Solution key

CHAPTER 1: THE INVESTMENT ENVIRONMENT

PROBLEM SETS

1. Ultimately, it is true that real assets determine the material well being of an economy. Nevertheless, individuals can benefit when financial engineering creates new products that allow them to manage their portfolios of financial assets more efficiently. Because bundling and unbundling creates financial products with new properties and sensitivities to various sources of risk, it allows investors to hedge particular sources of risk more efficiently.

2. Securitization requires access to a large number of potential investors. To attract these investors, the capital market needs:

    1. a safe system of business laws and low probability of confiscatory taxation/regulation;
    2. a well-developed investment banking industry;
    3. a well-developed system of brokerage and financial transactions, and;
    4. well-developed media, particularly financial reporting.

    These characteristics are found in (indeed make for) a well-developed financial market.

3. Securitization leads to disintermediation; that is, securitization provides a means for market participants to bypass intermediaries. For example, mortgage-backed securities channel funds to the housing market without requiring that banks or thrift institutions make loans from their own portfolios. As securitization progresses, financial intermediaries must increase other activities such as providing short-term liquidity to consumers and small business, and financial services.

4. Financial assets make it easy for large firms to raise the capital needed to finance their investments in real assets. If Ford, for example, could not issue stocks or bonds to the general public, it would have a far more difficult time raising capital. Contraction of the supply of financial assets would make financing more difficult, thereby increasing the cost of capital. A higher cost of capital results in less investment and lower real growth.

5. Even if the firm does not need to issue stock in any particular year, the stock market is still important to the financial manager. The stock price provides important information about how the market values the firm's investment projects. For example, if the stock price rises considerably, managers might conclude that the market believes the firm's future prospects are bright. This might be a useful signal to the firm to proceed with an investment such as an expansion of the firm's business.

   In addition, shares that can be traded in the secondary market are more attractive to initial investors since they know that they will be able to sell their shares. This in turn makes investors more willing to buy shares in a primary offering, and thus improves the terms on which firms can raise money in the equity market.

6. a. No. The increase in price did not add to the productive capacity of the economy.

   b. Yes, the value of the equity held in these assets has increased.

   c. Future homeowners as a whole are worse off, since mortgage liabilities have also increased. In addition, this housing price bubble will eventually burst and society as a whole (and most likely taxpayers) will endure the damage.

7. a. The bank loan is a financial liability for Lanni. (Lanni's IOU is the bank's financial asset.) The cash Lanni receives is a financial asset. The new financial asset created is Lanni's promissory note (that is, Lanni’s IOU to the bank).

   b. Lanni transfers financial assets (cash) to the software developers. In return, Lanni gets a real asset, the completed software. No financial assets are created or destroyed; cash is simply transferred from one party to another.

   c. Lanni gives the real asset (the software) to Microsoft in exchange for a financial asset, 1,500 shares of Microsoft stock. If Microsoft issues new shares in order to pay Lanni, then this would represent the creation of new financial assets.

   d. Lanni exchanges one financial asset (1,500 shares of stock) for another ($120,000). Lanni gives a financial asset ($50,000 cash) to the bank and gets back another financial asset (its IOU). The loan is "destroyed" in the transaction, since it is retired when paid off and no longer exists.

8. a.

Assets		*Liabilities & Shareholders' equity*	
Cash	$ 70,000	Bank loan	$ 50,000
Computers	30,000	Shareholders' equity	50,000
Total	$100,000	Total	$100,000

Ratio of real assets to total assets = $30,000/$100,000 = 0.30

b.

Assets		*Liabilities & Shareholders' equity*	
Software product*	$ 70,000	Bank loan	$ 50,000
Computers	30,000	Shareholders' equity	50,000
Total	$100,000	Total	$100,000

*Valued at cost

Ratio of real assets to total assets = $100,000/$100,000 = 1.0

c.

Assets		*Liabilities & Shareholders' equity*	
Microsoft shares	$120,000	Bank loan	$ 50,000
Computers	30,000	Shareholders' equity	100,000
Total	$150,000	Total	$150,000

Ratio of real assets to total assets = $30,000/$150,000 = 0.20

Conclusion: when the firm starts up and raises working capital, it is characterized by a low ratio of real assets to total assets. When it is in full production, it has a high ratio of real assets to total assets. When the project "shuts down" and the firm sells it off for cash, financial assets once again replace real assets.

9. For commercial banks, the ratio is: $140.1/$11,895.1 = 0.0118

For non-financial firms, the ratio is: $12,538/$26,572 = 0.4719

The difference should be expected primarily because the bulk of the business of financial institutions is to make loans; which are financial assets for financial institutions.

10. a. Primary-market transaction

b. Derivative assets

c. Investors who wish to hold gold without the complication and cost of physical storage.

11. a. A fixed salary means that compensation is (at least in the short run) independent of the firm's success. This salary structure does not tie the manager's immediate compensation to the success of the firm. However, the manager might view this as the safest compensation structure and therefore value it more highly.

b. A salary that is paid in the form of stock in the firm means that the manager earns the most when the shareholders' wealth is maximized. Five years of vesting helps align the interests of the employee with the long-term performance of the firm. This structure is therefore most likely to align the interests of managers and shareholders. If stock compensation is overdone, however, the manager might view it as overly risky since the manager's career is already linked to the firm, and this undiversified exposure would be exacerbated with a large stock position in the firm.

c. A profit-linked salary creates great incentives for managers to contribute to the firm's success. However, a manager whose salary is tied to short-term profits will be risk seeking, especially if these short-term profits determine salary or if the compensation structure does not bear the full cost of the project's risks. Shareholders, in contrast, bear the losses as well as the gains on the project, and might be less willing to assume that risk.

12. Even if an individual shareholder could monitor and improve managers' performance, and thereby increase the value of the firm, the payoff would be small, since the ownership share in a large corporation would be very small. For example, if you own \$10,000 of Ford stock and can increase the value of the firm by 5%, a very ambitious goal, you benefit by only: $0.05 \times \$10{,}000 = \$500$

In contrast, a bank that has a multimillion-dollar loan outstanding to the firm has a big stake in making sure that the firm can repay the loan. It is clearly worthwhile for the bank to spend considerable resources to monitor the firm.

13. Mutual funds accept funds from small investors and invest, on behalf of these investors, in the national and international securities markets.

Pension funds accept funds and then invest, on behalf of current and future retirees, thereby channeling funds from one sector of the economy to another.

Venture capital firms pool the funds of private investors and invest in start-up firms.

Banks accept deposits from customers and loan those funds to businesses, or use the funds to buy securities of large corporations.

14. Treasury bills serve a purpose for investors who prefer a low-risk investment. The lower average rate of return compared to stocks is the price investors pay for predictability of investment performance and portfolio value.

15. With a "top-down" investing style, you focus on asset allocation or the broad composition of the entire portfolio, which is the major determinant of overall performance. Moreover, top-down management is the natural way to establish a portfolio with a level of risk consistent with your risk tolerance. The disadvantage of an *exclusive* emphasis on top-down issues is that you may forfeit the potential high returns that could result from identifying and concentrating in undervalued securities or sectors of the market.

    With a "bottom-up" investing style, you try to benefit from identifying undervalued securities. The disadvantage is that you tend to overlook the overall composition of your portfolio, which may result in a non-diversified portfolio or a portfolio with a risk level inconsistent with your level of risk tolerance. In addition, this technique tends to require more active management, thus generating more transaction costs. Finally, your analysis may be incorrect, in which case you will have fruitlessly expended effort and money attempting to beat a simple buy-and-hold strategy.

16. You should be skeptical. If the author actually knows how to achieve such returns, one must question why the author would then be so ready to sell the secret to others. Financial markets are very competitive; one of the implications of this fact is that riches do not come easily. High expected returns require bearing some risk, and obvious bargains are few and far between. Odds are that the only one getting rich from the book is its author.

17. Financial assets provide for a means to acquire real assets as well as an expansion of these real assets. Financial assets provide a measure of liquidity to real assets and allow for investors to more effectively reduce risk through diversification.

18. Allowing traders to share in the profits increases the traders' willingness to assume risk. Traders will share in the upside potential directly but only in the downside indirectly (poor performance = potential job loss). Shareholders, by contrast, are affected directly by both the upside and downside potential of risk.

19. Answers may vary, however, students should touch on the following: increased transparency, regulations to promote capital adequacy by increasing the frequency of gain or loss settlement, incentives to discourage excessive risk taking, and the promotion of more accurate and unbiased risk assessment.

CHAPTER 2: ASSET CLASSES AND FINANCIAL INSTRUMENTS

PROBLEM SETS

1. Preferred stock is like long-term debt in that it typically promises a fixed payment each year. In this way, it is a perpetuity. Preferred stock is also like long-term debt in that it does not give the holder voting rights in the firm.

   Preferred stock is like equity in that the firm is under no contractual obligation to make the preferred stock dividend payments. Failure to make payments does not set off corporate bankruptcy. With respect to the priority of claims to the assets of the firm in the event of corporate bankruptcy, preferred stock has a higher priority than common equity but a lower priority than bonds.

2. Money market securities are called "cash equivalents" because of their great liquidity. The prices of money market securities are very stable, and they can be converted to cash (i.e., sold) on very short notice and with very low transaction costs.

3. (a) A repurchase agreement is an agreement whereby the seller of a security agrees to "repurchase" it from the buyer on an agreed upon date at an agreed upon price. Repos are typically used by securities dealers as a means for obtaining funds to purchase securities.

4. The spread will widen. Deterioration of the economy increases credit risk, that is, the likelihood of default. Investors will demand a greater premium on debt securities subject to default risk.

5.

	Corp. Bonds	Preferred Stock	Common Stock
Voting Rights (Typically)			Yes
Contractual Obligation	Yes		
Perpetual Payments		Yes	Yes
Accumulated Dividends		Yes	
Fixed Payments (Typically)	Yes	Yes	
Payment Preference	First	Second	Third

6. Municipal Bond interest is tax-exempt. When facing higher marginal tax rates, a high-income investor would be more inclined to pick tax-exempt securities.

7. a. You would have to pay the asked price of:

   86:14 = 86.43750% of par = $864.375

   b. The coupon rate is 3.5% implying coupon payments of $35.00 annually or, more precisely, $17.50 semiannually.

   c. Current yield = Annual coupon income/price

   = $35.00/$864.375 = 0.0405 = 4.05%

8. P = $10,000/1.02 = $9,803.92

9. The total before-tax income is $4. After the 70% exclusion for preferred stock dividends, the taxable income is: 0.30 × $4 = $1.20

   Therefore, taxes are: 0.30 × $1.20 = $0.36

   After-tax income is: $4.00 – $0.36 = $3.64

   Rate of return is: $3.64/$40.00 = 9.10%

10. a. You could buy: $5,000/$67.32 = 74.27 shares

   b. Your annual dividend income would be: 74.27 × $1.52 = $112.89

   c. The price-to-earnings ratio is 11 and the price is $67.32. Therefore:

   $67.32/Earnings per share = 11 Þ Earnings per share = $6.12

   d. General Dynamics closed today at $67.32, which was $0.47 higher than yesterday's price. Yesterday's closing price was: $66.85

11. a. At t = 0, the value of the index is: (90 + 50 + 100)/3 = 80

   At t = 1, the value of the index is: (95 + 45 + 110)/3 = 83.333

   The rate of return is: (83.333/80) – 1 = 4.17%

   b. In the absence of a split, Stock C would sell for 110, so the value of the index would be: 250/3 = 83.333

   After the split, Stock C sells for 55. Therefore, we need to find the divisor (d) such that: 83.333 = (95 + 45 + 55)/d ⇒ d = 2.340

   c. The return is zero. The index remains unchanged because the return for each stock separately equals zero.

12. a. Total market value at t = 0 is: ($9,000 + $10,000 + $20,000) = $39,000
Total market value at t = 1 is: ($9,500 + $9,000 + $22,000) = $40,500
Rate of return = ($40,500/$39,000) – 1 = 3.85%

b. The return on each stock is as follows:

r_A = (95/90) – 1 = 0.0556

r_B = (45/50) – 1 = –0.10

r_C = (110/100) – 1 = 0.10

The equally-weighted average is:

[0.0556 + (-0.10) + 0.10]/3 = 0.0185 = 1.85%

13. The after-tax yield on the corporate bonds is: 0.09 × (1 – 0.30) = 0.0630 = 6.30%
Therefore, municipals must offer at least 6.30% yields.

14. Equation (2.2) shows that the equivalent taxable yield is: $r = r_m /(1 - t)$

a. 4.00%

b. 4.44%

c. 5.00%

d. 5.71%

15. In an equally-weighted index fund, each stock is given equal weight regardless of its market capitalization. Smaller cap stocks will have the same weight as larger cap stocks. The challenges are as follows:

- Given equal weights placed to smaller cap and larger cap, equal-weighted indices (EWI) will tend to be more volatile than their market-capitalization counterparts;
- It follows that EWIs are not good reflectors of the broad market which they represent; EWIs underplay the economic importance of larger companies;
- Turnover rates will tend to be higher, as an EWI must be rebalanced back to its original target. By design, many of the transactions would be among the smaller, less-liquid stocks.

