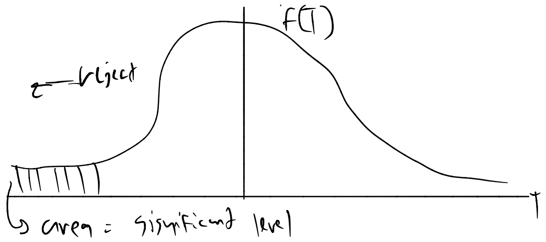
another possible hypothesis test (one-tailed alternative)

$H_0: \beta_1 = \beta_2 \rightarrow H_0: \beta_1 - \beta_2 = 0$

$H_a: \beta_1 < \beta_2 \rightarrow H_a: \beta_1 - \beta_2 < 0$

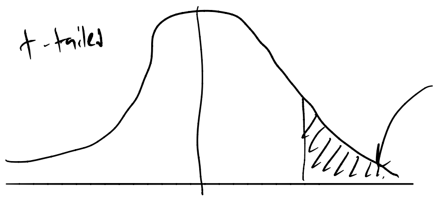
- it is assumed that β_1 would not be more than β_2 (returns to a 2-year college would never be more than return to university education)



$$t = \frac{(\hat\beta_1 - \hat\beta_2) - 0}{s.e.(\hat\beta_1 - \hat\beta_2)}$$

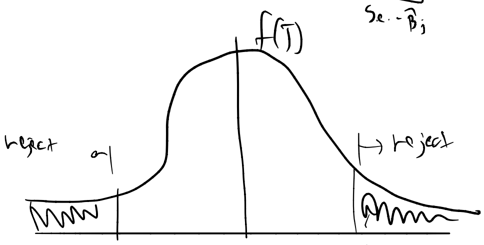
5 Computing p-Values for t-Tests

- What is the significance level given the computed t-statistics?



This shaded area in the rejection region is the p-value

$$t = \frac{\hat\beta_j - \beta_j}{se.\ \hat\beta_j}$$



t = computed $t = \frac{\hat\beta_j - \beta_j}{se.\ \hat\beta_j}$

- p-value : $P(|T| > |t|)$

T = t-distributed random variable with d.f. = n-k-1

t = computed t-statistic

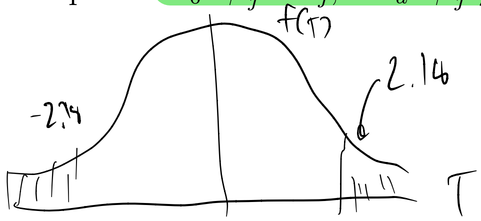
→ p-value = prob that a random t value will be greater than H_0 test

Example 1: $H_0 : \beta_j \geq 0, \quad H_a : \beta_j < 0, \quad$ d.f.$= 140$.

suppose the calculated $t_{\hat{\beta}_j} = -2.75$

- From the z-table, the value -2.75 corresponds to area = 0.003.
- Thus, p-value = 0.003.
- Would we reject H_0 if we use the significance level = 5%?

Example 2: $H_0 : \beta_j = a_j, \quad H_a : \beta_j \neq a_j, \quad$ d.f.$= 18$.



suppose the calculated $t_{\hat{\beta}_j} = -2.18$

- From the t-table, the value -2.18 corresponds to area = (0.02, 0.05).
- Thus, p-value = between 0.02 and 0.05.
- Would we reject H_0 if we use the significance level = 5%?

  Yes, reject H_0 because the area is less than 0.05 or p-value < 0.05

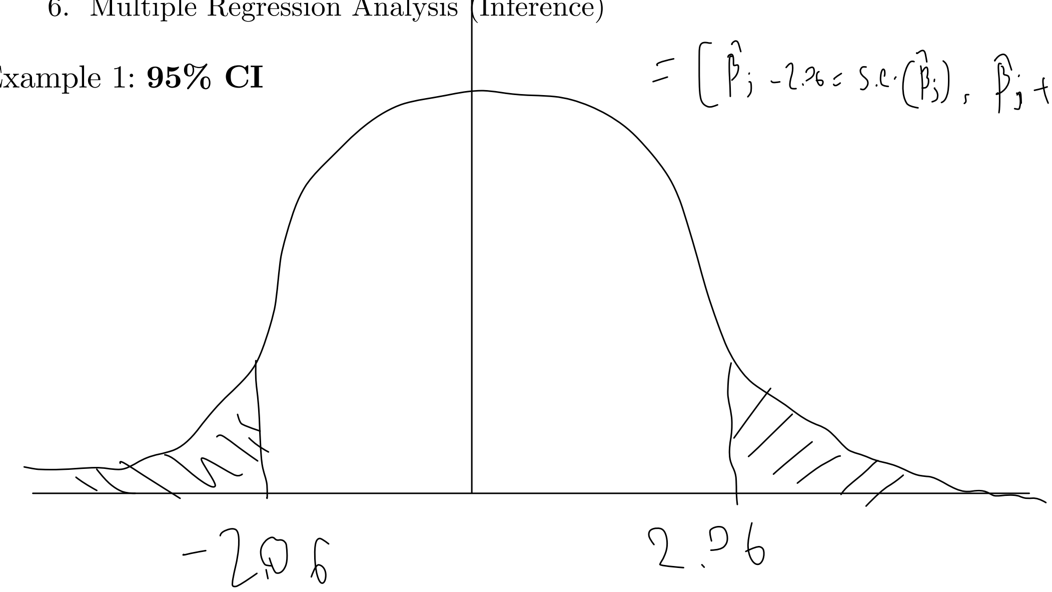
6 Confidence Intervals (CI)

