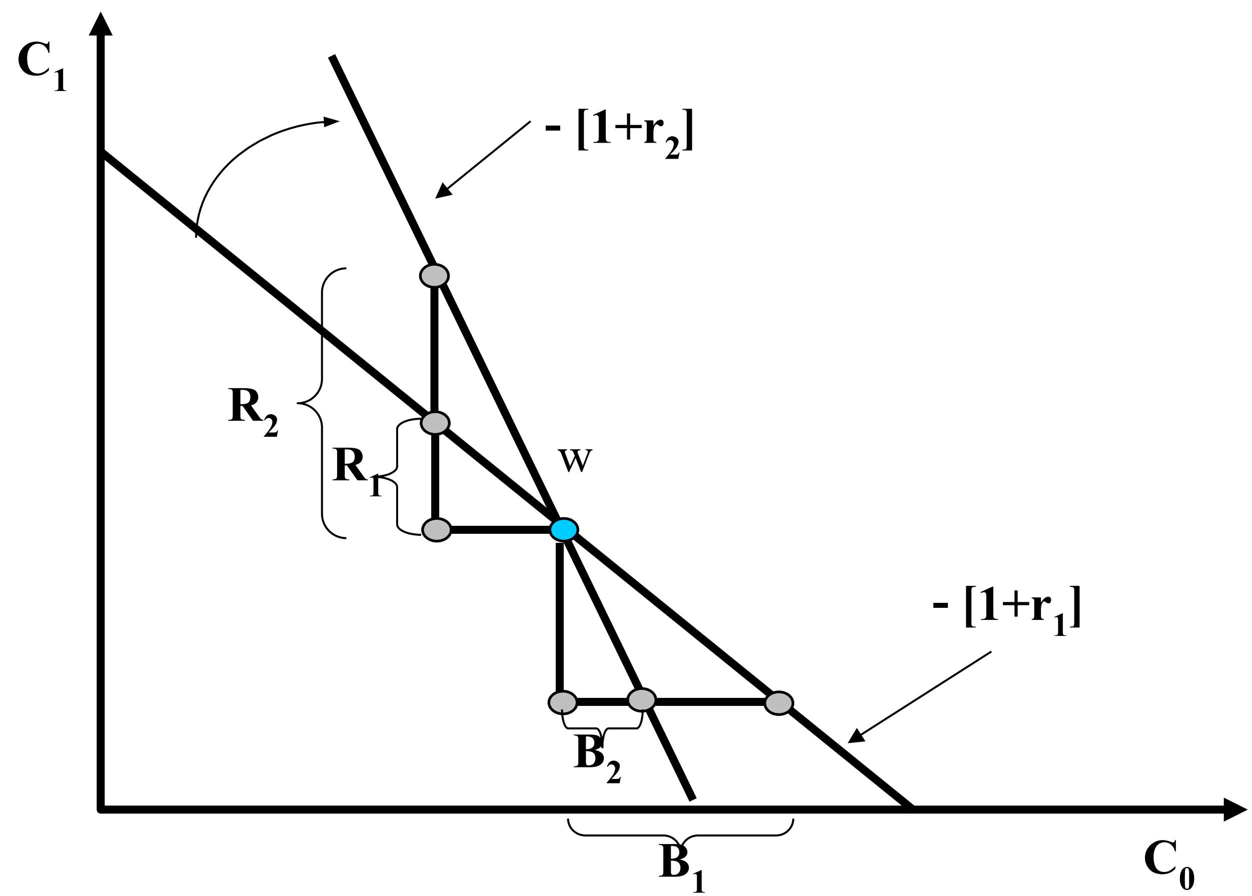
Comparative static analysis

- Changes in endowment
  - increase in Y_0
  - increase in Y_1
  - increase in both Y_0 and Y_1
  - increase in Y_1 but decrease in Y_0

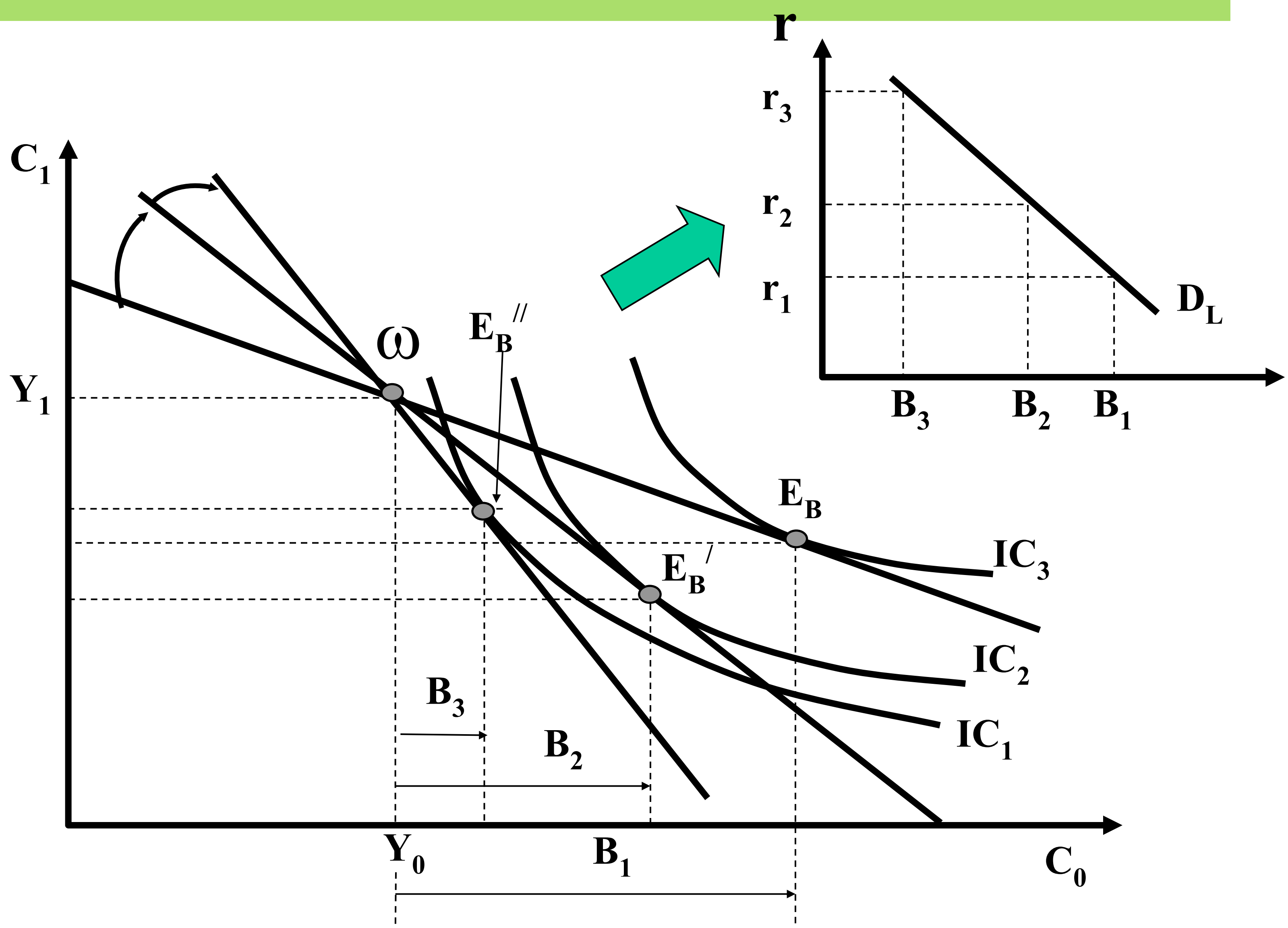
Question: A consumer initially borrows money. Show a new equilibrium with lending when Y_0 is increased.