16. a. The higher coupon bond.

b. The call with the lower exercise price.

c. The put on the lower priced stock.

17. a. You bought the contract when the futures price was $3.835 (see Figure 2.10). The contract closes at a price of $3.875, which is $0.04 more than the original futures price. The contract multiplier is 5000. Therefore, the gain will be: $\$0.04 \times 5000 = \$200.00$

b. Open interest is 177,561 contracts.

18. a. Since the stock price exceeds the exercise price, you exercise the call.

The payoff on the option will be: $21.75 – $21 = $0.75

The cost was originally $0.64, so the profit is: $0.75 – $0.64 = $0.11

b. If the call has an exercise price of $22, you would not exercise for any stock price of $22 or less. The loss on the call would be the initial cost: $0.30

c. Since the stock price is less than the exercise price, you will exercise the put. The payoff on the option will be: $22 – $21.75 = $0.25

The option originally cost $1.63 so the profit is: $0.25 − $1.63 = −$1.38

19. There is always a possibility that the option will be in-the-money at some time prior to expiration. Investors will pay something for this possibility of a positive payoff.

20.

	Value of call at expiration	Initial Cost	Profit
a.	0	4	-4
b.	0	4	-4
c.	0	4	-4
d.	5	4	1
e.	10	4	6

	Value of put at expiration	Initial Cost	Profit
a.	10	6	4
b.	5	6	-1
c.	0	6	-6
d.	0	6	-6
e.	0	6	-6

21. A put option conveys the *right* to sell the underlying asset at the exercise price. A short position in a futures contract carries an *obligation* to sell the underlying asset at the futures price.

22. A call option conveys the *right* to buy the underlying asset at the exercise price. A long position in a futures contract carries an *obligation* to buy the underlying asset at the futures price.

CFA PROBLEMS

1. (d)

2. The equivalent taxable yield is: 6.75%/(1 – 0.34) = 10.23%

3. (a) Writing a call entails unlimited potential losses as the stock price rises.

4. a. The taxable bond. With a zero tax bracket, the after-tax yield for the taxable bond is the same as the before-tax yield (5%), which is greater than the yield on the municipal bond.

   b. The taxable bond. The after-tax yield for the taxable bond is:
   0.05 × (1 – 0.10) = 4.5%

   c. You are indifferent. The after-tax yield for the taxable bond is:

   0.05 × (1 – 0.20) = 4.0%

   The after-tax yield is the same as that of the municipal bond.

   d. The municipal bond offers the higher after-tax yield for investors in tax brackets above 20%.

5. If the after-tax yields are equal, then: 0.056 = 0.08 × (1 – t)

   This implies that t = 0.30 =30%.

CHAPTER 5: INTRODUCTION TO RISK, RETURN, AND THE HISTORICAL RECORD

PROBLEM SETS

1. The Fisher equation predicts that the nominal rate will equal the equilibrium real rate plus the expected inflation rate. Hence, if the inflation rate increases from 3% to 5% while there is no change in the real rate, then the nominal rate will increase by 2%. On the other hand, it is possible that an increase in the expected inflation rate would be accompanied by a change in the real rate of interest. While it is conceivable that the nominal interest rate could remain constant as the inflation rate increased, implying that the real rate decreased as inflation increased, this is not a likely scenario.

2. If we assume that the distribution of returns remains reasonably stable over the entire history, then a longer sample period (i.e., a larger sample) increases the precision of the estimate of the expected rate of return; this is a consequence of the fact that the standard error decreases as the sample size increases. However, if we assume that the mean of the distribution of returns is changing over time but we are not in a position to determine the nature of this change, then the expected return must be estimated from a more recent part of the historical period. In this scenario, we must determine how far back, historically, to go in selecting the relevant sample. Here, it is likely to be disadvantageous to use the entire dataset back to 1880.

3. The true statements are (c) and (e). The explanations follow.

   Statement (c): Let σ = the annual standard deviation of the risky investments and σ_1 = the standard deviation of the first investment alternative over the two-year period. Then:

   $$\sigma_1 = \sqrt{2} \times \sigma$$

   Therefore, the annualized standard deviation for the first investment alternative is equal to:

   $$\frac{\sigma_1}{2} = \frac{\sigma}{\sqrt{2}} < \sigma$$

   Statement (e): The first investment alternative is more attractive to investors with lower degrees of risk aversion. The first alternative (entailing a sequence of two identically distributed and uncorrelated risky investments) is riskier than the second alternative (the risky investment followed by a risk-free investment). Therefore, the first alternative is more attractive to investors with lower degrees of risk aversion. Notice, however, that if you mistakenly believed that 'time diversification' can reduce the total risk of a sequence of risky investments, you would have been tempted to conclude that the first alternative is less risky and therefore more attractive to more risk-averse investors. This is clearly not the case; the two-year standard deviation of the first alternative is greater than the two-year standard deviation of the second alternative.

4. For the money market fund, your holding period return for the next year depends on the level of 30-day interest rates each month when the fund rolls over maturing securities. The one-year savings deposit offers a 7.5% holding period return for the year. If you forecast that the rate on money market instruments will increase significantly above the current 6% yield, then the money market fund might result in a higher HPR than the savings deposit. The 20-year Treasury bond offers a yield to maturity of 9% per year, which is 150 basis points higher than the rate on the one-year savings deposit; however, you could earn a one-year HPR much less than 7.5% on the bond if long-term interest rates increase during the year. If Treasury bond yields rise above 9%, then the price of the bond will fall, and the resulting capital loss will wipe out some or all of the 9% return you would have earned if bond yields had remained unchanged over the course of the year.

5. a. If businesses reduce their capital spending, then they are likely to decrease their demand for funds. This will shift the demand curve in Figure 5.1 to the left and reduce the equilibrium real rate of interest.

   b. Increased household saving will shift the supply of funds curve to the right and cause real interest rates to fall.

   c. Open market purchases of U.S. Treasury securities by the Federal Reserve Board are equivalent to an increase in the supply of funds (a shift of the supply curve to the right). The equilibrium real rate of interest will fall.

6. a. The "Inflation-Plus" CD is the safer investment because it guarantees the purchasing power of the investment. Using the approximation that the real rate equals the nominal rate minus the inflation rate, the CD provides a real rate of 1.5% regardless of the inflation rate.

   b. The expected return depends on the expected rate of inflation over the next year. If the expected rate of inflation is less than 3.5% then the conventional CD offers a higher real return than the Inflation-Plus CD; if the expected rate of inflation is greater than 3.5%, then the opposite is true.

c. If you expect the rate of inflation to be 3% over the next year, then the conventional CD offers you an expected real rate of return of 2%, which is 0.5% higher than the real rate on the inflation-protected CD. But unless you know that inflation will be 3% with certainty, the conventional CD is also riskier. The question of which is the better investment then depends on your attitude towards risk versus return. You might choose to diversify and invest part of your funds in each.

d. No. We cannot assume that the entire difference between the risk-free nominal rate (on conventional CDs) of 5% and the real risk-free rate (on inflation-protected CDs) of 1.5% is the expected rate of inflation. Part of the difference is probably a risk premium associated with the uncertainty surrounding the real rate of return on the conventional CDs. This implies that the expected rate of inflation is less than 3.5% per year.

7. $E(r) = [0.35 \times 44.5\%] + [0.30 \times 14.0\%] + [0.35 \times (–16.5\%)] = 14\%$

$\sigma^2 = [0.35 \times (44.5 – 14)^2] + [0.30 \times (14 – 14)^2] + [0.35 \times (–16.5 – 14)^2] = 651.175$

$\sigma = 25.52\%$

The mean is unchanged, but the standard deviation has increased, as the probabilities of the high and low returns have increased.

8. Probability distribution of price and one-year holding period return for a 30-year U.S. Treasury bond (which will have 29 years to maturity at year's end):

Economy	Probability	YTM	Price	Capital Gain	Coupon Interest	HPR
Boom	0.20	11.0%	$ 74.05	–$25.95	$8.00	–17.95%
Normal Growth	0.50	8.0%	$100.00	$ 0.00	$8.00	8.00%
Recession	0.30	7.0%	$112.28	$12.28	$8.00	20.28%

9. $E(q) = (0 \times 0.25) + (1 \times 0.25) + (2 \times 0.50) = 1.25$

$\sigma_q = [0.25 \times (0 – 1.25)^2 + 0.25 \times (1 – 1.25)^2 + 0.50 \times (2 – 1.25)^2]^{1/2} = 0.8292$

10. (a) With probability 0.9544, the value of a normally distributed variable will fall within two standard deviations of the mean; that is, between –40% and 80%.

11. From Table 5.3 and Figure 5.6, the average risk premium for the period 1926-2009 was: (11.63% – 3.71%) = 7.92% per year

Adding 7.92% to the 3% risk-free interest rate, the expected annual HPR for the S&P 500 stock portfolio is: 3.00% + 7.92% = 10.92%

12. The average rates of return and standard deviations are quite different in the sub periods:

	STOCKS			
	Mean	Standard Deviation	Skewness	Kurtosis
1926 – 2005	12.15%	20.26%	-0.3605	-0.0673
1976 – 2005	13.85%	15.68%	-0.4575	-0.6489
1926 – 1941	6.39%	30.33%	-0.0022	-1.0716

	Mean	Standard Deviation	Skewness	Kurtosis
1926 – 2005	5.68%	8.09%	0.9903	1.6314
1976 – 2005	9.57%	10.32%	0.3772	-0.0329
1926 – 1941	4.42%	4.32%	-0.5036	0.5034

The most relevant statistics to use for projecting into the future would seem to be the statistics estimated over the period 1976-2005, because this later period seems to have been a different economic regime. After 1955, the U.S. economy entered the Keynesian era, when the Federal government actively attempted to stabilize the economy and to prevent extremes in boom and bust cycles. Note that the standard deviation of stock returns has decreased substantially in the later period while the standard deviation of bond returns has increased.

13. a $$r = \frac{1+R}{1+i} - 1 = \frac{R-i}{1+i} = \frac{0.80-0.70}{1.70} = 0.0588 = 5.88\%$$

b. $r \approx R - i = 80\% - 70\% = 10\%$

Clearly, the approximation gives a real HPR that is too high.

14. From Table 5.2, the average real rate on T-bills has been: 0.70%

a. T-bills: 0.72% real rate + 3% inflation = 3.70%

b. Expected return on large stocks:

3.70% T-bill rate + 8.40% historical risk premium = 12.10%

c. The risk premium on stocks remains unchanged. A premium, the difference between two rates, is a real value, unaffected by inflation.

15. Real interest rates are expected to rise. The investment activity will shift the demand for funds curve (in Figure 5.1) to the right. Therefore the equilibrium real interest rate will increase.

16. a. Probability Distribution of the HPR on the Stock Market and Put:

State of the Economy	Probability	STOCK Ending Price + Dividend	STOCK HPR	PUT Ending Value	PUT HPR
Excellent	0.25	$ 131.00	31.00%	$ 0.00	–100%
Good	0.45	$ 114.00	14.00%	$ 0.00	–100%
Poor	0.25	$ 93.25	−6.75%	$ 20.25	68.75%
Crash	0.05	$ 48.00	–52.00%	$ 64.00	433.33%

Remember that the cost of the index fund is $100 per share, and the cost of the put option is $12.

b. The cost of one share of the index fund plus a put option is $112. The probability distribution of the HPR on the portfolio is:

State of the Economy	Probability	Ending Price + Put + Dividend	HPR	
Excellent	0.25	$ 131.00	17.0%	= (131 – 112)/112
Good	0.45	$ 114.00	1.8%	= (114 – 112)/112
Poor	0.25	$ 113.50	1.3%	= (113.50 – 112)/112
Crash	0.05	$ 112.00	0.0%	= (112 – 112)/112

c. Buying the put option guarantees the investor a minimum HPR of 0.0% regardless of what happens to the stock's price. Thus, it offers insurance against a price decline.

17. The probability distribution of the dollar return on CD plus call option is:

State of the Economy	Probability	Ending Value of CD	Ending Value of Call	Combined Value
Excellent	0.25	$ 114.00	$16.50	$130.50
Good	0.45	$ 114.00	$ 0.00	$114.00
Poor	0.25	$ 114.00	$ 0.00	$114.00
Crash	0.05	$ 114.00	$ 0.00	$114.00

CFA PROBLEMS

1. The expected dollar return on the investment in equities is $18,000 compared to the $5,000 expected return for T-bills. Therefore, the expected risk premium is $13,000.

