

### EE320 Placement test

1. Attempt all.
2. Submit your work (in .pdf) on the Moodle. The required format of your filename is **studentID\_PT**
3. You will get **TWO** bonus points if you submit this placement test by the deadline.
4. **This placement test is due on Friday 14<sup>th</sup>, at 11 AM. Late submission will not be accepted.**

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1. Suppose that market demand is given by  $P = 10 - Q^2$  and the market supply is given by  $Q = a + P$ , where  $P$  is the unit price,  $Q$  is the quantity of output, and  $a$  is the coefficient in the supply equation.
    - 1.1) Graph the market demand and market supply curve in a P-Q diagram. Set the value of  $a$  equal to  $-14$ .
    - 1.2) Solve for the market equilibrium quantity ( $Q^*$ ) and price ( $P^*$ ) when  $a = -14$ . Show your work.
    - 1.3) If " $a$ " increases to  $-12$ , what would happen to the market equilibrium quantity and price? State the qualitative predictions without redoing the algebra.

demand equation :  $P = 10 - Q^2$

find P-intercept ( $Q=0$ )  $P = 10 - 0$

$$P = 10$$

$$\therefore \text{P-intercept} = (0, 10)$$

supply equation :  $Q = -14 + P$

find P-intercept ( $Q=0$ )  $0 = -14 + P$

$$P = 14$$

$$\therefore \text{P-intercept} = (0, 14)$$

find intersection point  $P = 10 - Q^2$  — ①

$$Q = -14 + P$$

$$P = Q + 14$$
 — ②

$$\textcircled{1} = \textcircled{2} ; 10 - Q^2 = Q + 14$$

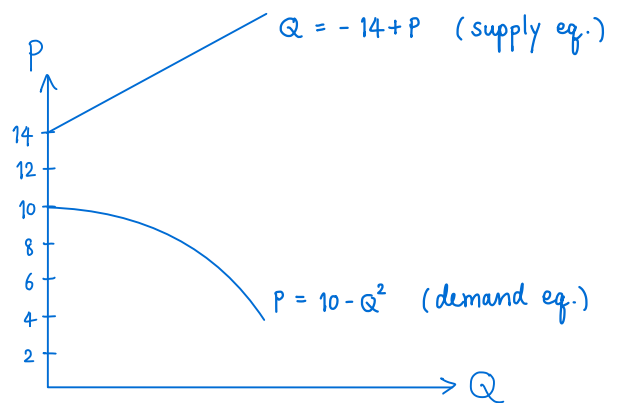
$$-4 = Q + Q^2$$

$$0 = Q^2 + Q + 4$$

$$Q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$Q = \frac{-1 \pm \sqrt{1 - 4(1)(4)}}{2} = \frac{-1 \pm \sqrt{-15}}{2}$$

There's no intersection point



undefined

2. Suppose that the revenue function is given by  $R(Q) = \ln(Q^2 + 1) + 3\left(\frac{Q}{Q+1}\right)$ ,  $Q \geq 0$ . Use the derivative technique and calculate the marginal revenue function. Is the revenue function an increasing or decreasing function?

if  $R'(Q) > 0$  it will be an increasing function

$R'(Q) < 0$  it will be a decreasing function

$$MR = R'(Q) = \left(\frac{1}{Q^2+1}\right)(2Q) + 3 \left[ \frac{(Q+1)(1) - (Q)(1)}{(Q+1)^2} \right]$$

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$$R'(Q) = \frac{2Q}{Q^2+1} + 3 \left[ \frac{Q+1-Q}{(Q+1)^2} \right]$$

$$R'(Q) = \frac{2Q}{Q^2+1} + \frac{3}{(Q+1)^2}$$

because  $Q > 0 \therefore 2Q$  is more than zero

$Q > 0 \therefore Q^2 > 0$  and  $Q^2 + 1 > 0$

$Q > 0 ; Q+1 > 1 \therefore (Q+1)^2 > 1$

$$\frac{2Q}{Q^2+1} > 0 \text{ and } \frac{3Q}{(Q+1)^2} > 0 \therefore R'(Q) > 0$$

3. Suppose that the profit function is given by  $\pi(Q) = -\frac{1}{3}Q^3 - Q^2 + 8Q - 1$  where Q is the level of output. Use the calculus and solve for the level of profit-maximizing output. Confirm your answer with the second derivative.

$$\pi'(Q) = -\frac{3}{3}Q^2 - 2Q + 8$$

$$\pi'(Q) = -Q^2 - 2Q + 8$$

$$\text{FOC : } \pi'(Q) = 0$$

$$-Q^2 - 2Q + 8 = 0$$

$$\text{let } -1 \text{ multiplied ; } Q^2 - 2Q - 8 = 0$$

$$(Q+4)(Q-2) = 0$$

$$Q = -4, 2 \quad \text{because quantity have to higher than zero}$$

To confirm my answer with the second derivative,

$Q = 2$  in  $\pi''(Q)$  must **lower than 0**

$$\text{SOC : } \pi''(Q) < 0$$

$$\text{find } \pi''(Q) \text{ from } \pi'(Q) = -Q^2 - 2Q + 8$$

$$\pi''(Q) = -2Q - 2$$

$$\text{Plug in } Q = 2 \text{ ; } \pi''(2) = -4 - 2 = -6 < 0$$

$\therefore$  The level of profit maximizing output is 2 unit #

4. Suppose that  $A = \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix}$ ,  $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$  and  $C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ , calculate the following object. Show your work.

4.1  $A+B$  because the size of  $A$  is  $2 \times 2$  but  $B$  is  $2 \times 3$  so they cannot be combined.

4.2  $A \cdot B$

$$A = \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$AB_{2 \times 3} = \begin{bmatrix} R_1C_1 & R_1C_2 & R_1C_3 \\ 8+36=44 & 61 & 78 \\ R_2C_1 & R_2C_2 & R_2C_3 \\ 54 & 75 & 96 \end{bmatrix}$$

4.3  $\det(A)$

$$A = \begin{bmatrix} 8 & 9 \\ 10 & 11 \end{bmatrix} = 88 - 90 = -2$$

4.4  $\det(B)$

because  $B$  is not a square matrix, so  $\det(B)$  cannot be found.

4.5  $\det(C)$

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} = [45 + 84 + 96] - [105 + 48 + 72] = 225 - 225 = 0$$

$$U(x, y) = x^a y^b + \ln x - \ln(x+y)$$

$$\frac{\partial U}{\partial x} = \text{wavy} + \frac{1}{x} - \left(\frac{1}{x+y}\right)(1+0)$$

5. Suppose that  $U(x, y) = x^a y^b + \ln\left(\frac{x}{x+y}\right)$ . Use the partial derivative technique, calculate  $\frac{\partial U}{\partial x}$  and  $\frac{\partial U}{\partial y}$ .

$$\frac{\partial U}{\partial x} = ax^{a-1} \cdot y^b + \frac{1}{x} - \left(\frac{1}{x+y}\right)(1+0)$$

$$\frac{\partial U}{\partial x} = ax^{a-1} \cdot y^b + \frac{1}{x} - \frac{1}{x+y}$$

$$\frac{\partial U}{\partial x} = x^a \cdot by^{b-1} + 0 - \left(\frac{1}{x+y}\right)(0+1)$$

$$\frac{\partial U}{\partial x} = x^a by^{b-1} - \frac{1}{x+y}$$