Comparative static analysis

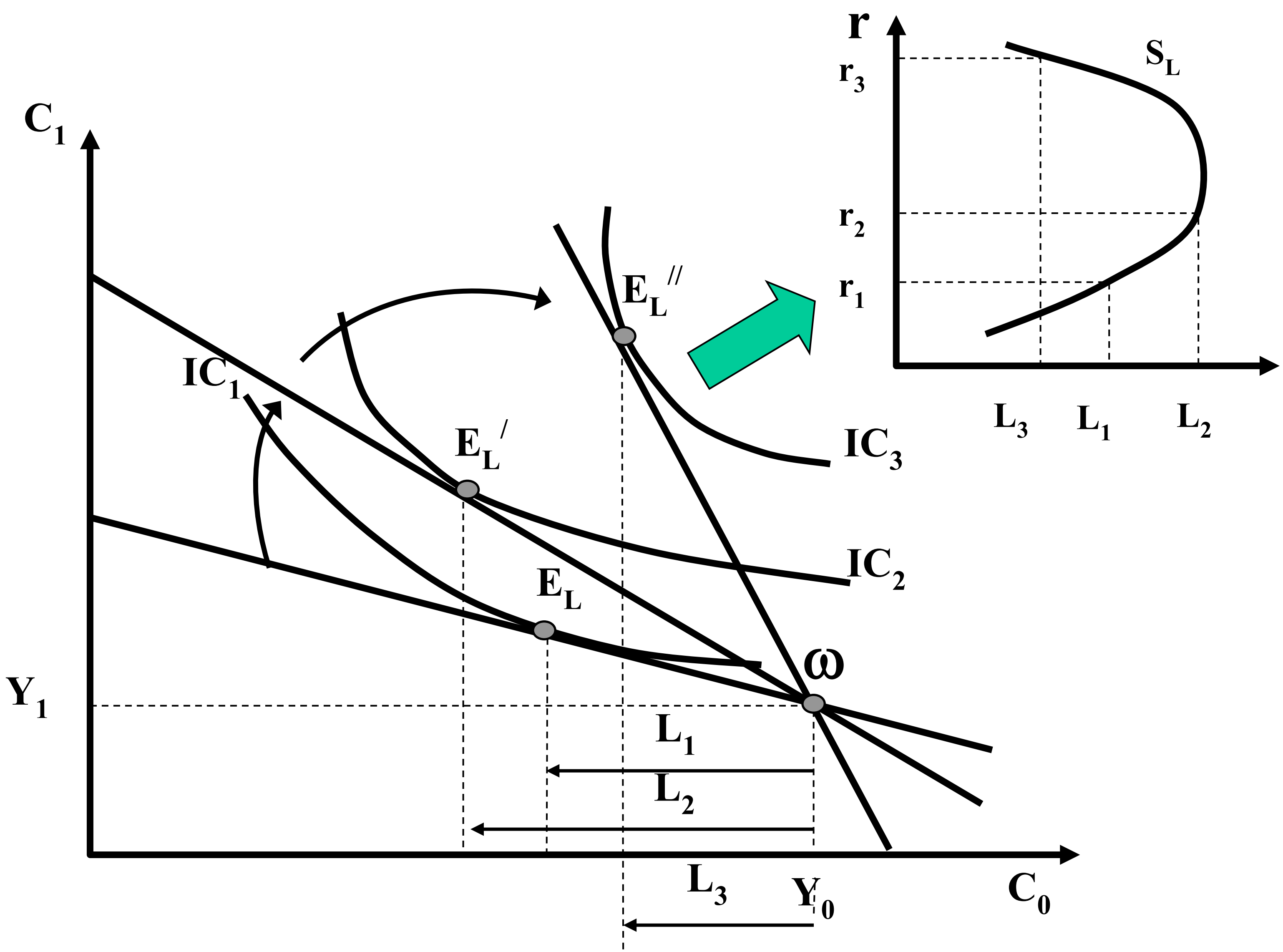
- Changes in the interest rate: r_1 increases



Borrowing case



Lending case



Why does the supply of loan bend backward?

When r increases,

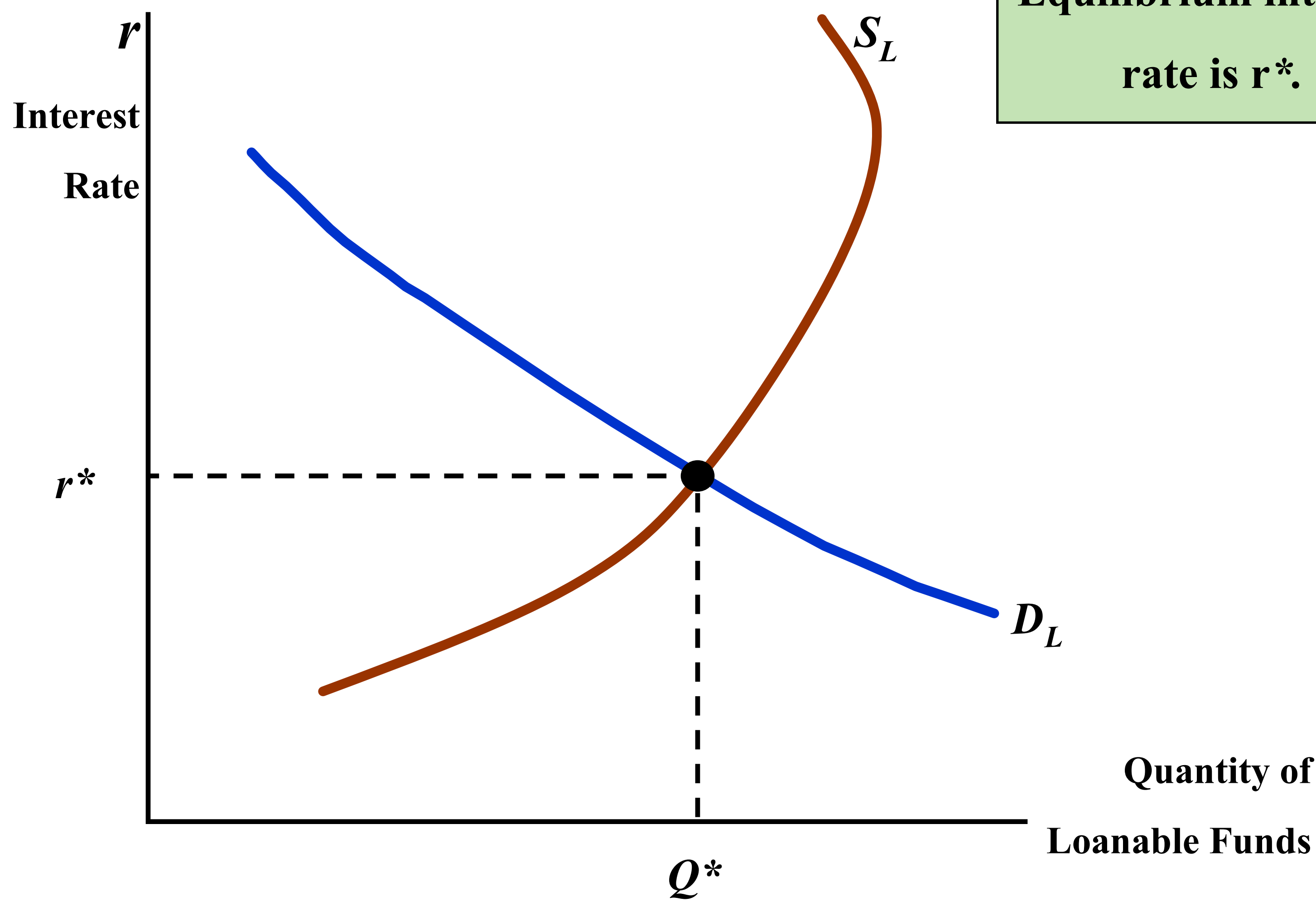
- the opportunity cost of C_0 increases, substitute C_0 with C_1; C_0 decreases causing L to increase --> Substitution effect
- income increases, C_0 increases (since it is normal) causing L to decrease --> Income effect
- When Income effect is stronger, S_L bend backward

How are Interest Rates Determined?