2. $E(r) = [0.2 \times (-25\%)] + [0.3 \times 10\%] + [0.5 \times 24\%] = 10\%$

3. $E(r_X) = [0.2 \times (-20\%)] + [0.5 \times 18\%] + [0.3 \times 50\%] = 20\%$

   $E(r_Y) = [0.2 \times (-15\%)] + [0.5 \times 20\%] + [0.3 \times 10\%] = 10\%$

4. $\sigma_X{}^2 = [0.2 \times (-20 - 20)^2] + [0.5 \times (18 - 20)^2] + [0.3 \times (50 - 20)^2] = 592$

   $\sigma_X = 24.33\%$

   $\sigma_Y{}^2 = [0.2 \times (-15 - 10)^2] + [0.5 \times (20 - 10)^2] + [0.3 \times (10 - 10)^2] = 175$

   $\sigma_Y = 13.23\%$

5. $E(r) = (0.9 \times 20\%) + (0.1 \times 10\%) = 19\%$ → $1,900 in returns

6. The probability that the economy will be neutral is 0.50, or 50%. *Given* a neutral economy, the stock will experience poor performance 30% of the time. The probability of both poor stock performance and a neutral economy is therefore:

   $0.30 \times 0.50 = 0.15 = 15\%$

7. $E(r) = (0.1 \times 15\%) + (0.6 \times 13\%) + (0.3 \times 7\%) = 11.4\%$

CHAPTER 6: RISK AVERSION AND CAPITAL ALLOCATION TO RISKY ASSETS

PROBLEM SETS

1. (e)

2. (b) A higher borrowing rate is a consequence of the risk of the borrowers' default. In perfect markets with no additional cost of default, this increment would equal the value of the borrower's option to default, and the Sharpe measure, with appropriate treatment of the default option, would be the same. However, in reality there are costs to default so that this part of the increment lowers the Sharpe ratio. Also, notice that answer (c) is not correct because doubling the expected return with a fixed risk-free rate will more than double the risk premium and the Sharpe ratio.

3. Assuming no change in risk tolerance, that is, an unchanged risk aversion coefficient (A), then higher perceived volatility increases the denominator of the equation for the optimal investment in the risky portfolio (Equation 6.7). The proportion invested in the risky portfolio will therefore decrease.

4. a. The expected cash flow is: (0.5 × \$70,000) + (0.5 × 200,000) = \$135,000

   With a risk premium of 8% over the risk-free rate of 6%, the required rate of return is 14%. Therefore, the present value of the portfolio is:

   \$135,000/1.14 = \$118,421

   b. If the portfolio is purchased for \$118,421, and provides an expected cash inflow of \$135,000, then the expected rate of return [E(r)] is as follows:

   \$118,421 × [1 + E(r)] = \$135,000

   Therefore, E(r) = 14%. The portfolio price is set to equate the expected rate of return with the required rate of return.

   c. If the risk premium over T-bills is now 12%, then the required return is:

   6% + 12% = 18%

   The present value of the portfolio is now:

   \$135,000/1.18 = \$114,407

   d. For a given expected cash flow, portfolios that command greater risk premia must sell at lower prices. The extra discount from expected value is a penalty for risk.

5. When we specify utility by $U = E(r) - 0.5A\sigma^2$, the utility level for T-bills is: 0.07

The utility level for the risky portfolio is:

$$U = 0.12 - 0.5 \times A \times (0.18)^2 = 0.12 - 0.0162 \times A$$

In order for the risky portfolio to be preferred to bills, the following must hold:

$$0.12 - 0.0162A > 0.07 \Rightarrow A < 0.05/0.0162 = 3.09$$

A must be less than 3.09 for the risky portfolio to be preferred to bills.

6. Points on the curve are derived by solving for E(r) in the following equation:

$$U = 0.05 = E(r) - 0.5A\sigma^2 = E(r) - 1.5\sigma^2$$

The values of E(r), given the values of σ^2, are therefore:

σ	σ^2	E(r)
0.00	0.0000	0.05000
0.05	0.0025	0.05375
0.10	0.0100	0.06500
0.15	0.0225	0.08375
0.20	0.0400	0.11000
0.25	0.0625	0.14375

The bold line in the graph on the next page (labeled Q6, for Question 6) depicts the indifference curve.

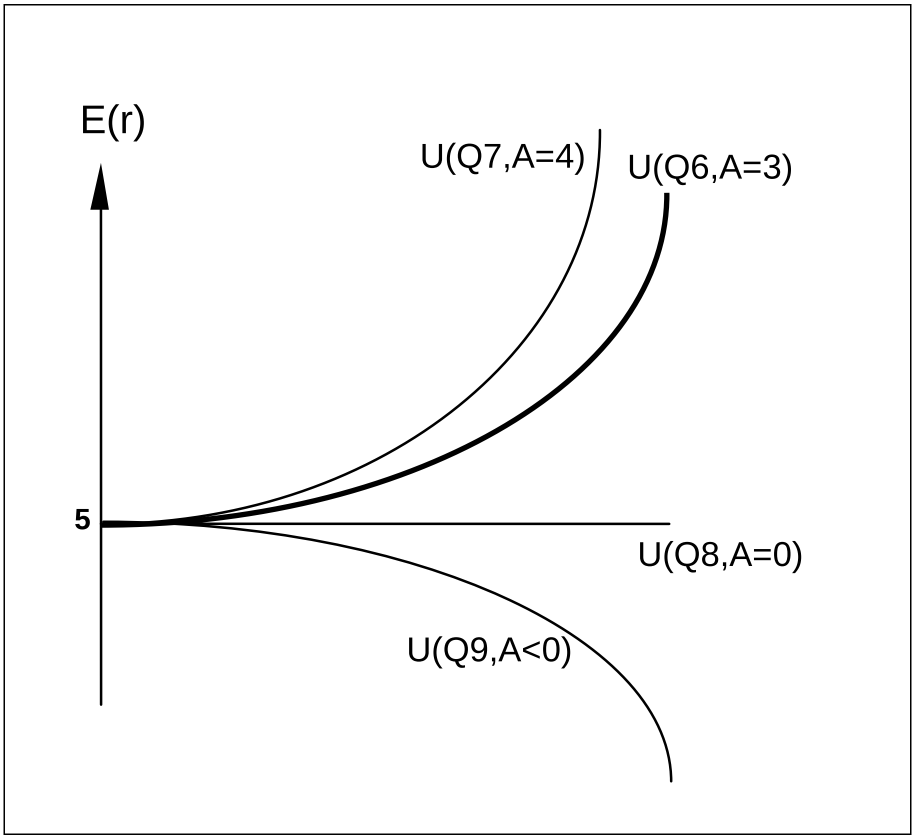
7. Repeating the analysis in Problem 6, utility is now:

$$U = E(r) - 0.5A\sigma^2 = E(r) - 2.0\sigma^2 = 0.05$$

The equal-utility combinations of expected return and standard deviation are presented in the table below. The indifference curve is the upward sloping line in the graph on the next page, labeled Q7 (for Question 7).

σ	σ^2	E(r)
0.00	0.0000	0.0500
0.05	0.0025	0.0550
0.10	0.0100	0.0700
0.15	0.0225	0.0950
0.20	0.0400	0.1300
0.25	0.0625	0.1750

The indifference curve in Problem 7 differs from that in Problem 6 in slope. When A increases from 3 to 4, the increased risk aversion results in a greater slope for the indifference curve since more expected return is needed in order to compensate for additional σ.



8. The coefficient of risk aversion for a risk neutral investor is zero. Therefore, the corresponding utility is equal to the portfolio's expected return. The corresponding indifference curve in the expected return-standard deviation plane is a horizontal line, labeled Q8 in the graph above (see Problem 6).

9. A risk lover, rather than penalizing portfolio utility to account for risk, derives greater utility as variance increases. This amounts to a negative coefficient of risk aversion. The corresponding indifference curve is downward sloping in the graph above (see Problem 6), and is labeled Q9.

10. The portfolio expected return and variance are computed as follows:

(1) W_{Bills}	(2) r_{Bills}	(3) W_{Index}	(4) r_{Index}	$r_{Portfolio}$ (1)×(2)+(3)×(4)	$\sigma_{Portfolio}$ (3) × 20%	$\sigma^2_{Portfolio}$
0.0	5%	1.0	13.0%	13.0% = 0.130	20% = 0.20	0.0400
0.2	5%	0.8	13.0%	11.4% = 0.114	16% = 0.16	0.0256
0.4	5%	0.6	13.0%	9.8% = 0.098	12% = 0.12	0.0144
0.6	5%	0.4	13.0%	8.2% = 0.082	8% = 0.08	0.0064
0.8	5%	0.2	13.0%	6.6% = 0.066	4% = 0.04	0.0016
1.0	5%	0.0	13.0%	5.0% = 0.050	0% = 0.00	0.0000

11. Computing utility from $U = E(r) - 0.5 \times A\sigma^2 = E(r) - \sigma^2$, we arrive at the values in the column labeled U(A = 2) in the following table:

W_{Bills}	W_{Index}	$r_{Portfolio}$	$\sigma_{Portfolio}$	$\sigma^2_{Portfolio}$	U(A = 2)	U(A = 3)
0.0	1.0	0.130	0.20	0.0400	0.0900	.0700
0.2	0.8	0.114	0.16	0.0256	0.0884	.0756
0.4	0.6	0.098	0.12	0.0144	0.0836	.0764
0.6	0.4	0.082	0.08	0.0064	0.0756	.0724
0.8	0.2	0.066	0.04	0.0016	0.0644	.0636
1.0	0.0	0.050	0.00	0.0000	0.0500	.0500

The column labeled U(A = 2) implies that investors with A = 2 prefer a portfolio that is invested 100% in the market index to any of the other portfolios in the table.

12. The column labeled U(A = 3) in the table above is computed from:

$$U = E(r) - 0.5A\sigma^2 = E(r) - 1.5\sigma^2$$

The more risk averse investors prefer the portfolio that is invested 40% in the market, rather than the 100% market weight preferred by investors with A = 2.

13. Expected return = (0.7 × 18%) + (0.3 × 8%) = 15%

Standard deviation = 0.7 × 28% = 19.6%

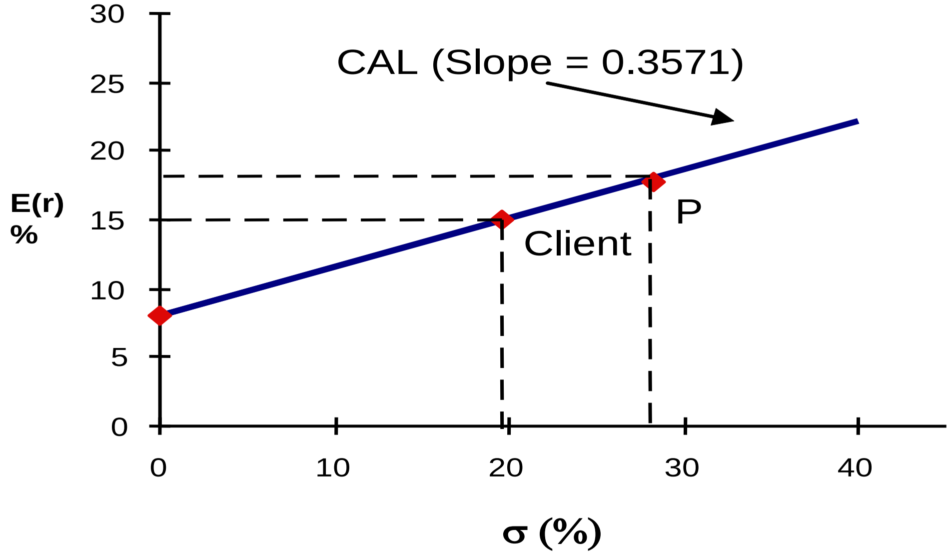
14. Investment proportions:

	30.0% in T-bills
0.7 × 25% =	17.5% in Stock A
0.7 × 32% =	22.4% in Stock B
0.7 × 43% =	30.1% in Stock C

15. Your reward-to-volatility ratio: $S = \frac{.18 - .08}{.28} = 0.3571$

Client's reward-to-volatility ratio: $S = \frac{.15 - .08}{.196} = 0.3571$

16.



17. a. $E(r_C) = r_f + y \times [E(r_P) - r_f] = 8 + y \times (18 - 8)$

If the expected return for the portfolio is 16%, then:

$$16\% = 8\% + 10\% \times y \Rightarrow y = \frac{.16 - .08}{.10} = 0.8$$

Therefore, in order to have a portfolio with expected rate of return equal to 16%, the client must invest 80% of total funds in the risky portfolio and 20% in T-bills.

b.

Client's investment proportions:		20.0% in T-bills
	0.8 × 25% =	20.0% in Stock A
	0.8 × 32% =	25.6% in Stock B
	0.8 × 43% =	34.4% in Stock C

c. $\sigma_C = 0.8 \times \sigma_P = 0.8 \times 28\% = 22.4\%$

18. a. $\sigma_C = y \times 28\%$

If your client prefers a standard deviation of at most 18%, then:

y = 18/28 = 0.6429 = 64.29% invested in the risky portfolio

b. $E(r_C) = .08 + .1 \times y = .08 + (0.6429 \times .1) = 14.429\%$

19. a. $y^* = \frac{E(r_P) - r_f}{A\sigma_P^2} = \frac{0.18 - 0.08}{3.5 \times 0.28^2} = \frac{0.10}{0.2744} = 0.3644$

Therefore, the client's optimal proportions are: 36.44% invested in the risky portfolio and 63.56% invested in T-bills.

b. $E(r_C) = 8 + 10 \times y^* = 8 + (0.3644 \times 10) = 11.644\%$

$\sigma_C = 0.3644 \times 28 = 10.203\%$

20. a. If the period 1926 - 2009 is assumed to be representative of future expected performance, then we use the following data to compute the fraction allocated to equity: A = 4, $E(r_M) - r_f = 7.93\%$, $\sigma_M = 20.81\%$ (we use the standard deviation of the risk premium from Table 6.7). Then y^* is given by:

$$y^* = \frac{E(r_M) - r_f}{A\sigma_M^2} = \frac{0.0793}{4 \times 0.2081^2} = 0.4578$$

That is, 45.78% of the portfolio should be allocated to equity and 54.22% should be allocated to T-bills.

b. If the period 1968 - 1988 is assumed to be representative of future expected performance, then we use the following data to compute the fraction allocated to equity: A = 4, $E(r_M) - r_f = 3.44\%$, $\sigma_M = 16.71\%$ and y* is given by:

c.

$$y^* = \frac{E(r_M) - r_f}{A\sigma_M^2} = \frac{0.0344}{4 \times 0.1671^2} = 0.3080$$

Therefore, 30.80% of the complete portfolio should be allocated to equity and 69.20% should be allocated to T-bills.

c. In part (b), the market risk premium is expected to be lower than in part (a) and market risk is higher. Therefore, the reward-to-volatility *ratio* is expected to be lower in part (b), which explains the greater proportion invested in T-bills.

21. a. $E(r_C) = 8\% = 5\% + y \times (11\% - 5\%) \Rightarrow y = \frac{.08 - .05}{.11 - .05} = 0.5$

b. $\sigma_C = y \times \sigma_P = 0.50 \times 15\% = 7.5\%$

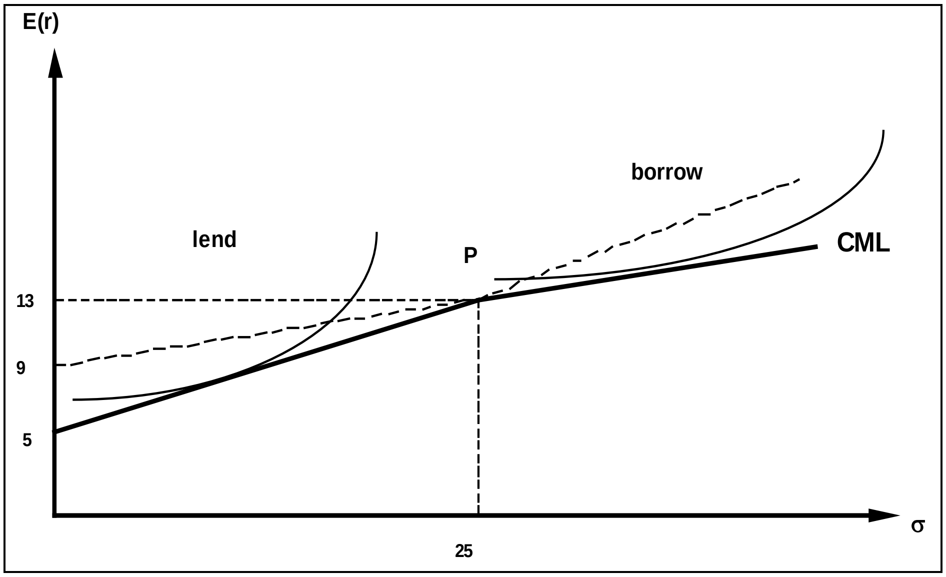
c. The first client is more risk averse, allowing a smaller standard deviation.

22. Johnson requests the portfolio standard deviation to equal one half the market portfolio standard deviation. The market portfolio $\sigma_M = 20\%$ which implies $\sigma_P = 10\%$. The intercept of the CML equals $r_f = 0.05$ and the slope of the CML equals the Sharpe ratio for the market portfolio (35%). Therefore using the CML:

$$E(r_P) = r_f + \frac{E(r_M) - r_f}{\sigma_M}\sigma_P = 0.05 + 0.35 \times 0.10 = 0.085 = 8.5\%$$

23. Data: $r_f = 5\%$, $E(r_M) = 13\%$, $\sigma_M = 25\%$, and $r_f^B = 9\%$

The CML and indifference curves are as follows:



24. For *y* to be less than 1.0 (that the investor is a lender), risk aversion (A) must be large enough such that:

$$y = \frac{E(r_M) - r_f}{A\sigma_M^2} < 1 \Rightarrow A > \frac{0.13 - 0.05}{0.25^2} = 1.28$$

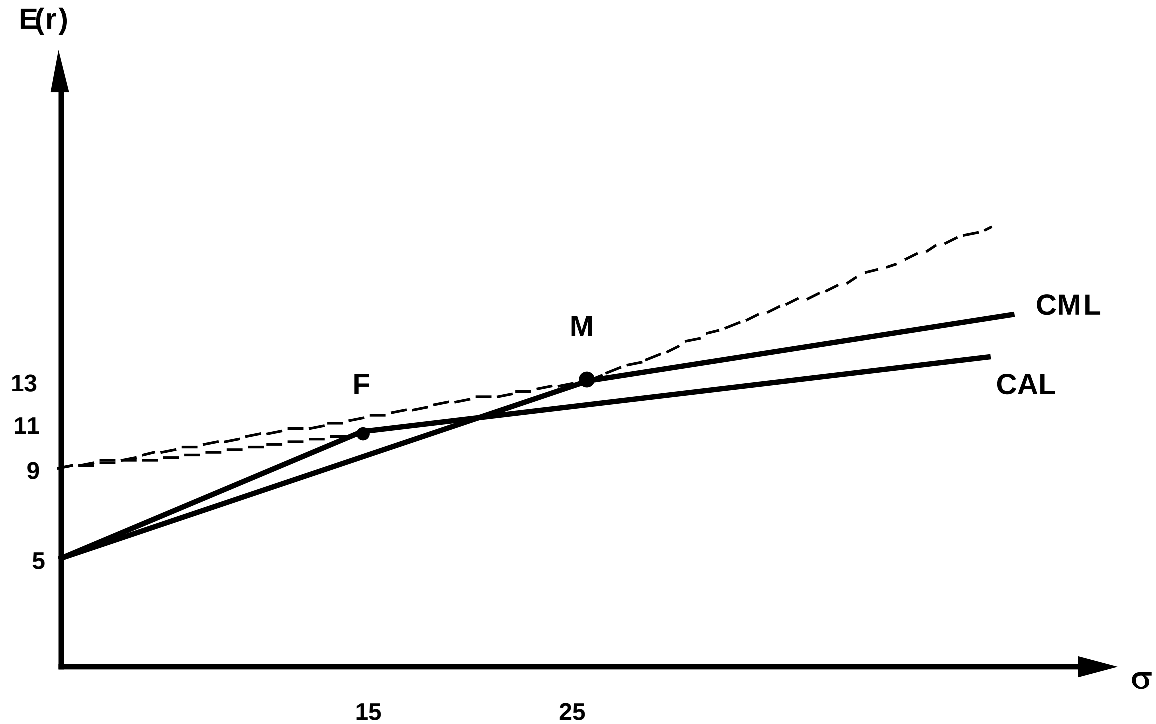
For *y* to be greater than 1 (the investor is a borrower), A must be small enough:

$$y = \frac{E(r_M) - r_f}{A\sigma_M^2} > 1 \Rightarrow A < \frac{0.13 - 0.09}{0.25^2} = 0.64$$

For values of risk aversion within this range, the client will neither borrow nor lend, but will hold a portfolio comprised only of the optimal risky portfolio:

$$y = 1 \text{ for } 0.64 \le A \le 1.28$$

25. a. The graph for Problem 23 has to be redrawn here, with:
$E(r_P) = 11\%$ and $\sigma_P = 15\%$



b. For a lending position: $A > \frac{0.11 - 0.05}{0.15^2} = 2.67$

For a borrowing position: $A < \frac{0.11 - 0.09}{0.15^2} = 0.89$

Therefore, y = 1 for $0.89 \le A \le 2.67$

26. The maximum feasible fee, denoted f, depends on the reward-to-variability ratio.

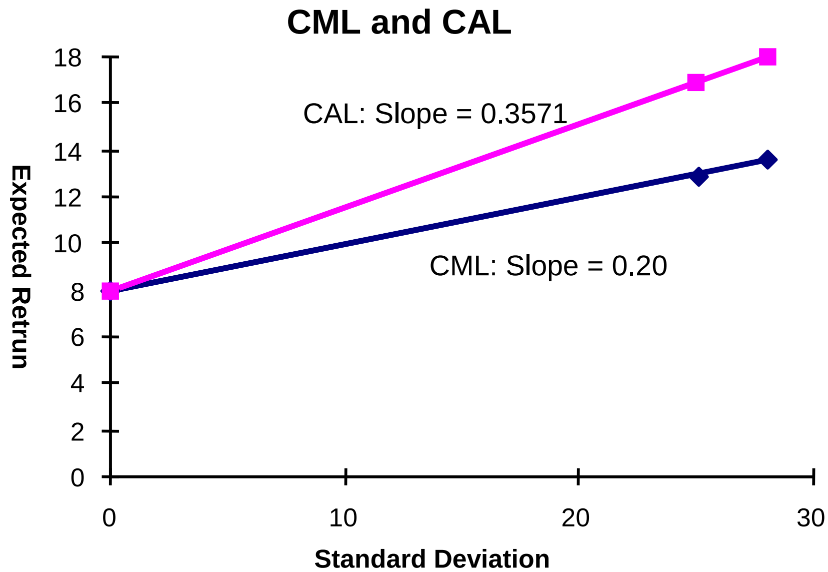
For y < 1, the lending rate, 5%, is viewed as the relevant risk-free rate, and we solve for f as follows:

$$\frac{.11 - .05 - f}{.15} = \frac{.13 - .05}{.25} \Rightarrow f = .06 - \frac{.15 \times .08}{.25} = .012 = 1.2\%$$

For y > 1, the borrowing rate, 9%, is the relevant risk-free rate. Then we notice that, even without a fee, the active fund is inferior to the passive fund because: More risk tolerant investors (who are more inclined to borrow) will not be clients of the fund. We find that f is negative: that is, you would need to *pay* investors to choose your active fund. These investors desire higher risk-higher return complete portfolios and thus are in the borrowing range of the relevant CAL. In this range, the reward-to-variability ratio of the index (the passive fund) is better than that of the managed fund.

27. a. Slope of the CML $= \dfrac{.13 - .08}{.25} = 0.20$

The diagram follows.



b. My fund allows an investor to achieve a higher mean for any given standard deviation than would a passive strategy, i.e., a higher expected return for any given level of risk.

28. a. With 70% of his money invested in my fund's portfolio, the client's expected return is 15% per year and standard deviation is 19.6% per year. If he shifts that money to the passive portfolio (which has an expected return of 13% and standard deviation of 25%), his overall expected return becomes:

$E(r_C) = r_f + 0.7 \times [E(r_M) - r_f] = .08 + [0.7 \times (.13 - .08)] = .115 = 11.5\%$

The standard deviation of the complete portfolio using the passive portfolio would be:

$\sigma_C = 0.7 \times \sigma_M = 0.7 \times 25\% = 17.5\%$

Therefore, the shift entails a decrease in mean from 15% to 11.5% and a decrease in standard deviation from 19.6% to 17.5%. Since both mean return *and* standard deviation decrease, it is not yet clear whether the move is beneficial. The disadvantage of the shift is that, if the client is willing to accept a mean return on his total portfolio of 11.5%, he can achieve it with a lower standard deviation using my fund rather than the passive portfolio.

To achieve a target mean of 11.5%, we first write the mean of the complete portfolio as a function of the proportion invested in my fund (*y*):

$$E(r_C) = .08 + y \times (.18 - .08) = .08 + .10 \times y$$

Our target is: $E(r_C) = 11.5\%$. Therefore, the proportion that must be invested in my fund is determined as follows:

$$.115 = .08 + .10 \times y \Rightarrow y = \frac{.115 - .08}{.10} = 0.35$$

The standard deviation of this portfolio would be:

$$\sigma_C = y \times 28\% = 0.35 \times 28\% = 9.8\%$$

Thus, by using my portfolio, the same 11.5% expected return can be achieved with a standard deviation of only 9.8% as opposed to the standard deviation of 17.5% using the passive portfolio.

b. The fee would reduce the reward-to-volatility ratio, i.e., the slope of the CAL. The client will be indifferent between my fund and the passive portfolio if the slope of the after-fee CAL and the CML are equal. Let f denote the fee:

$$\text{Slope of CAL with fee} = \frac{.18 - .08 - f}{.28} = \frac{.10 - f}{.28}$$

$$\text{Slope of CML (which requires no fee)} = \frac{.13 - .08}{.25} = 0.20$$

Setting these slopes equal we have:

$$\frac{.10 - f}{.28} = 0.20 \Rightarrow f = 0.044 = 4.4\% \text{ per year}$$

29. a. The formula for the optimal proportion to invest in the passive portfolio is:

$$y^* = \frac{E(r_M) - r_f}{A\sigma_M^2}$$

Substitute the following: $E(r_M) = 13\%$; $r_f = 8\%$; $\sigma_M = 25\%$; $A = 3.5$:

$$y^* = \frac{0.13 - 0.08}{3.5 \times 0.25^2} = 0.2286 = 22.86\% \text{ in the passive portfolio}$$

b. The answer here is the same as the answer to Problem 28(b). The fee that you can charge a client is the same regardless of the asset allocation mix of the client's portfolio. You can charge a fee that will equate the reward-to-volatility *ratio* of your portfolio to that of your competition.