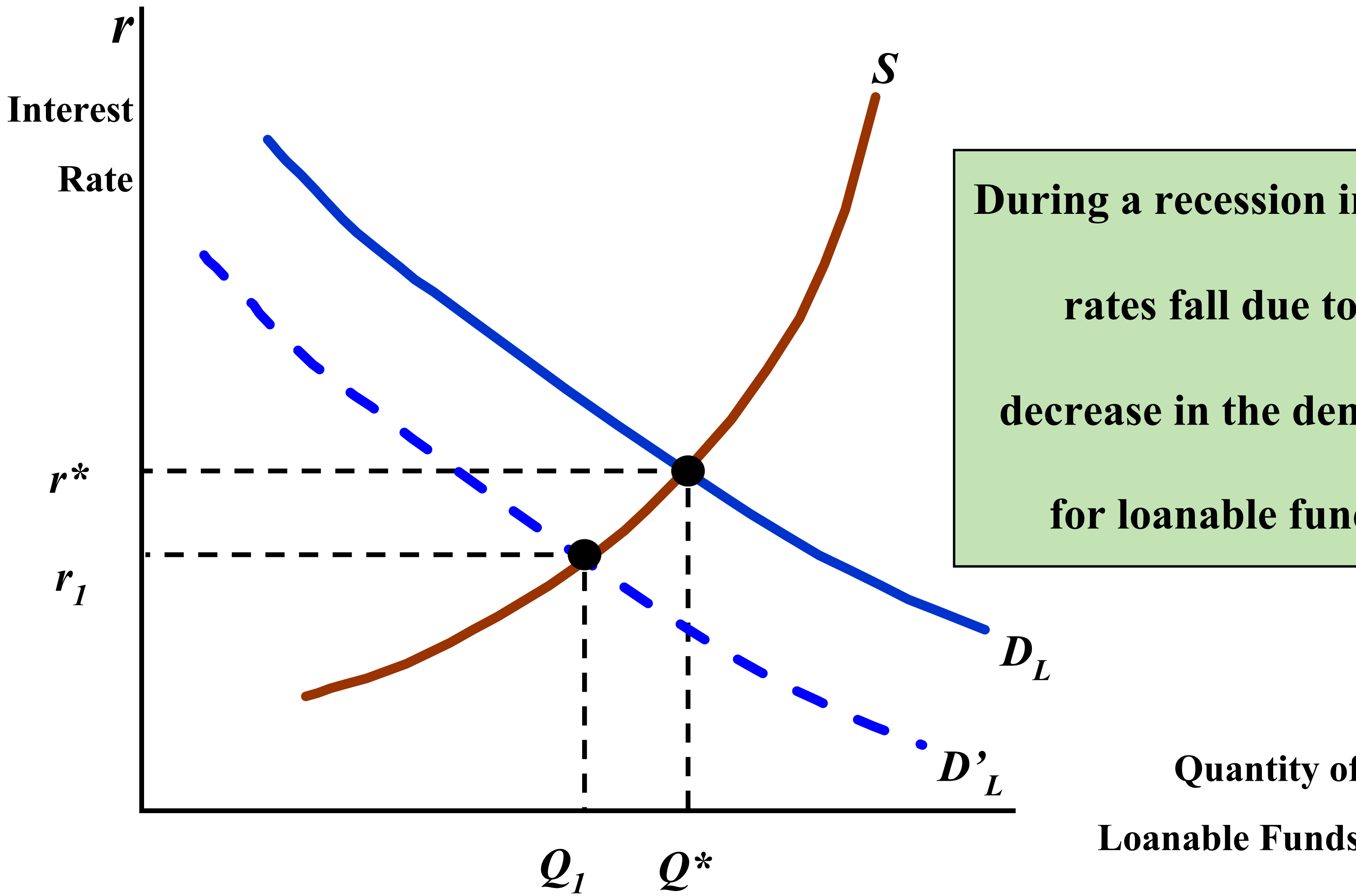
- For households, the higher the interest rate, the greater the cost of consuming
  - Less willing to borrow
  - Demand is declining function of interest rate
- Firms invest in project when NPV > 0
  - Higher interest rate means lower NPV
  - Demand is downward sloping
- Total demand for loanable funds is sum of household demand and firm demand

Supply and Demand for Loanable Funds Determines Interest Rates

Equilibrium interest rate is r*.

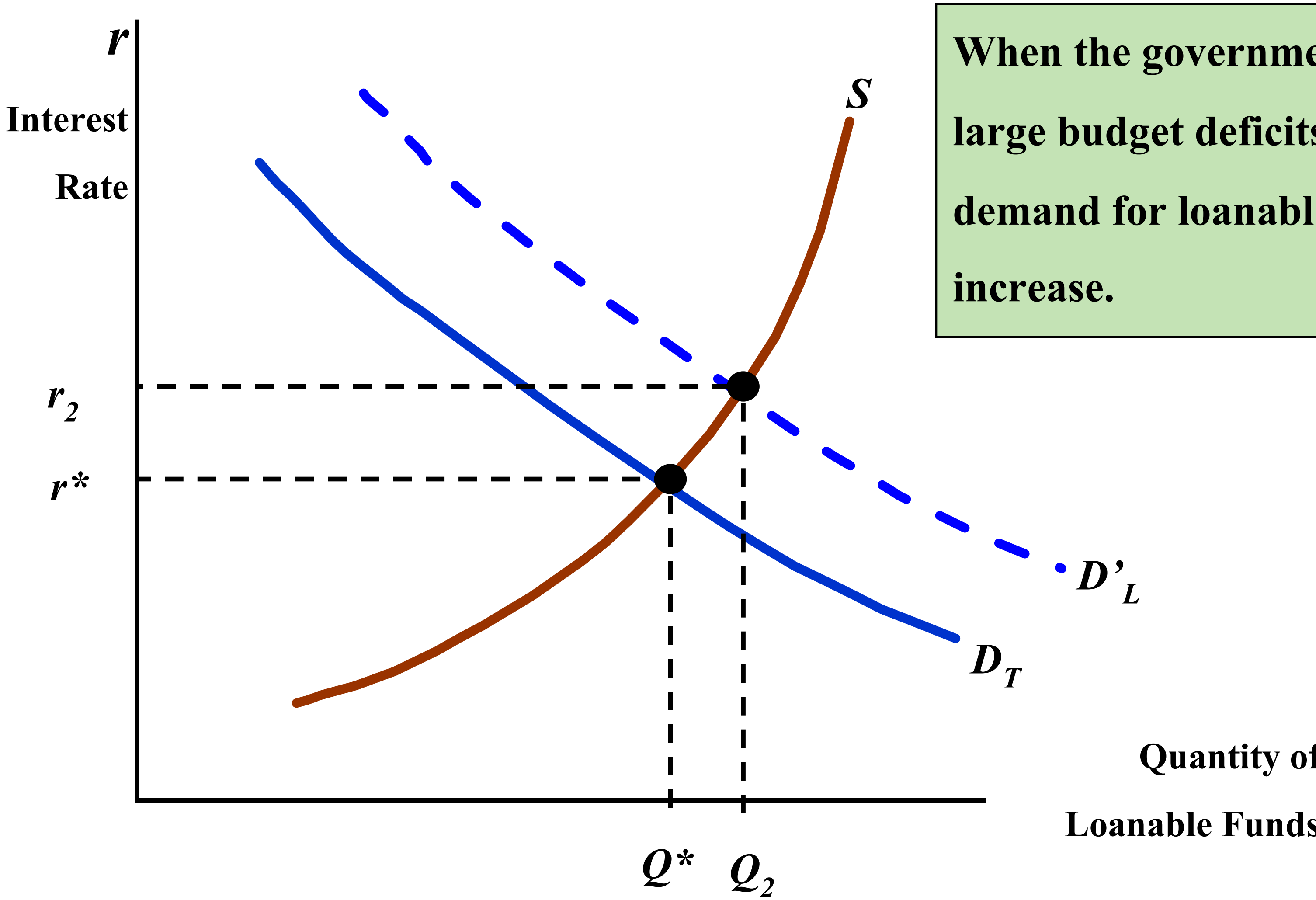


Changes In The Equilibrium



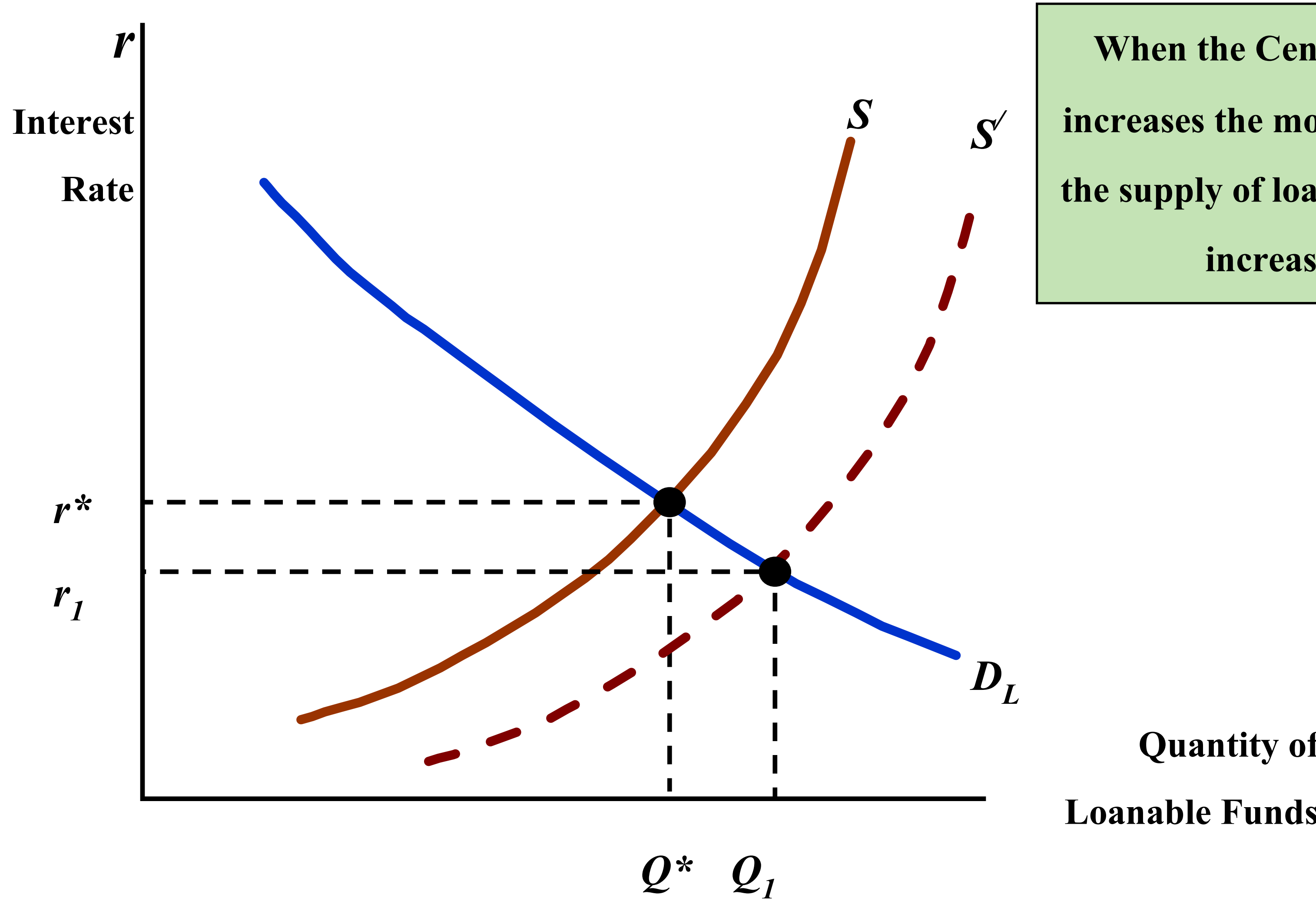
During a recession interest rates fall due to a decrease in the demand for loanable funds.

Changes In The Equilibrium



When the government runs large budget deficits the demand for loanable funds increase.

Changes In The Equilibrium

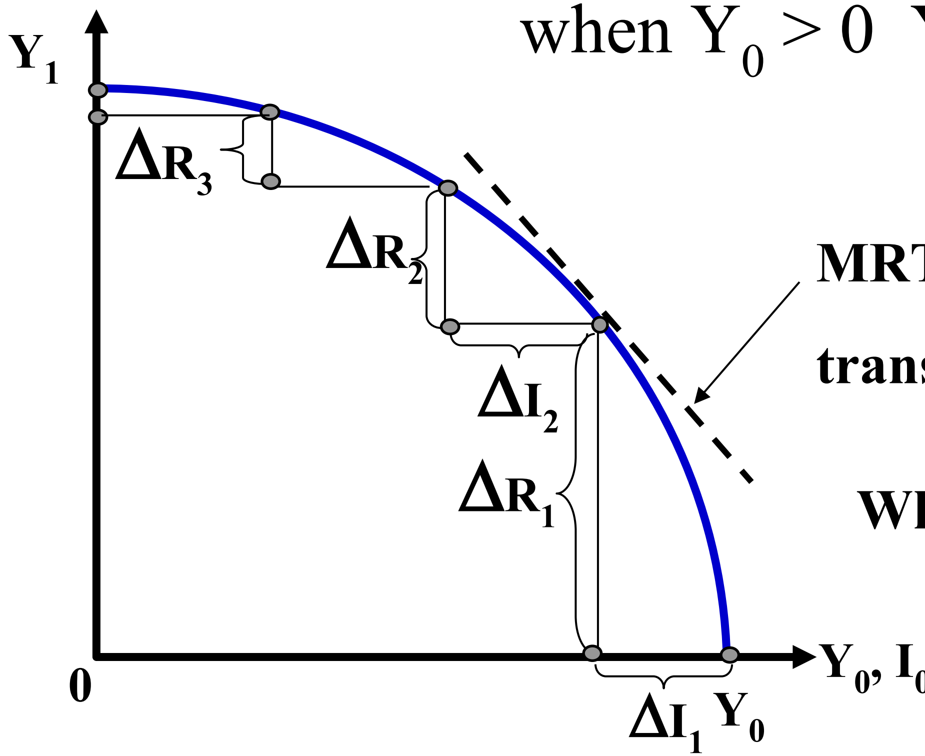


When the Central Bank increases the money supply, the supply of loanable funds increases.

Consumption, saving and investment equilibrium

1) Assume no money market but there are investment opportunities.