CFA PROBLEMS

1. Utility for each investment = E(r) – 0.5 × 4 × σ^2

   We choose the investment with the highest utility value, Investment 3.

Investment	Expected return E(r)	Standard deviation σ	Utility U
1	0.12	0.30	-0.0600
2	0.15	0.50	-0.3500
3	0.21	0.16	0.1588
4	0.24	0.21	0.1518

2. When investors are risk neutral, then A = 0; the investment with the highest utility is Investment 4 because it has the highest expected return.

3. (b)

4. Indifference curve 2

5. Point E

6. (0.6 × $50,000) + [0.4 × (–$30,000)] – $5,000 = $13,000

7. (b)

8. Expected return for equity fund = T-bill rate + risk premium = 6% + 10% = 16%
   Expected rate of return of the client's portfolio = (0.6 × 16%) + (0.4 × 6%) = 12%

   Expected return of the client's portfolio = 0.12 × $100,000 = $12,000

   (which implies expected total wealth at the end of the period = $112,000)

   Standard deviation of client's overall portfolio = 0.6 × 14% = 8.4%

9. Reward-to-volatility ratio = $\frac{.10}{.14} = 0.71$

CHAPTER 6: APPENDIX

1. By year end, the \$50,000 investment will grow to: \$50,000 × 1.06 = \$53,000
Without insurance, the probability distribution of end-of-year wealth is:

	Probability	Wealth
No fire	0.999	\$253,000
Fire	0.001	\$ 53,000

For this distribution, expected utility is computed as follows:

$$E[U(W)] = [0.999 \times \ln(253{,}000)] + [0.001 \times \ln(53{,}000)] = 12.439582$$

The certainty equivalent is:

$$W_{CE} = e^{12.439582} = \$252{,}604.85$$

With fire insurance, at a cost of \$P, the investment in the risk-free asset is:

$$\$(50{,}000 - P)$$

Year-end wealth will be certain (since you are fully insured) and equal to:

$$[\$(50{,}000 - P) \times 1.06] + \$200{,}000$$

Solve for P in the following equation:

$$[\$(50{,}000 - P) \times 1.06] + \$200{,}000 = \$252{,}604.85 \Rightarrow P = \$372.78$$

This is the most you are willing to pay for insurance. Note that the expected loss is “only” \$200, so you are willing to pay a substantial risk premium over the expected value of losses. The primary reason is that the value of the house is a large proportion of your wealth.

2. a. With insurance coverage for one-half the value of the house, the premium is $100, and the investment in the safe asset is $49,900.
By year end, the investment of $49,900 will grow to: $49,900 × 1.06 = $52,894
If there is a fire, your insurance proceeds will be $100,000, and the probability distribution of end-of-year wealth is:

	Probability	Wealth
No fire	0.999	$252,894
Fire	0.001	$152,894

For this distribution, expected utility is computed as follows:

$$E[U(W)] = [0.999 \times \ln(252{,}894)] + [0.001 \times \ln(152{,}894)] = 12.4402225$$

The certainty equivalent is:

$$W_{CE} = e^{12.4402225} = \$252{,}766.77$$

b. With insurance coverage for the full value of the house, costing $200, end-of-year wealth is certain, and equal to:

$$[(\$50{,}000 - \$200) \times 1.06] + \$200{,}000 = \$252{,}788$$

Since wealth is certain, this is also the certainty equivalent wealth of the fully insured position.

c. With insurance coverage for 1½ times the value of the house, the premium is $300, and the insurance pays off $300,000 in the event of a fire. The investment in the safe asset is $49,700. By year end, the investment of $49,700 will grow to: $49,700 × 1.06 = $52,682
The probability distribution of end-of-year wealth is:

	Probability	Wealth
No fire	0.999	$252,682
Fire	0.001	$352,682

For this distribution, expected utility is computed as follows:

$$E[U(W)] = [0.999 \times \ln(252{,}682)] + [0.001 \times \ln(352{,}682)] = 12.4402205$$

The certainty equivalent is:

$$W_{CE} = e^{12.440222} = \$252{,}766.27$$

Therefore, full insurance dominates both over- and under-insurance. Over-insuring creates a gamble (you actually gain when the house burns down). Risk is minimized when you insure exactly the value of the house.

CHAPTER 7: OPTIMAL RISKY PORTFOLIOS

PROBLEM SETS

1. (a) and (e).

2. (a) and (c). After real estate is added to the portfolio, there are four asset classes in the portfolio: stocks, bonds, cash and real estate. Portfolio variance now includes a variance term for real estate returns and a covariance term for real estate returns with returns for each of the other three asset classes. Therefore, portfolio risk is affected by the variance (or standard deviation) of real estate returns and the correlation between real estate returns and returns for each of the other asset classes. (Note that the correlation between real estate returns and returns for cash is most likely zero.)

3. (a) Answer (a) is valid because it provides the definition of the minimum variance portfolio.

4. The parameters of the opportunity set are:

   $E(r_S) = 20\%$, $E(r_B) = 12\%$, $\sigma_S = 30\%$, $\sigma_B = 15\%$, $\rho = 0.10$

   From the standard deviations and the correlation coefficient we generate the covariance matrix [note that $Cov(r_S, r_B) = \rho \times \sigma_S \times \sigma_B$]:

	Bonds	Stocks
Bonds	225	45
Stocks	45	900

   The minimum-variance portfolio is computed as follows:

$$w_{Min}(S) = \frac{\sigma_B^2 - Cov(r_S, r_B)}{\sigma_S^2 + \sigma_B^2 - 2Cov(r_S, r_B)} = \frac{225 - 45}{900 + 225 - (2 \times 45)} = 0.1739$$

$$w_{Min}(B) = 1 - 0.1739 = 0.8261$$

   The minimum variance portfolio mean and standard deviation are:

$$E(r_{Min}) = (0.1739 \times .20) + (0.8261 \times .12) = .1339 = 13.39\%$$

$$\sigma_{Min} = [w_S^2\sigma_S^2 + w_B^2\sigma_B^2 + 2w_S w_B Cov(r_S, r_B)]^{1/2}$$

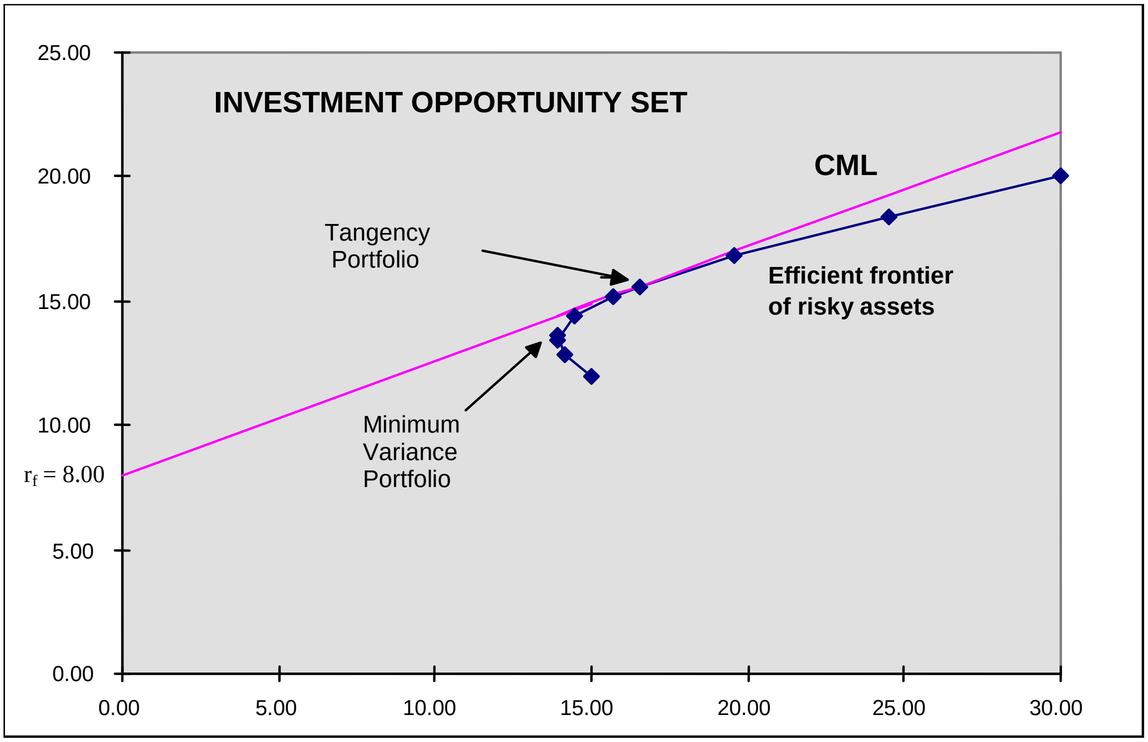
$$= [(0.1739^2 \times 900) + (0.8261^2 \times 225) + (2 \times 0.1739 \times 0.8261 \times 45)]^{1/2}$$

$$= 13.92\%$$

5.

Proportion in stock fund	Proportion in bond fund	Expected return	Standard Deviation	
0.00%	100.00%	12.00%	15.00%	
17.39%	82.61%	13.39%	13.92%	minimum variance
20.00%	80.00%	13.60%	13.94%	
40.00%	60.00%	15.20%	15.70%	
45.16%	54.84%	15.61%	16.54%	tangency portfolio
60.00%	40.00%	16.80%	19.53%	
80.00%	20.00%	18.40%	24.48%	
100.00%	0.00%	20.00%	30.00%	

Graph shown below.



6. The above graph indicates that the optimal portfolio is the tangency portfolio with expected return approximately 15.6% and standard deviation approximately 16.5%.

7. The proportion of the optimal risky portfolio invested in the stock fund is given by:

$$w_S = \frac{[E(r_S)-r_f]\times\sigma_B^2 - [E(r_B)-r_f]\times Cov(r_S,r_B)}{[E(r_S)-r_f]\times\sigma_B^2 + [E(r_B)-r_f]\times\sigma_S^2 - [E(r_S)-r_f + E(r_B)-r_f]\times Cov(r_S,r_B)}$$

$$= \frac{[(.20-.08)\times 225]-[(.12-.08)\times 45]}{[(.20-.08)\times 225]+[(.12-.08)\times 900]-[(.20-.08+.12-.08)\times 45]} = 0.4516$$

$$w_B = 1-0.4516 = 0.5484$$

The mean and standard deviation of the optimal risky portfolio are:

$E(r_P)$ = (0.4516 × .20) + (0.5484 × .12) = .1561

= 15.61%

$\sigma_p = [(0.4516^2 \times 900) + (0.5484^2 \times 225) + (2 \times 0.4516 \times 0.5484 \times 45)]^{1/2}$

= 16.54%

8. The reward-to-volatility ratio of the optimal CAL is:

$$\frac{E(r_p)-r_f}{\sigma_p} = \frac{.1561-.08}{.1654} = 0.4601$$ .4601 should be .4603 (rounding)

9. a. If you require that your portfolio yield an expected return of 14%, then you can find the corresponding standard deviation from the optimal CAL. The equation for this CAL is:

$$E(r_C) = r_f + \frac{E(r_p)-r_f}{\sigma_P}\sigma_C = .08 + 0.4601\sigma_C$$ .4601 should be .4603 (rounding)

If $E(r_C)$ is equal to 14%, then the standard deviation of the portfolio is 13.03%.

b. To find the proportion invested in the T-bill fund, remember that the mean of the complete portfolio (i.e., 14%) is an average of the T-bill rate and the optimal combination of stocks and bonds (P). Let y be the proportion invested in the portfolio P. The mean of any portfolio along the optimal CAL is:

$$E(r_C) = (1-y)\times r_f + y\times E(r_P) = r_f + y\times[E(r_P)-r_f] = .08 + y\times(.1561-.08)$$

Setting $E(r_C)$ = 14% we find: y = 0.7881 and (1 − y) = 0.2119 (the proportion invested in the T-bill fund).