Investment Possibility Frontier

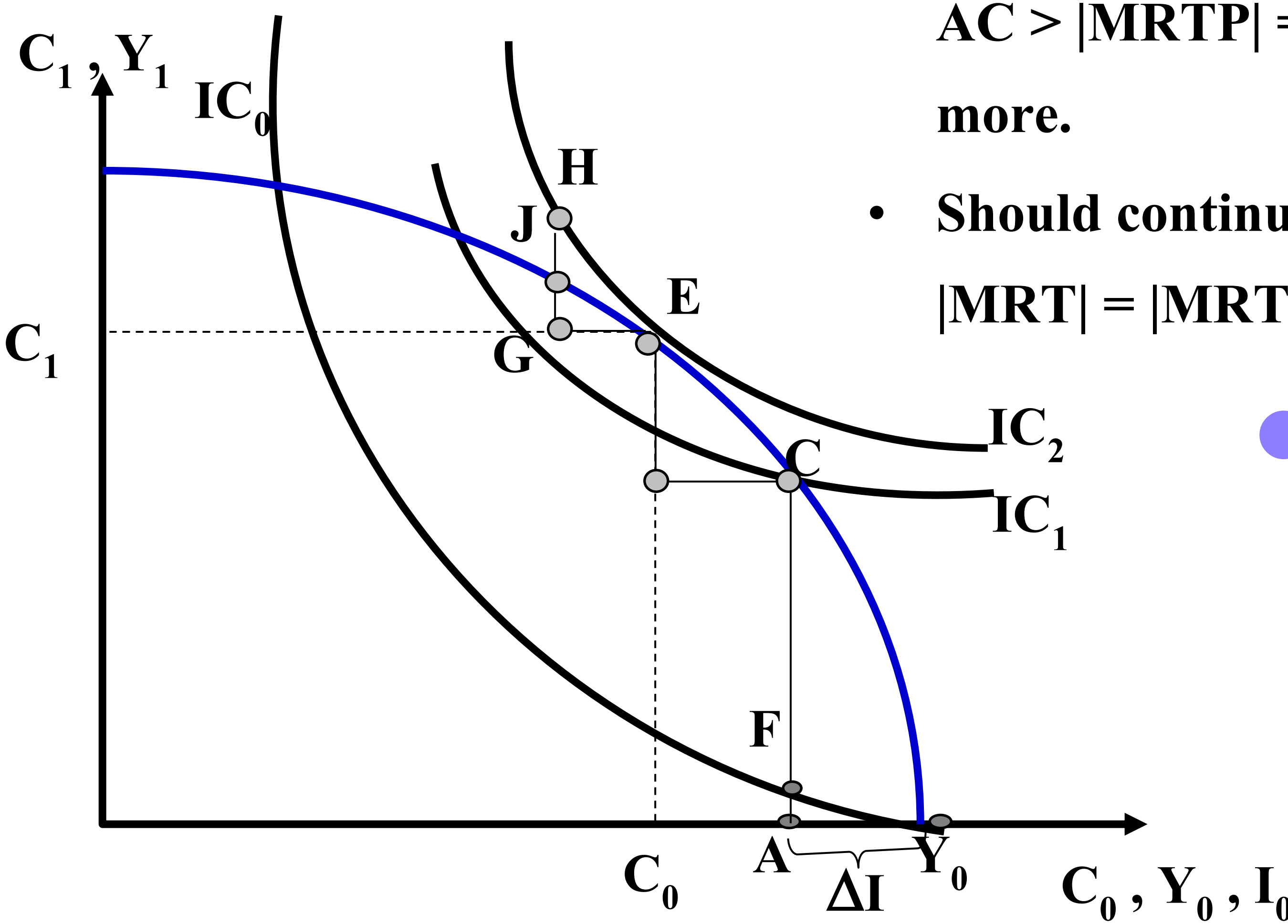


when $Y_0 > 0$ $Y_1 = 0$

MRT: marginal rate of transformation

What if the $Y_1 > 0$?

Consumption, saving and investment equilibrium



- The return on investment is AC > |MRTP| = FA --> invest more.
- Should continue to invest until |MRT| = |MRTP| at point E.

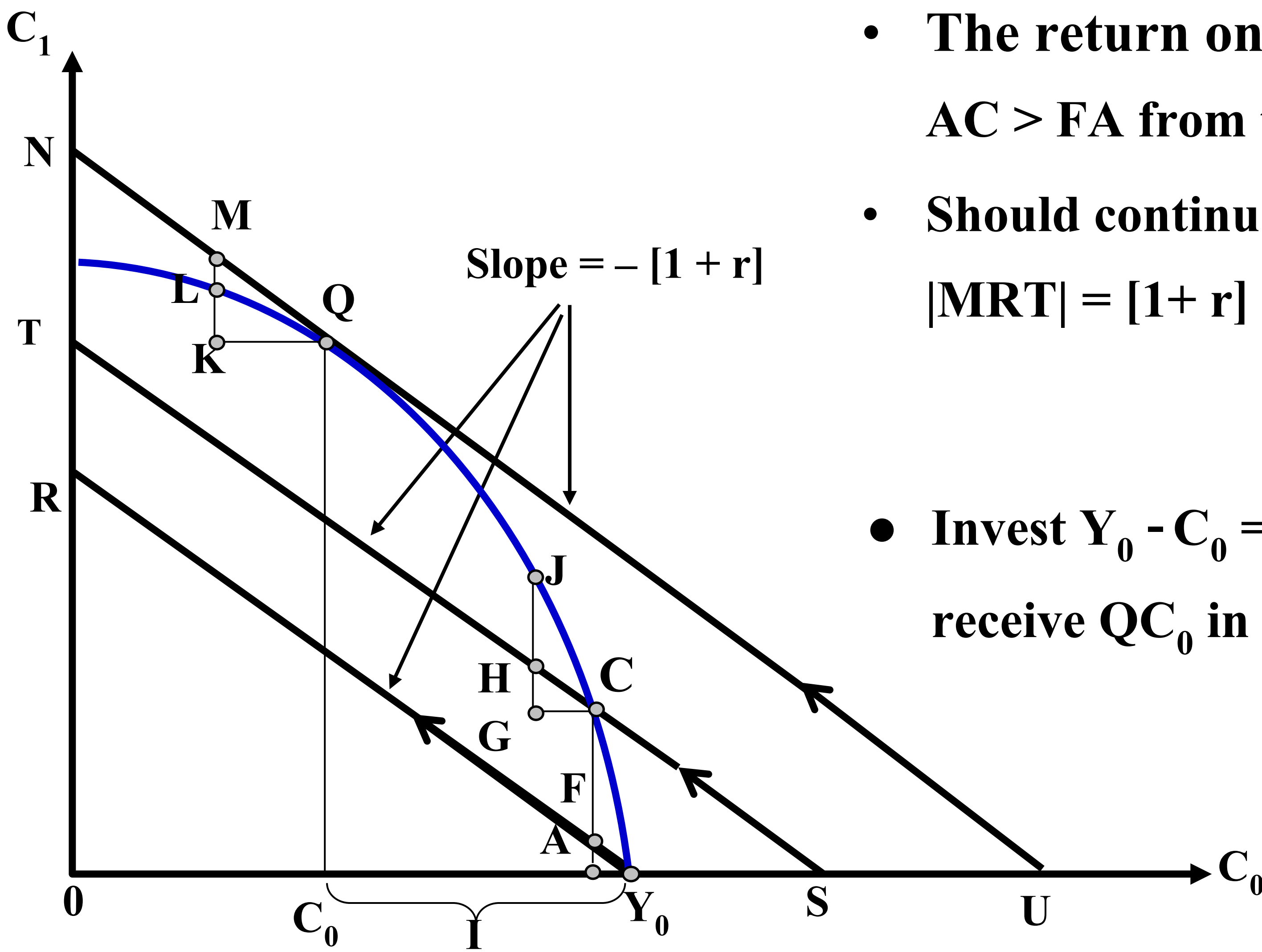
- Consume C_0, C_1 by investing Y_0 - C_0 now and receiving EC_0 in the future.

Consumption, saving and investment equilibrium

2) There are money market and investment opportunities.

- The interest rate is r, the rate of returns on investment depends on the size of investment, given by MRT.
- The consumer needs to choose among consumption, investment, lending or borrowing.

Consumption, saving and investment equilibrium



- The return on investment is AC > FA from the money market.
- Should continue to invest until |MRT| = [1+ r] at point Q.

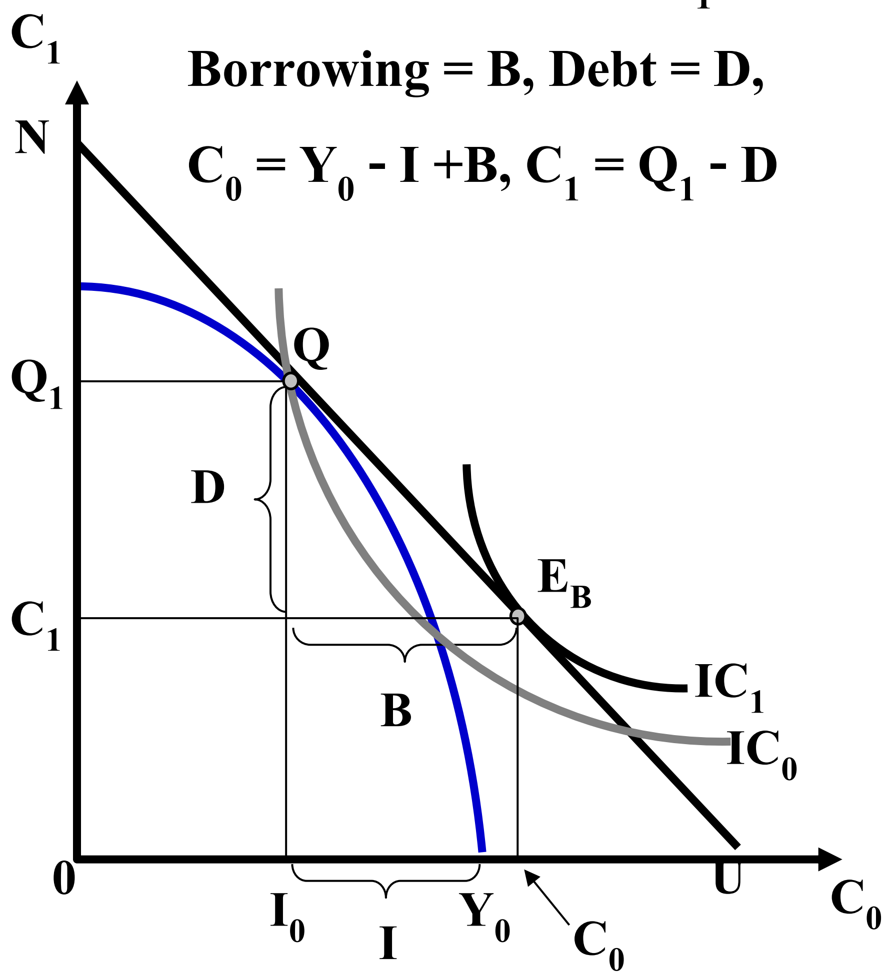
- Invest $Y_0 - C_0 = I$ now, and receive QC_0 in the future.