To find the proportions invested in each of the funds, multiply 0.7884 times the respective proportions of stocks and bonds in the optimal risky portfolio:

Proportion of stocks in complete portfolio = 0.7881 × 0.4516 = 0.3559

Proportion of bonds in complete portfolio = 0.7881 × 0.5484 = 0.4322

10. Using only the stock and bond funds to achieve a portfolio expected return of 14%, we must find the appropriate proportion in the stock fund (w_S) and the appropriate proportion in the bond fund ($w_B = 1 - w_S$) as follows:

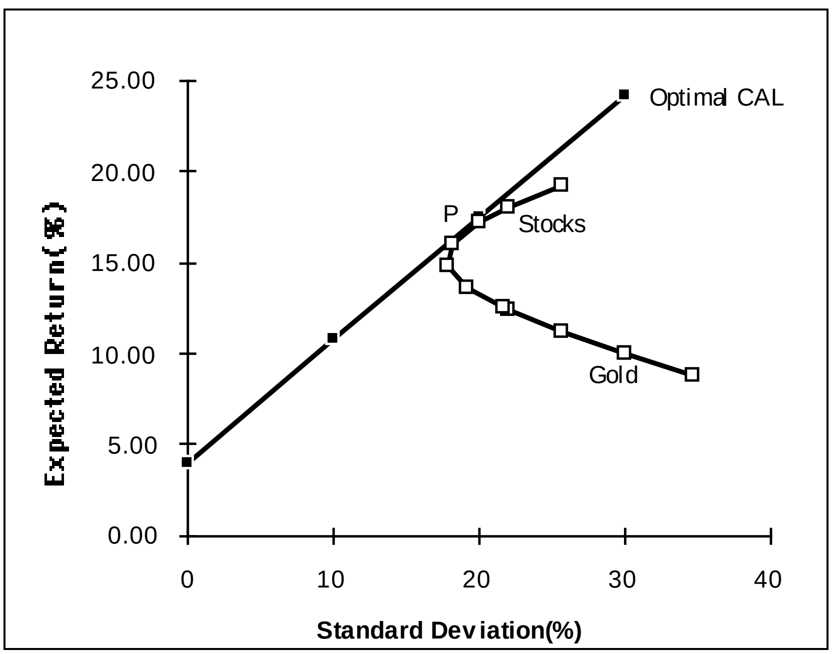
$$.14 = .20 \times w_S + .12 \times (1 - w_S) = .12 + .08 \times w_S \Rightarrow w_S = 0.25$$

So the proportions are 25% invested in the stock fund and 75% in the bond fund. The standard deviation of this portfolio will be:

$$\sigma_P = [(0.25^2 \times 900) + (0.75^2 \times 225) + (2 \times 0.25 \times 0.75 \times 45)]^{1/2} = 14.13\%$$

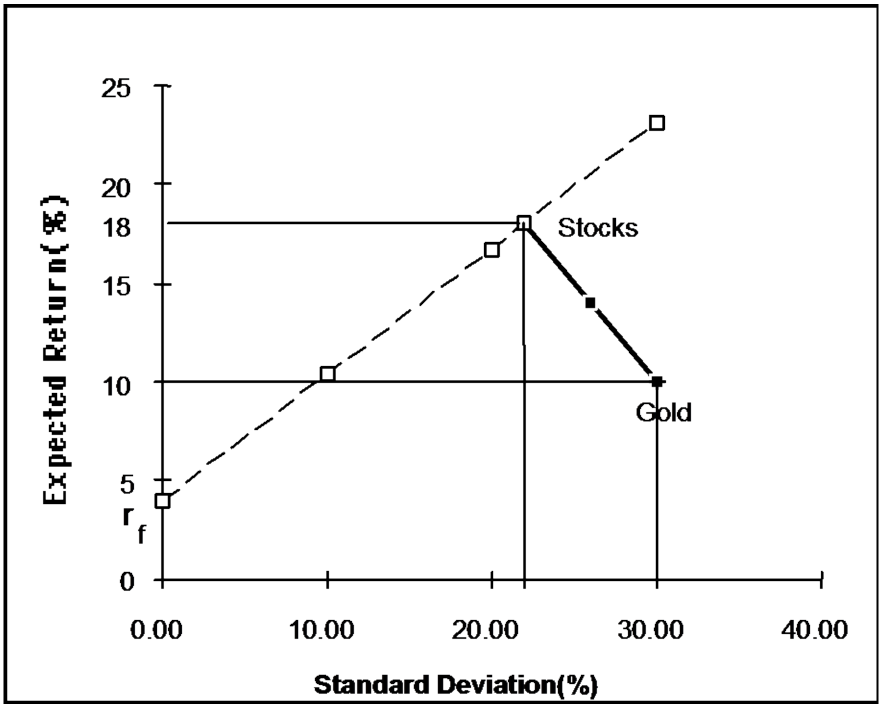
This is considerably greater than the standard deviation of 13.04% achieved using T-bills and the optimal portfolio.

11. a.



Even though it seems that gold is dominated by stocks, gold might still be an attractive asset to hold as a *part* of a portfolio. If the correlation between gold and stocks is sufficiently low, gold will be held as a component in a portfolio, specifically, the optimal tangency portfolio.

b. If the correlation between gold and stocks equals +1, then no one would hold gold. The optimal CAL would be comprised of bills and stocks only. Since the set of risk/return combinations of stocks and gold would plot as a straight line with a negative slope (see the following graph), these combinations would be dominated by the stock portfolio. Of course, this situation could not persist. If no one desired gold, its price would fall and its expected rate of return would increase until it became sufficiently attractive to include in a portfolio.



12. Since Stock A and Stock B are perfectly negatively correlated, a risk-free portfolio can be created and the rate of return for this portfolio, in equilibrium, will be the risk-free rate. To find the proportions of this portfolio [with the proportion w_A invested in Stock A and $w_B = (1 - w_A)$ invested in Stock B], set the standard deviation equal to zero. With perfect negative correlation, the portfolio standard deviation is:

σ_P = Absolute value $[w_A\sigma_A - w_B\sigma_B]$

$0 = 5 \times w_A - [10 \times (1 - w_A)] \Rightarrow w_A = 0.6667$

The expected rate of return for this risk-free portfolio is:

$E(r) = (0.6667 \times 10) + (0.3333 \times 15) = 11.667\%$

Therefore, the risk-free rate is: 11.667%

13. False. If the borrowing and lending rates are not identical, then, depending on the tastes of the individuals (that is, the shape of their indifference curves), borrowers and lenders could have different optimal risky portfolios.

14. False. The portfolio standard deviation equals the weighted average of the component-asset standard deviations *only* in the special case that all assets are perfectly positively correlated. Otherwise, as the formula for portfolio standard deviation shows, the portfolio standard deviation is *less* than the weighted average of the component-asset standard deviations. The portfolio *variance* is a weighted *sum* of the elements in the covariance matrix, with the products of the portfolio proportions as weights.

15. The probability distribution is:

Probability	Rate of Return
0.7	100%
0.3	−50%

Mean = [0.7 × 100%] + [0.3 × (-50%)] = 55%

Variance = $[0.7 \times (100 - 55)^2] + [0.3 \times (-50 - 55)^2] = 4725$

Standard deviation = $4725^{1/2} = 68.74\%$

16. $\sigma_P = 30 = y \times \sigma = 40 \times y \Rightarrow y = 0.75$

$E(r_P) = 12 + 0.75(30 - 12) = 25.5\%$

17. The correct choice is c. Intuitively, we note that since all stocks have the same expected rate of return and standard deviation, we choose the stock that will result in lowest risk. This is the stock that has the lowest correlation with Stock A.

More formally, we note that when all stocks have the same expected rate of return, the optimal portfolio for any risk-averse investor is the global minimum variance portfolio (G). When the portfolio is restricted to Stock A and one additional stock, the objective is to find G for any pair that includes Stock A, and then select the combination with the lowest variance. With two stocks, I and J, the formula for the weights in G is:

$$w_{Min}(I) = \frac{\sigma_J^2 - Cov(r_I, r_J)}{\sigma_I^2 + \sigma_J^2 - 2Cov(r_I, r_J)}$$

$$w_{Min}(J) = 1 - w_{Min}(I)$$

Since all standard deviations are equal to 20%:

$$Cov(r_I, r_J) = \rho\sigma_I\sigma_J = 400\rho \text{ and } w_{Min}(I) = w_{Min}(J) = 0.5$$

This intuitive result is an implication of a property of any efficient frontier, namely, that the covariances of the global minimum variance portfolio with all other assets on the frontier are identical and equal to its own variance. (Otherwise, additional diversification would further reduce the variance.) In this case, the standard deviation of G(I, J) reduces to:

$$\sigma_{Min}(G) = [200 \times (1 + \rho_{IJ})]^{1/2}$$

This leads to the intuitive result that the desired addition would be the stock with the lowest correlation with Stock A, which is Stock D. The optimal portfolio is equally invested in Stock A and Stock D, and the standard deviation is 17.03%.

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$$\sigma_{Min}(G) = [200 \times (1 + \rho_{IJ})]^{1/2}$$

This leads to the intuitive result that the desired addition would be the stock with the lowest correlation with Stock A, which is Stock D. The optimal portfolio is equally invested in Stock A and Stock D, and the standard deviation is 17.03%.

18. No, the answer to Problem 17 would not change, at least as long as investors are not risk lovers. Risk neutral investors would not care which portfolio they held since all portfolios have an expected return of 8%.

19. Yes, the answers to Problems 17 and 18 would change. The efficient frontier of risky assets is horizontal at 8%, so the optimal CAL runs from the risk-free rate through G. This implies risk-averse investors will just hold Treasury Bills.

20. Rearranging the table (converting rows to columns), and computing serial correlation results in the following table:

Nominal Rates

	Small company stocks	Large company stocks	Long-term government bonds	Intermed-term government bonds	Treasury bills	Inflation
1920s	-3.72	18.36	3.98	3.77	3.56	-1.00
1930s	7.28	-1.25	4.60	3.91	0.30	-2.04
1940s	20.63	9.11	3.59	1.70	0.37	5.36
1950s	19.01	19.41	0.25	1.11	1.87	2.22
1960s	13.72	7.84	1.14	3.41	3.89	2.52
1970s	8.75	5.90	6.63	6.11	6.29	7.36
1980s	12.46	17.60	11.50	12.01	9.00	5.10
1990s	13.84	18.20	8.60	7.74	5.02	2.93
Serial Correlation	0.46	-0.22	0.60	0.59	0.63	0.23

For example: to compute serial correlation in decade nominal returns for large-company stocks, we set up the following two columns in an Excel spreadsheet. Then, use the Excel function "CORREL" to calculate the correlation for the data.

	Decade	Previous
1930s	-1.25%	18.36%
1940s	9.11%	-1.25%
1950s	19.41%	9.11%
1960s	7.84%	19.41%
1970s	5.90%	7.84%
1980s	17.60%	5.90%
1990s	18.20%	17.60%

Note that each correlation is based on only seven observations, so we cannot arrive at any statistically significant conclusions. Looking at the results, however, it appears that, with the exception of large-company stocks, there is persistent serial correlation. (This conclusion changes when we turn to real rates in the next problem.)

21. The table for real rates (using the approximation of subtracting a decade's average inflation from the decade's average nominal return) is:

Real Rates

	Small company stocks	Large company stocks	Long-term government bonds	Intermed-term government bonds	Treasury bills
1920s	-2.72	19.36	4.98	4.77	4.56
1930s	9.32	0.79	6.64	5.95	2.34
1940s	15.27	3.75	-1.77	-3.66	-4.99
1950s	16.79	17.19	-1.97	-1.11	-0.35
1960s	11.20	5.32	-1.38	0.89	1.37
1970s	1.39	-1.46	-0.73	-1.25	-1.07
1980s	7.36	12.50	6.40	6.91	3.90
1990s	10.91	15.27	5.67	4.81	2.09
Serial Correlation	0.29	-0.27	0.38	0.11	0.00

While the serial correlation in decade *nominal* returns seems to be positive, it appears that real rates are serially uncorrelated. The decade time series (although again too short for any definitive conclusions) suggest that real rates of return are independent from decade to decade.

CFA PROBLEMS

1. a. Restricting the portfolio to 20 stocks, rather than 40 to 50 stocks, will increase the risk of the portfolio, but it is possible that the increase in risk will be minimal. Suppose that, for instance, the 50 stocks in a universe have the same standard deviation (σ) and the correlations between each pair are identical, with correlation coefficient ρ. Then, the covariance between each pair of stocks would be $\rho\sigma^2$, and the variance of an equally weighted portfolio would be:

$$\sigma_P^2 = \frac{1}{n}\sigma^2 + \frac{n-1}{n}\rho\sigma^2$$

The effect of the reduction in n on the second term on the right-hand side would be relatively small (since 49/50 is close to 19/20 and $\rho\sigma^2$ is smaller than σ^2), but the denominator of the first term would be 20 instead of 50. For example, if $\sigma = 45\%$ and $\rho = 0.2$, then the standard deviation with 50 stocks would be 20.91%, and would rise to 22.05% when only 20 stocks are held. Such an increase might be acceptable if the expected return is increased sufficiently.

b. Hennessy could contain the increase in risk by making sure that he maintains reasonable diversification among the 20 stocks that remain in his portfolio. This entails maintaining a low correlation among the remaining stocks. For example, in part (a), with $\rho = 0.2$, the increase in portfolio risk was minimal. As a practical matter, this means that Hennessy would have to spread his portfolio among many industries; concentrating on just a few industries would result in higher correlations among the included stocks.