Consumption, saving and investment equilibrium

Invest =I, Returns = Q_1,

Borrowing = B, Debt = D,

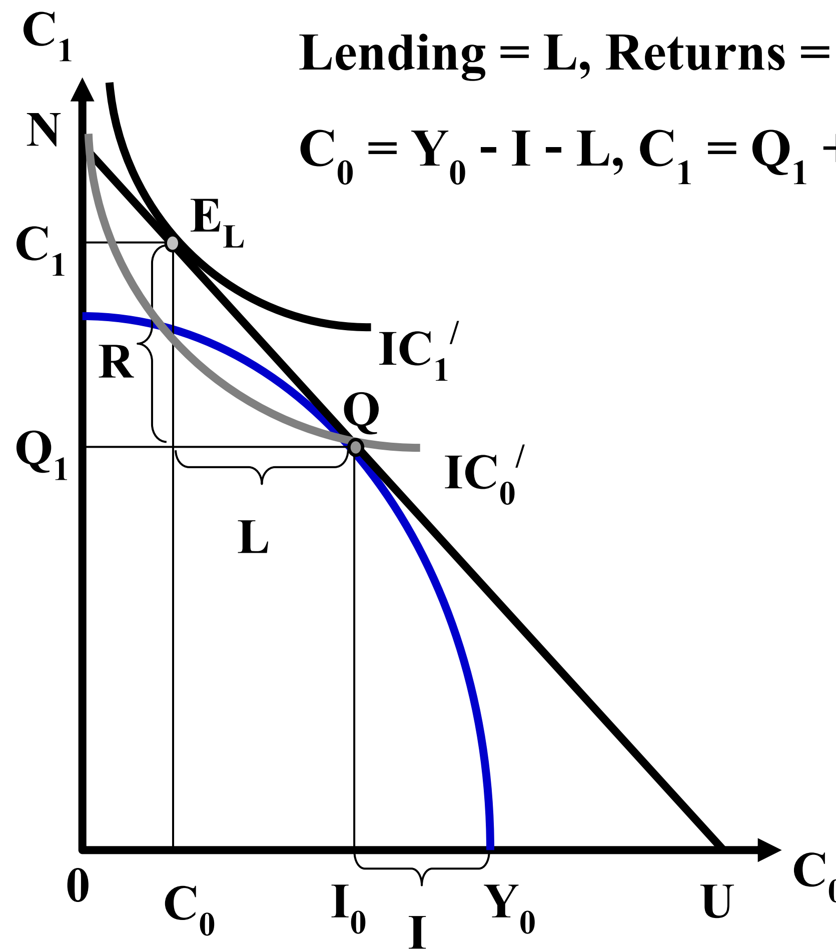
$C_0 = Y_0 - I + B$, $C_1 = Q_1 - D$



Invest =I, Returns = Q_1,

Lending = L, Returns = R,

$C_0 = Y_0 - I - L$, $C_1 = Q_1 + R$



Present Value (PV)

- Determining the value today of a future flow of income
  - The value of a future payment must be discounted for the time period and interest rate that could be earned.
    - Interest rate – rate at which one can borrow or lend money

Present Value (PV)

$$\text{Future Dollar Value of \$1 invested today} = (1 + r)^n$$

where n = Number of year in future.

PV = Present dollar value of \$1 received

$$\text{in the future} = \frac{1}{(1+r)^n};$$ (how much would you have to invest today to have a dollar in the future)

Example: Put \$X in the bank for 2 years at $r = 0.1$

--> $(1+0.1)(1+0.1)\$X = \$1$ or $X = 1/(1+0.1)^2 = 0.83$

Present Value (PV)

$$= \frac{\$}{(1+r)}$$

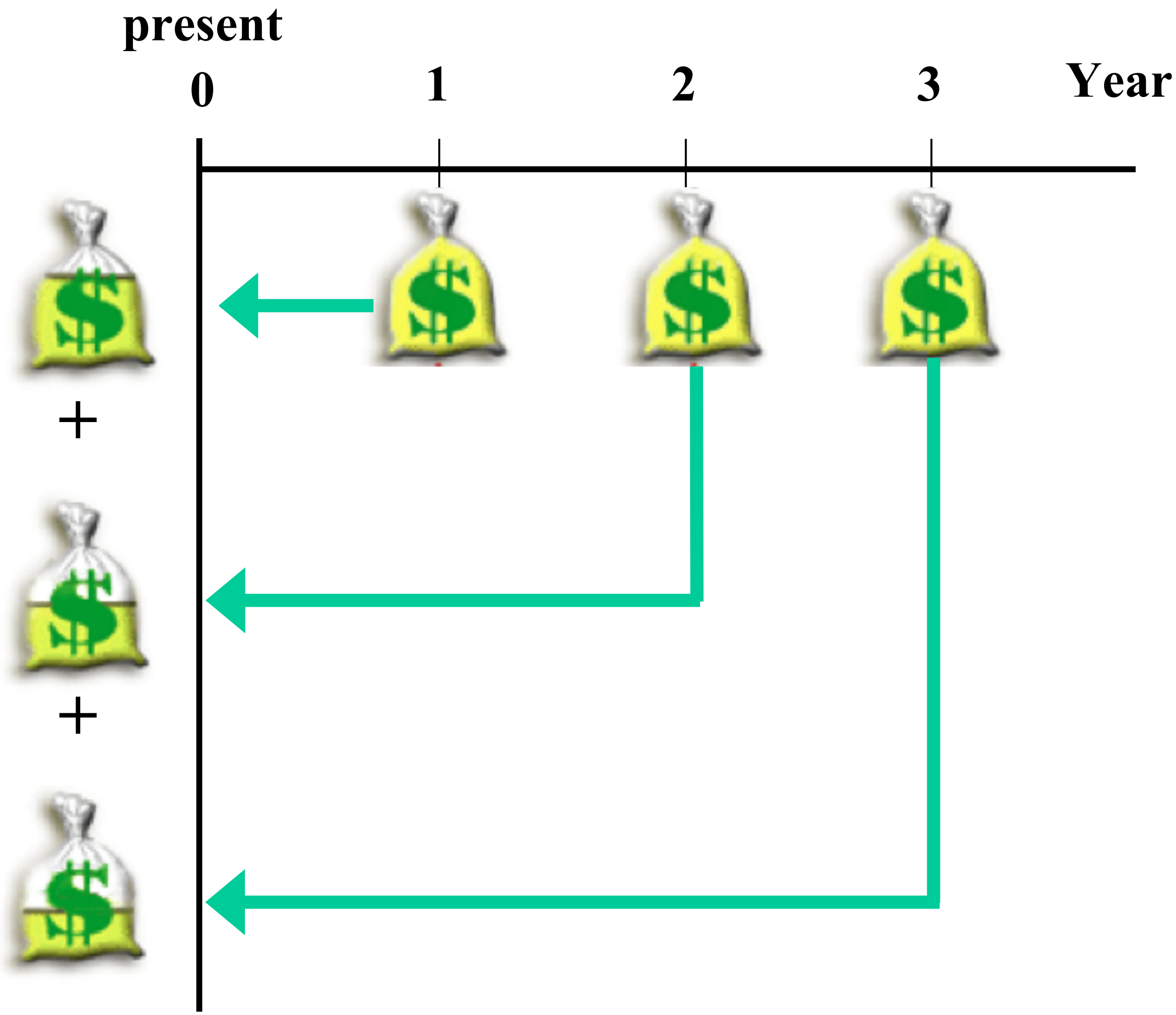
+

$$= \frac{\$}{(1+r)^2}$$

+

$$= \frac{\$}{(1+r)^3}$$

= **Present value**



The Value of a Bond

- A bond is a contract in which a borrower agrees to pay the bondholder (the lender) a stream of money

- Example: A bond issued by a company may make a "coupon" payment of $100 per year for the next 10 years and a final payment of $1000.
  - How much would you pay for this bond?

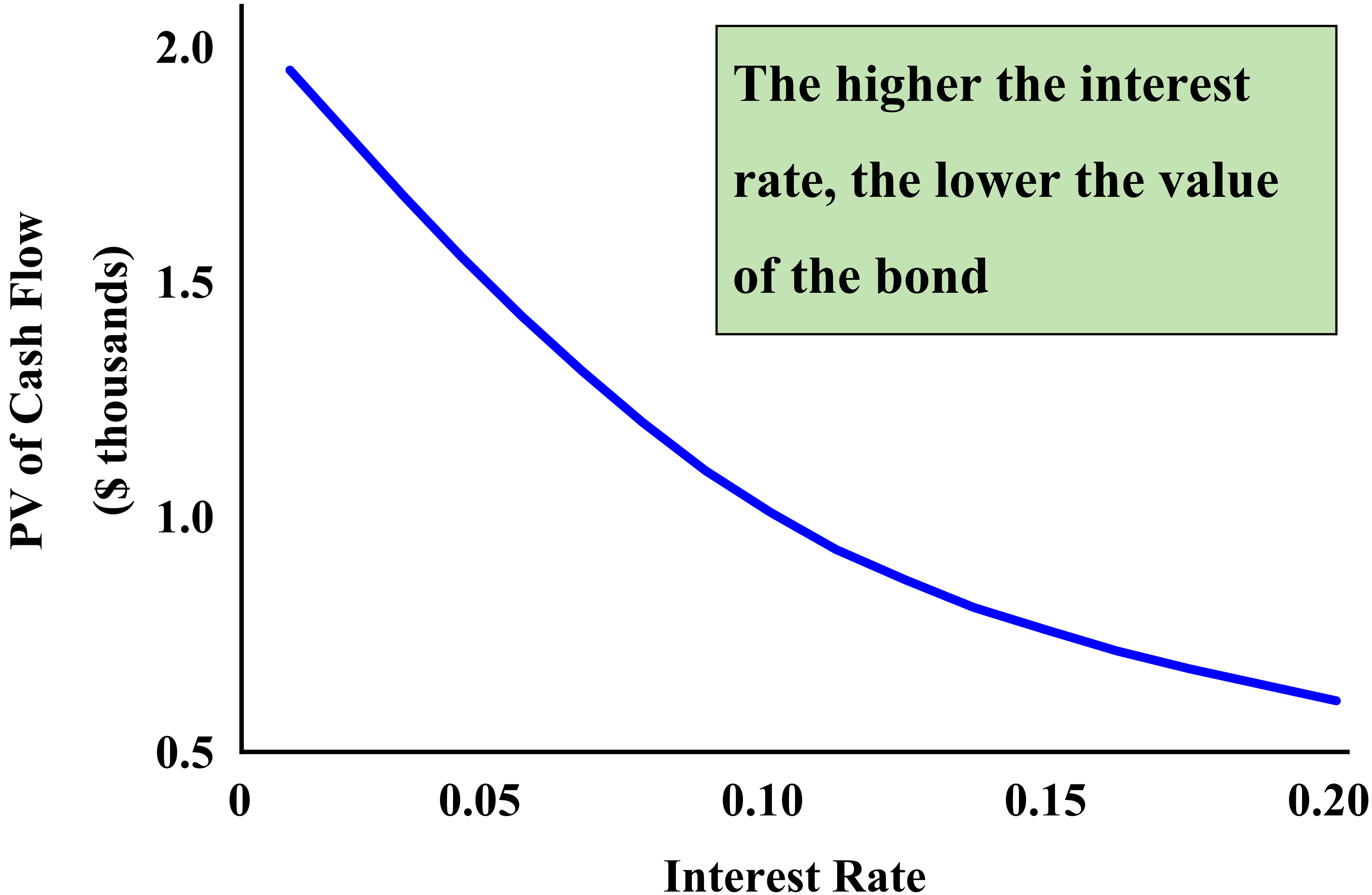
    --> Present value of payment stream

The Value of a Bond

- Determining the Price of a Bond
  - Coupon Payments = \$100/yr. for 10 yrs.
  - Principal Payment = \$1,000 in 10 yrs.

$$PV = \frac{\$100}{(1+r)} + \frac{\$100}{(1+r)^2} + \ldots + \frac{\$100}{(1+r)^{10}} + \frac{\$1000}{(1+r)^{10}}$$

Present Value of the Cash Flow from a Bond



The Value of a Bond

- Perpetuity is a bond that pays out a fixed amount of money each year forever.
- Present value of a perpetuity is an infinite summation.
- Can express the value of a perpetuity by

$$PV = \$100/r$$

- In general, PV = payment/r

Let $d = \dfrac{1}{[1+r]}$, the infinite summation of a perpetuity is

$$PV = dR + d^2R + d^3R + \ldots\ldots$$

$$dPV = d^2R + d^3R + \ldots\ldots$$

$$PV - dPV = dR$$

$$[1-d]PV = dR$$

$$PV = \frac{dR}{[1-d]} = \frac{R}{r}$$

since $\dfrac{d}{[1-d]} = \dfrac{1/[1+r]}{1-1/[1+r]} = \dfrac{1/[1+r]}{\dfrac{1+r-1}{[1+r]}} = \dfrac{1/[1+r]}{r/[1+r]} = \dfrac{1}{r}$

The Effective Yield on a Bond

- Corporate and government bonds are often traded on the bond market
- The price of the bond can be determined by looking at the market price
  - The value placed on it by buyers and sellers
- To compare the bond with other investment options, can determine the interest rate consistent with that value

Effective Yield on a Bond

- Calculating the Rate of Return From a Bond
- P is value of perpetuity – market price
- Can use equations to find value of r given P and the payment

$$P = \frac{\text{Payment}}{r} \quad \text{then} \quad r = \frac{\text{Payment}}{P}$$

Effective Yield on a Bond

- From previous example, P = Payment/r where P = $1000, Payment = $100, hence

  $$r = \$100/\$1000 = 0.10 = 10\%$$

- The interest rate calculated here is the effective yield
  - Rate of return one received by investing in the bond