2. Risk reduction benefits from diversification are not a linear function of the number of issues in the portfolio. Rather, the incremental benefits from additional diversification are most important when you are least diversified. Restricting Hennessy to 10 instead of 20 issues would increase the risk of his portfolio by a greater amount than would a reduction in the size of the portfolio from 30 to 20 stocks. In our example, restricting the number of stocks to 10 will increase the standard deviation to 23.81%. The 1.76% increase in standard deviation resulting from giving up 10 of 20 stocks is greater than the 1.14% increase that results from giving up 30 of 50 stocks.

3. The point is well taken because the committee should be concerned with the volatility of the entire portfolio. Since Hennessy's portfolio is only one of six well-diversified portfolios and is smaller than the average, the concentration in fewer issues might have a minimal effect on the diversification of the total fund. Hence, unleashing Hennessy to do stock picking may be advantageous.

4. d. Portfolio Y cannot be efficient because it is dominated by another portfolio. For example, Portfolio X has both higher expected return and lower standard deviation.

5. c.

6. d.

7. b.

8. a.

9. c.

10. Since we do not have any information about expected returns, we focus exclusively on reducing variability. Stocks A and C have equal standard deviations, but the correlation of Stock B with Stock C (0.10) is less than that of Stock A with Stock B (0.90). Therefore, a portfolio comprised of Stocks B and C will have lower total risk than a portfolio comprised of Stocks A and B.

11. Fund D represents the single *best* addition to complement Stephenson's current portfolio, given his selection criteria. Fund D's expected return (14.0 percent) has the potential to increase the portfolio's return somewhat. Fund D's relatively low correlation with his current portfolio (+0.65) indicates that Fund D will provide greater diversification benefits than any of the other alternatives except Fund B. The result of adding Fund D should be a portfolio with approximately the same expected return and somewhat lower volatility compared to the original portfolio.

The other three funds have shortcomings in terms of expected return enhancement or volatility reduction through diversification. Fund A offers the potential for increasing the portfolio's return, but is too highly correlated to provide substantial volatility reduction benefits through diversification. Fund B provides substantial volatility reduction through diversification benefits, but is expected to generate a return well below the current portfolio's return. Fund C has the greatest potential to increase the portfolio's return, but is too highly correlated with the current portfolio to provide substantial volatility reduction benefits through diversification.

12. a. Subscript OP refers to the original portfolio, ABC to the new stock, and NP to the new portfolio.

i. $E(r_{NP}) = w_{OP}\, E(r_{OP}) + w_{ABC}\, E(r_{ABC}) = (0.9 \times 0.67) + (0.1 \times 1.25) = 0.728\%$

ii. $Cov = \rho \times \sigma_{OP} \times \sigma_{ABC} = 0.40 \times 2.37 \times 2.95 = 2.7966 \cong 2.80$

iii. $\sigma_{NP} = [w_{OP}^2\, \sigma_{OP}^2 + w_{ABC}^2\, \sigma_{ABC}^2 + 2\, w_{OP}\, w_{ABC}\, (Cov_{OP,\, ABC})]^{1/2}$

$= [(0.9^2 \times 2.37^2) + (0.1^2 \times 2.95^2) + (2 \times 0.9 \times 0.1 \times 2.80)]^{1/2}$

$= 2.2673\% \cong 2.27\%$

b. Subscript OP refers to the original portfolio, GS to government securities, and NP to the new portfolio.

i. $E(r_{NP}) = w_{OP}\, E(r_{OP}) + w_{GS}\, E(r_{GS}) = (0.9 \times 0.67) + (0.1 \times 0.42) = 0.645\%$

ii. $Cov = \rho \times \sigma_{OP} \times \sigma_{GS} = 0 \times 2.37 \times 0 = 0$

iii. $\sigma_{NP} = [w_{OP}^2\, \sigma_{OP}^2 + w_{GS}^2\, \sigma_{GS}^2 + 2\, w_{OP}\, w_{GS}\, (Cov_{OP,\, GS})]^{1/2}$

$= [(0.9^2 \times 2.37^2) + (0.1^2 \times 0) + (2 \times 0.9 \times 0.1 \times 0)]^{1/2} = 2.133\% \cong 2.13\%$

c. Adding the risk-free government securities would result in a lower beta for the new portfolio. The new portfolio beta will be a weighted average of the individual security betas in the portfolio; the presence of the risk-free securities would lower that weighted average.

d. The comment is not correct. Although the respective standard deviations and expected returns for the two securities under consideration are equal, the covariances between each security and the original portfolio are unknown, making it impossible to draw the conclusion stated. For instance, if the covariances are different, selecting one security over the other may result in a lower standard deviation for the portfolio as a whole. In such a case, that security would be the preferred investment, assuming all other factors are equal.

e. i. Grace clearly expressed the sentiment that the risk of loss was more important to her than the opportunity for return. Using variance (or standard deviation) as a measure of risk in her case has a serious limitation because standard deviation does not distinguish between positive and negative price movements.

ii. Two alternative risk measures that could be used instead of variance are:

Range of returns, which considers the highest and lowest expected returns in the future period, with a larger range being a sign of greater variability and therefore of greater risk.

Semivariance can be used to measure expected deviations of returns below the mean, or some other benchmark, such as zero.

Either of these measures would potentially be superior to variance for Grace. Range of returns would help to highlight the full spectrum of risk she is assuming, especially the downside portion of the range about which she is so concerned. Semivariance would also be effective, because it implicitly assumes that the investor wants to minimize the likelihood of returns falling below some target rate; in Grace's case, the target rate would be set at zero (to protect against negative returns).

13. a. Systematic risk refers to fluctuations in asset prices caused by macroeconomic factors that are common to all risky assets; hence systematic risk is often referred to as market risk. Examples of systematic risk factors include the business cycle, inflation, monetary policy and technological changes.

Firm-specific risk refers to fluctuations in asset prices caused by factors that are independent of the market, such as industry characteristics or firm characteristics. Examples of firm-specific risk factors include litigation, patents, management, and financial leverage.

b. Trudy should explain to the client that picking only the top five best ideas would most likely result in the client holding a much more risky portfolio. The total risk of a portfolio, or portfolio variance, is the combination of systematic risk and firm-specific risk.

The systematic component depends on the sensitivity of the individual assets to market movements as measured by beta. Assuming the portfolio is well diversified, the number of assets will not affect the systematic risk component of portfolio variance. The portfolio beta depends on the individual security betas and the portfolio weights of those securities.

On the other hand, the components of firm-specific risk (sometimes called nonsystematic risk) are not perfectly positively correlated with each other and, as more assets are added to the portfolio, those additional assets tend to reduce portfolio risk. Hence, increasing the number of securities in a portfolio reduces firm-specific risk. For example, a patent expiration for one company would not affect the other securities in the portfolio. An increase in oil prices might hurt an airline stock but aid an energy stock. As the number of randomly selected securities increases, the total risk (variance) of the portfolio approaches its systematic variance.

CHAPTER 9: THE CAPITAL ASSET PRICING MODEL

PROBLEM SETS

1. $E(r_P) = r_f + \beta_P \times [E(r_M) - r_f]$

   $.18 = .06 + \beta_P \times [.14 - .06] \rightarrow \beta_P = \frac{.12}{.08} = 1.5$

2. If the security's correlation coefficient with the market portfolio doubles (with all other variables such as variances unchanged), then beta, and therefore the risk premium, will also double. The current risk premium is: 14% – 6% = 8%

   The new risk premium would be 16%, and the new discount rate for the security would be: 16% + 6% = 22%

   If the stock pays a constant perpetual dividend, then we know from the original data that the dividend (D) must satisfy the equation for the present value of a perpetuity:

   Price = Dividend/Discount rate

   $50 = D/0.14 \Rightarrow D = 50 \times 0.14 = \$7.00$

   At the new discount rate of 22%, the stock would be worth: \$7/0.22 = \$31.82 The increase in stock risk has lowered its value by 36.36%.

3. a. False. $\beta = 0$ implies $E(r) = r_f$, not zero.

   b. False. Investors require a risk premium only for bearing systematic (undiversifiable or market) risk. Total volatility includes diversifiable risk.

   c. False. Your portfolio should be invested 75% in the market portfolio and 25% in T-bills. Then: $\beta_P = (0.75 \times 1) + (0.25 \times 0) = 0.75$

4. The expected return is the return predicted by the CAPM for a given level of systematic risk.

   $$E(r_i) = r_f + \beta_i \times [E(r_M) - r_f]$$
   $$E(r_{\$1\ Discount}) = .04 + 1.5 \times (.10 - .04) = .13 = 13\%$$
   $$E(r_{Everything\ \$5}) = .04 + 1.0 \times (.10 - .04) = .10 = 10\%$$

5. According to the CAPM, \$1 Discount Stores requires a return of 13% based on its systematic risk level of $\beta = 1.5$. However, the forecasted return is only 12%. Therefore, the security is currently overvalued.

   Everything \$5 requires a return of 10% based on its systematic risk level of $\beta = 1.0$. However, the forecasted return is 11%. Therefore, the security is currently undervalued.

6. The expected return of a stock with a $\beta = 1.0$ must, on average, be the same as the expected return of the market which also has a $\beta = 1.0$.

7. Beta is a measure of systematic risk. Since only systematic risk is rewarded, it is safe to conclude that the expected return will be higher for Kaskin's stock than for Quinn's stock.

8. The appropriate discount rate for the project is:

$$r_f + \beta \times [E(r_M) - r_f] = .08 + [1.8 \times (.16 - .08)] = .224 = 22.4\%$$

Using this discount rate:

$$NPV = -\$40 + \sum_{t=1}^{10} \frac{\$15}{1.224^t} = -\$40 + [\$15 \times \text{Annuity factor } (22.4\%, 10 \text{ years})] =$$

$18.09

The internal rate of return (IRR) for the project is 35.73%. Recall from your introductory finance class that NPV is positive if IRR > discount rate (or, equivalently, hurdle rate). The highest value that beta can take before the hurdle rate exceeds the IRR is determined by:

$$.3573 = .08 + \beta \times (.16 - .08) \Rightarrow \beta = .2773/.08 = 3.47$$

9. a. Call the aggressive stock A and the defensive stock D. Beta is the sensitivity of the stock's return to the market return, i.e., the change in the stock return per unit change in the market return. Therefore, we compute each stock's beta by calculating the difference in its return across the two scenarios divided by the difference in the market return:

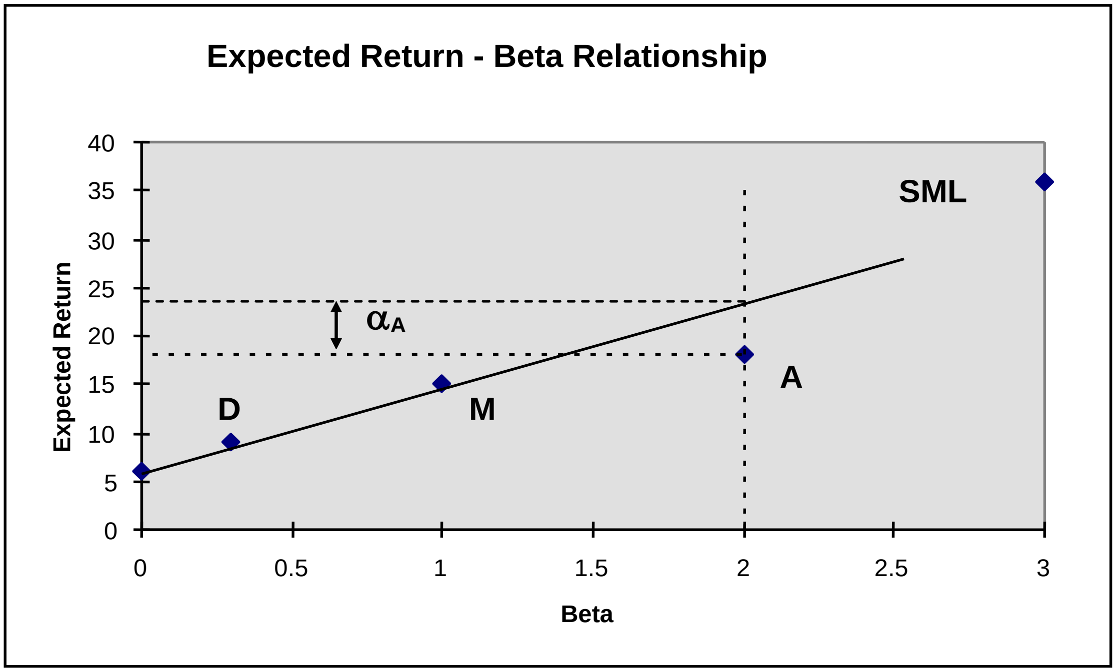
$$\beta_A = \frac{-.02 - .38}{.05 - .25} = 2.00 \qquad \beta_D = \frac{.06 - .12}{.05 - .25} = 0.30$$

b. With the two scenarios equally likely, the expected return is an average of the two possible outcomes:

$$E(r_A) = 0.5 \times (-.02 + .38) = .18 = 18\%$$

$$E(r_D) = 0.5 \times (.06 + .12) = .09 = 9\%$$

c. The SML is determined by the market expected return of [0.5 × (.25 + .05)] = 15%, with $\beta_M = 1$, and $r_f = 6\%$ (which has $\beta_f = 0$). See the following graph:



The equation for the security market line is:

$$E(r) = .06 + \beta \times (.15 - .06)$$

d. Based on its risk, the aggressive stock has a required expected return of:

$$E(r_A) = .06 + 2.0 \times (.15 - .06) = .24 = 24\%$$

The analyst's forecast of expected return is only 18%. Thus the stock's alpha is:

$$\alpha_A = \text{actually expected return} - \text{required return (given risk)}$$

$$= 18\% - 24\% = -6\%$$

Similarly, the required return for the defensive stock is:

$$E(r_D) = .06 + 0.3 \times (.15 - .06) = 8.7\%$$

The analyst's forecast of expected return for D is 9%, and hence, the stock has a positive alpha:

$$\alpha_D = \text{actually expected return} - \text{required return (given risk)}$$

$$= .09 - .087 = +0.003 = +0.3\%$$

The points for each stock plot on the graph as indicated above.

e. The hurdle rate is determined by the project beta (0.3), not the firm's beta. The correct discount rate is 8.7%, the fair rate of return for stock D.

10. Not possible. Portfolio A has a higher beta than Portfolio B, but the expected return for Portfolio A is lower than the expected return for Portfolio B. Thus, these two portfolios cannot exist in equilibrium.

11. Possible. If the CAPM is valid, the expected rate of return compensates only for systematic (market) risk, represented by beta, rather than for the standard deviation, which includes nonsystematic risk. Thus, Portfolio A's lower rate of return can be paired with a higher standard deviation, as long as A's beta is less than B's.

12. Not possible. The reward-to-variability ratio for Portfolio A is better than that of the market. This scenario is impossible according to the CAPM because the CAPM predicts that the market is the most efficient portfolio. Using the numbers supplied:

$$S_A = \frac{.16 - .10}{.12} = 0.5 \qquad S_M = \frac{.18 - .10}{.24} = 0.33$$

Portfolio A provides a better risk-reward tradeoff than the market portfolio.

13. Not possible. Portfolio A clearly dominates the market portfolio. Portfolio A has both a lower standard deviation and a higher expected return.

14. Not possible. The SML for this scenario is: $E(r) = 10 + \beta \times (18 - 10)$

Portfolios with beta equal to 1.5 have an expected return equal to:

$$E(r) = 10 + [1.5 \times (18 - 10)] = 22\%$$

The expected return for Portfolio A is 16%; that is, Portfolio A plots below the SML ($\alpha_A = -6\%$), and hence, is an overpriced portfolio. This is inconsistent with the CAPM.

15. Not possible. The SML is the same as in Problem 14. Here, Portfolio A's required return is: $.10 + (0.9 \times .08) = 17.2\%$

This is greater than 16%. Portfolio A is overpriced with a negative alpha: $\alpha_A = -1.2\%$

16. Possible. The CML is the same as in Problem 12. Portfolio A plots below the CML, as any asset is expected to. This scenario is not inconsistent with the CAPM.

17. Since the stock's beta is equal to 1.2, its expected rate of return is:

$.06 + [1.2 \times (.16 - .06)] = 18\%$

$$E(r) = \frac{D_1 + P_1 - P_0}{P_0} \rightarrow 0.18 = \frac{P_1 - \$50 + \$6}{\$50} \rightarrow P_1 = \$53$$

18. The series of $1,000 payments is a perpetuity. If beta is 0.5, the cash flow should be discounted at the rate:

$.06 + [0.5 \times (.16 - .06)] = .11 = 11\%$

PV = $1,000/0.11 = $9,090.91

If, however, beta is equal to 1, then the investment should yield 16%, and the price paid for the firm should be:

PV = $1,000/0.16 = $6,250

The difference, $2,840.91, is the amount you will overpay if you erroneously assume that beta is 0.5 rather than 1.

19. Using the SML: $.04 = .06 + \beta \times (.16 - .06) \Rightarrow \beta = -.02/.10 = -0.2$

20. $r_1 = 19\%$; $r_2 = 16\%$; $\beta_1 = 1.5$; $\beta_2 = 1$

a. To determine which investor was a better selector of individual stocks we look at abnormal return, which is the ex-post alpha; that is, the abnormal return is the difference between the actual return and that predicted by the SML. Without information about the parameters of this equation (risk-free rate and market rate of return) we cannot determine which investor was more accurate.

b. If $r_f = 6\%$ and $r_M = 14\%$, then (using the notation alpha for the abnormal return):

$\alpha_1 = .19 - [.06 + 1.5 \times (.14 - .06)] = .19 - .18 = 1\%$

$\alpha_2 = .16 - [.06 + 1 \times (.14 - .06)] = .16 - .14 = 2\%$

Here, the second investor has the larger abnormal return and thus appears to be the superior stock selector. By making better predictions, the second investor appears to have tilted his portfolio toward underpriced stocks.

c. If $r_f = 3\%$ and $r_M = 15\%$, then:

$\alpha_1 = .19 - [.03 + 1.5 \times (.15 - .03)] = .19 - .21 = -2\%$

$\alpha_2 = .16 - [.03 + 1 \times (.15 - .03)] = .16 - .15 = 1\%$

Here, not only does the second investor appear to be the superior stock selector, but the first investor's predictions appear valueless (or worse).

21. a. Since the market portfolio, by definition, has a beta of 1, its expected rate of return is 12%.

b. $\beta = 0$ means no systematic risk. Hence, the stock's expected rate of return in market equilibrium is the risk-free rate, 5%.

c. Using the SML, the *fair* expected rate of return for a stock with $\beta = -0.5$ is:

$$E(r) = 0.05 + [(-0.5) \times (0.12 - 0.05)] = 1.5\%$$

The *actually* expected rate of return, using the expected price and dividend for next year is:

$$E(r) = \frac{\$41 + \$3}{\$40} - 1 = 0.10 = 10\%$$

Because the actually expected return exceeds the fair return, the stock is underpriced.

22. In the zero-beta CAPM the zero-beta portfolio replaces the risk-free rate, and thus:E(r) = 8 + 0.6(17 – 8) = 13.4%

23. a. $E(r_P) = r_f + \beta_P \times [E(r_M) - r_f] = 5\% + 0.8\ (15\% - 5\%) = 13\%$

$\alpha = 14\% - 13\% = 1\%$

You should invest in this fund because alpha is positive.

b. The passive portfolio with the same beta as the fund should be invested 80% in the market-index portfolio and 20% in the money market account. For this portfolio:

$E(r_P) = (0.8 \times 15\%) + (0.2 \times 5\%) = 13\%$

$14\% - 13\% = 1\% = \alpha$

24. a. We would incorporate liquidity into the CCAPM in a manner analogous to the way in which liquidity is incorporated into the conventional CAPM. In the latter case, in addition to the market risk premium, expected return is also dependent on the expected cost of illiquidity and three liquidity-related betas which measure the sensitivity of: (1) the security's illiquidity to market illiquidity; (2) the security's return to market illiquidity; and, (3) the security's illiquidity to the market return. A similar approach can be used for the CCAPM, except that the liquidity betas would be measured relative to consumption growth rather than the usual market index.

b. As in part (a), non-traded assets would be incorporated into the CCAPM in a fashion similar to part (a). Replace the market portfolio with consumption growth. The issue of liquidity is more acute with non traded-assets such as privately-held businesses and labor income.

While ownership of a privately-held business is analogous to ownership of an illiquid stock, expect a greater degree of illiquidity for the typical private business. If the owner of a privately-held business is satisfied with the dividends paid out from the business, then the lack of liquidity is not an issue. If the owner seeks to realize income greater than the business can pay out, then selling ownership, in full or part, typically entails a substantial liquidity discount. The illiquidity correction should be treated as suggested in part (a).

The same general considerations apply to labor income, although it is probable that the lack of liquidity for labor income has an even greater impact on security market equilibrium values. Labor income has a major impact on portfolio decisions. While it is possible to borrow against labor income to some degree, and some of the risk associated with labor income can be ameliorated with insurance, it is plausible that the liquidity betas of consumption streams are quite significant, as the need to borrow against labor income is likely cyclical.

CFA PROBLEMS

1. a. Agree; Regan's conclusion is correct. By definition, the market portfolio lies on the capital market line (CML). Under the assumptions of capital market theory, all portfolios on the CML dominate, in a risk-return sense, portfolios that lie on the Markowitz efficient frontier because, given that leverage is allowed, the CML creates a portfolio possibility line that is higher than all points on the efficient frontier except for the market portfolio, which is Rainbow's portfolio. Because Eagle's portfolio lies on the Markowitz efficient frontier at a point other than the market portfolio, Rainbow's portfolio dominates Eagle's portfolio.

b. Nonsystematic risk is the unique risk of individual stocks in a portfolio that is diversified away by holding a well-diversified portfolio. Total risk is composed of systematic (market) risk and nonsystematic (firm-specific) risk.

Disagree; Wilson's remark is incorrect. Because both portfolios lie on the Markowitz efficient frontier, neither Eagle nor Rainbow has any nonsystematic risk. Therefore, nonsystematic risk does not explain the different expected returns. The determining factor is that Rainbow lies on the (straight) line (the CML) connecting the risk-free asset and the market portfolio (Rainbow), at the point of tangency to the Markowitz efficient frontier having the highest return per unit of risk. Wilson's remark is also countered by the fact that, since nonsystematic risk can be eliminated by diversification, the expected return for bearing nonsystematic is zero. This is a result of the fact that well-diversified investors bid up the price of every asset to the point where only systematic risk earns a positive return (nonsystematic risk earns no return).

2. $E(r) = r_f + \beta \times [E(r_M) - r_f]$

   Furhman Labs: E(r) = .05 + 1.5 × [.115 − .05] = 14.75%

   Garten Testing: E(r) = .05 + 0.8 × [.115 − .05] = 10.20%

   If the forecast rate of return is less than (greater than) the required rate of return, then the security is overvalued (undervalued).

   Furhman Labs: Forecast return – Required return = 13.25% − 14.75% = −1.50%

   Garten Testing: Forecast return – Required return = 11.25% − 10.20% = 1.05%

   Therefore, Furhman Labs is overvalued and Garten Testing is undervalued.

3. a.

4. d. From CAPM, the fair expected return = 8 + 1.25 × (15 – 8) = 16.75%
   Actually expected return = 17%

   α = 17 – 16.75 = 0.25%

5. d.

6. c.

7. d.

8. d. [You need to know the risk-free rate]

9. d. [You need to know the risk-free rate]

10. Under the CAPM, the only risk that investors are compensated for bearing is the risk that cannot be diversified away (systematic risk). Because systematic risk (measured by beta) is equal to 1.0 for both portfolios, an investor would expect the same rate of return from both portfolios A and B. Moreover, since both portfolios are well diversified, it doesn't matter if the specific risk of the individual securities is high or low. The firm-specific risk has been diversified away for both portfolios.

11. a. McKay should borrow funds and invest those funds proportionately in Murray's existing portfolio (i.e., buy more risky assets on margin). In addition to increased expected return, the alternative portfolio on the capital market line will also have increased risk, which is caused by the higher proportion of risky assets in the total portfolio.

b. McKay should substitute low beta stocks for high beta stocks in order to reduce the overall beta of York's portfolio. By reducing the overall portfolio beta, McKay will reduce the systematic risk of the portfolio, and therefore reduce its volatility relative to the market. The security market line (SML) suggests such action (i.e., moving down the SML), even though reducing beta may result in a slight loss of portfolio efficiency unless full diversification is maintained. York's primary objective, however, is not to maintain efficiency, but to reduce risk exposure; reducing portfolio beta meets that objective. Because York does not want to engage in borrowing or lending, McKay cannot reduce risk by selling equities and using the proceeds to buy risk-free assets (i.e., lending part of the portfolio).

12. a.

	Expected Return	Alpha
Stock X	5% + 0.8 × (14% – 5%) = 12.2%	14.0% – 12.2% = 1.8%
Stock Y	5% + 1.5 × (14% – 5%) = 18.5%	17.0% – 18.5% = –1.5%

b. i. Kay should recommend Stock X because of its positive alpha, compared to Stock Y, which has a negative alpha. In graphical terms, the expected return/risk profile for Stock X plots above the security market line (SML), while the profile for Stock Y plots below the SML. Also, depending on the individual risk preferences of Kay's clients, the lower beta for Stock X may have a beneficial effect on overall portfolio risk.

ii. Kay should recommend Stock Y because it has higher forecasted return and lower standard deviation than Stock X. The respective Sharpe ratios for Stocks X and Y and the market index are:

Stock X: (14% – 5%)/36% = 0.25

Stock Y: (17% – 5%)/25% = 0.48

Market index: (14% – 5%)/15% = 0.60

The market index has an even more attractive Sharpe ratio than either of the individual stocks, but, given the choice between Stock X and Stock Y, Stock Y is the superior alternative.

When a stock is held as a single stock portfolio, standard deviation is the relevant risk measure. For such a portfolio, beta as a risk measure is irrelevant.

Although holding a single asset is not a typically recommended investment strategy, some investors may hold what is essentially a single-asset portfolio when they hold the stock of their employer company. For such investors, the relevance of standard deviation versus beta is an important issue.