Effective Yield on a Bond

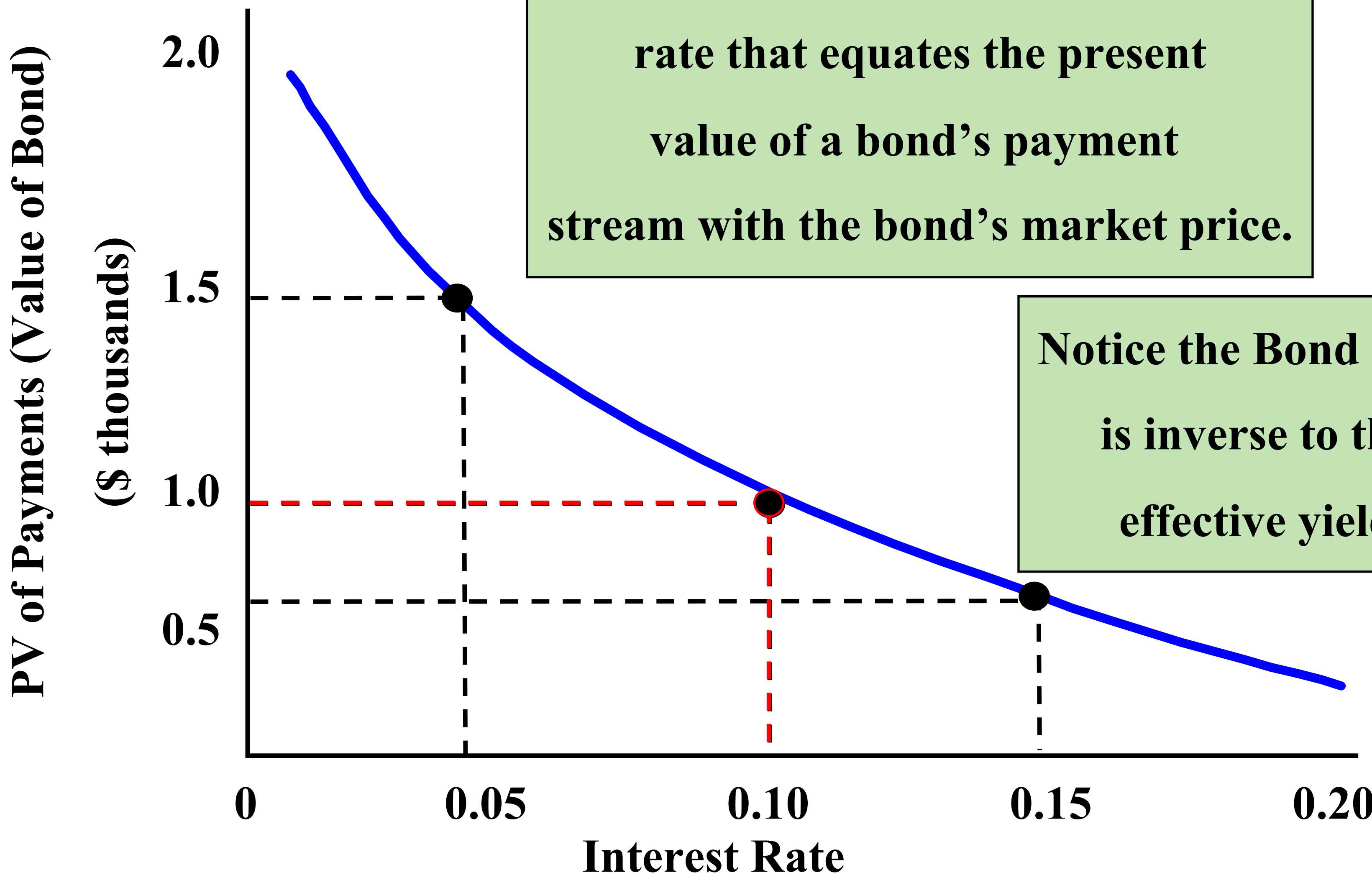
Calculating the Rate of Return From a Bond

$$\text{Coupon Bond : PV} = \frac{\$100}{(1+r)} + \frac{\$100}{(1+r)^2} + \ldots + \frac{\$100}{(1+r)^{10}} + \frac{\$1000}{(1+r)^{10}}$$

Calculate r in terms of P

Effective Yield on a Bond

The effective yield is the interest rate that equates the present value of a bond's payment stream with the bond's market price.



Notice the Bond Price is inverse to the effective yield

The Net Present Value Criterion for Capital Investment Decisions

- Firms have to decide when and how much capital to invest in
- Comparing the present value (PV) of the cash flows from the investment to the cost of the investment can give firm information needed to make worthwhile decisions.
- NPV Criterion: Firms should invest if the present value of the expected future cash flows from an investment exceeds the cost of the investment.

The Net Present Value Criterion for Capital Investment Decisions

$C =$ capital cost

$\pi_n =$ profits for n years $(n = 10)$

$$NPV = -C + \frac{\pi_1}{(1+r)} + \frac{\pi_2}{(1+r)^2} + ... + \frac{\pi_{10}}{(1+r)^{10}}$$

r = discount rate on opportunity cost of capital

with a similar risk

Invest if NPV > 0

The Net Present Value Criterion for Capital Investment Decisions

- The Electric Motor Factory (choosing to build a $10 million factory)
  - 8,000 motors/ month for 20 yrs
    - Cost = $42.50 each
    - Price = $52.50/motor
    - Profit = $10/motor or $80,000/month , $960,000/year
    - Factory life is 20 years with a scrap value of $1 million
  - Should the company invest?

The Net Present Value Criterion for Capital Investment Decisions

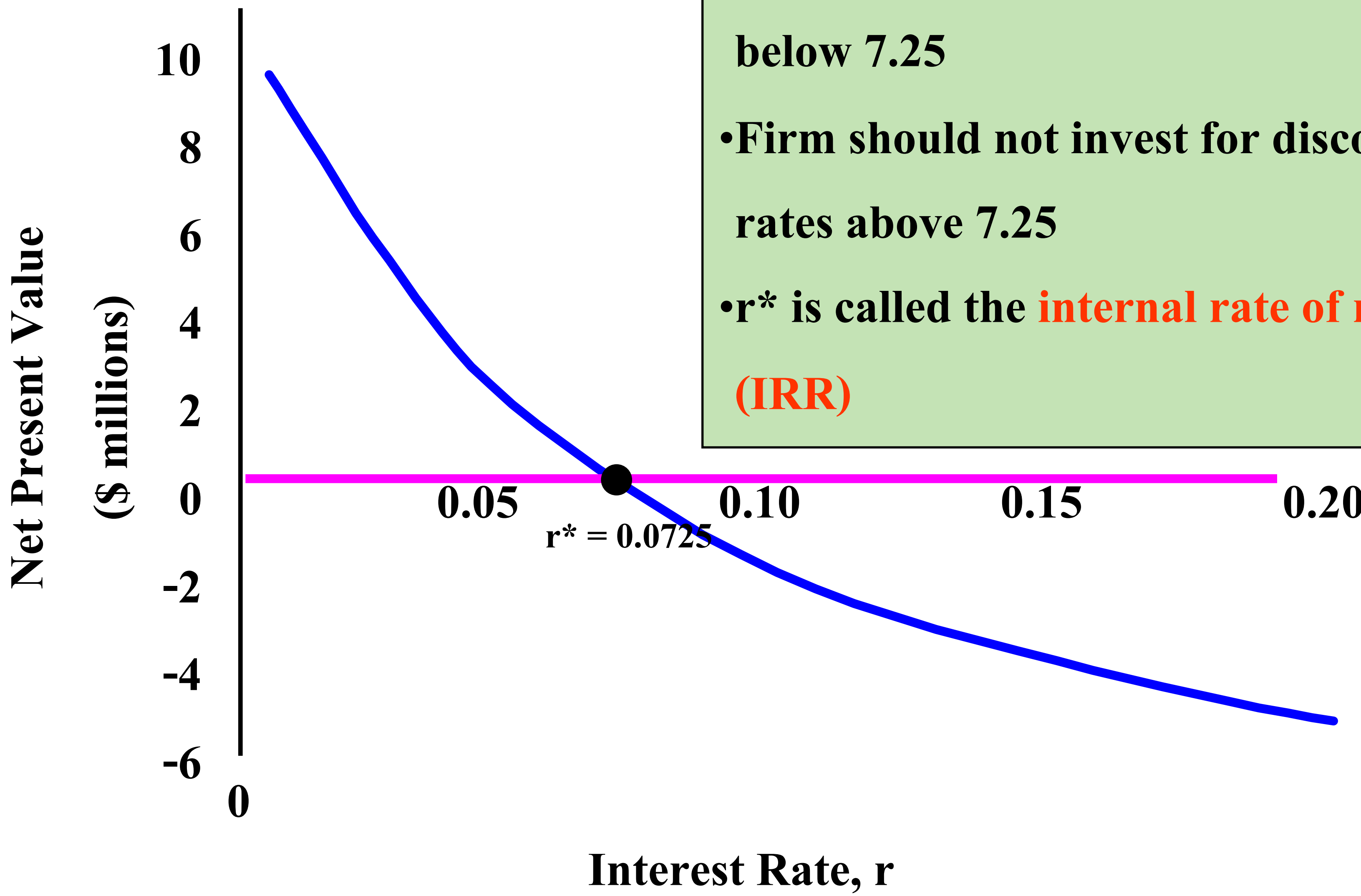
- Assume all information is certain (no risk)
  - r = government bond rate

$$NPV = -10 + \frac{.96}{(1+r)} + \frac{.96}{(1+r)^2} + \ldots + \frac{.96}{(1+r)^{20}} + \frac{1}{(1+r)^{20}}$$

$$r^* = 7.25\%$$

- Discount rates below 7.25, NPV is positive
- Discount rates above 7.25, NPV is negative

Net Present Value of a Factory



- Firm should invest for discount rates below 7.25
- Firm should not invest for discount rates above 7.25
- r* is called the **internal rate of return (IRR)**