- Confidence Intervales for the POPULATION PARAMETER (β_j)
- A 95% CI of β_j is given by

The 95% CI for $\hat{\beta}_j$

$= [\hat{\beta}_j - 2.26 \times s.e.(\hat{\beta}_j),\ \hat{\beta}_j + 2.26 \times s.e.(\hat{\beta}_j)]$

Example 1: **95% CI**

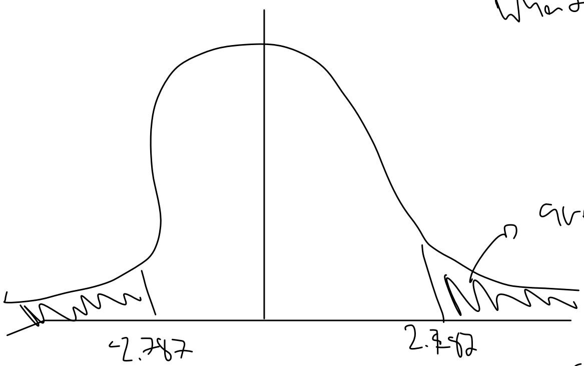


Example 2: **99% CI**

d.f. = 24

what percentile 99.5

area = $\frac{1\%}{2}$ = 0.005



The 99% CI for $\hat{\beta}_j = [\hat{\beta}_j - 2.787 \times s.e.\hat{\beta}_j,\ \hat{\beta}_j + 2.747 \times se\ \hat{\beta}_j]$

7 Testing Multiple Linear Restrictions: The F-test

Suppose the model is specified by

$$
\begin{aligned}
y &= \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + u \\
H_0 &: \beta_1 = 0 \text{ and } \beta_2 = 0 \\
H_1 &: H_0 \text{ is not true}
\end{aligned}
$$

→ want to test if x_1, x_2 both have no impact on y.

We can use the F-test to test this type of "multiple hypotheses".

1. Our full model is called the "unrestricted" model (ur). Suppose it can be expressed as:

   $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + u$ is true → reject H_0

   $$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + ... + \beta_k x_k + u$$

2. The model which takes out x (which we think its associated $\beta = 0$) is called the restricted model (r).

$y = \beta_0 + \beta_1 x_1 + u$ is true → do not reject H_0

Suppose there are "q" number of β that we would like to perform

→ joint-test of $= 0$

e.g. in this model $q = 2$

$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots \beta_{2-q} x_{k-q} + u$$

$$H_0: \beta_{k-q+1} = \beta_{k-q+2} = \dots = \beta_k = 0$$

H_a : H_0 is not true

$$F = \frac{\dfrac{(SSR_r - SSR_{ur})}{q}}{\dfrac{SSR_{ur}}{(n-k-1)}}$$

→ this term always positive and $SSR_{ur} < SSR_r$

df of the "ur" model

3. Some useful facts

① $R^2_{ur} > R^2_r$ because any additional x could ↑ R^2

② By including more x, the model is certainly better explained

However, we would like to reject H_0 if the inclusion of extra variables doesn't improve the model enough.

4. Other ways to calculate the F-statistics:

from R^2 s $1 - \frac{SSR}{SST}$

we have $F = \dfrac{(R^2_{ur} - R^2_r)/q}{(1-R^2_{ur})/(n-k-1)}$

q: # of β that are set to "0"

n: # obs

k: # of slope β

intercept

Example: Suppose we are interested in understanding the determinant of a baseball player's salary.

salary	= season salary
years	= years in major leagues
gamesyr	= games per year in the league
bavg	= career batting average
hrunsyr	= homeruns per year
rbisyr	= runs batted in per year

- the unrestricted model (ur) is defined by ur model

y x

```
. regress log_salary years gamesyr bavg hrunsyr rbisyr

      Source |       SS       df       MS              Number of obs =     353
-------------+------------------------------           F(  5,   347) =  117.06
SSE    Model |  308.989208     5  61.7978416           Prob > F      =  0.0000
SSR Residual |  183.186327   347  .527914487           R-squared     =  0.6278
-------------+------------------------------           Adj R-squared =  0.6224
SST    Total |  492.175535   352  1.39822595           Root MSE      =  .72658

------------------------------------------------------------------------------
  log_salary |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       years |   .0688626   .0121145     5.68   0.000     .0450355    .0926898
     gamesyr |   .0125521   .0026468     4.74   0.000     .0073464    .0177578
        bavg |   .0009786   .0011035     0.89   0.376    -.0011918     .003149
     hrunsyr |   .0144295    .016057     0.90   0.369    -.0171518    .0460107
      rbisyr |   .0107657    .007175     1.50   0.134    -.0033462    .0248776
       _cons |   11.19242   .2888229    38.75   0.000     10.62435    11.76048
------------------------------------------------------------------------------
```

q = 3

When considering each of the performance x one-by-one none of them has significant impact at 5%.

- the restricted model (r) is defined by

y x

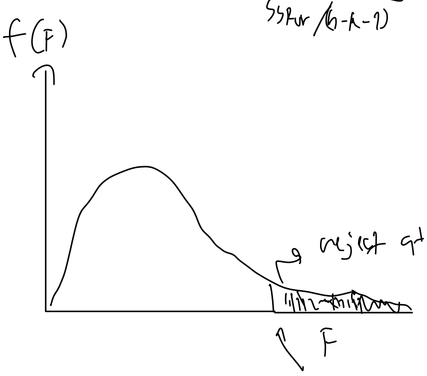
```
. regress log_salary years gamesyr

      Source |       SS       df       MS              Number of obs =     353
-------------+------------------------------           F(  2,   350) =  259.32
SSE    Model |  293.864058     2  146.932029           Prob > F      =  0.0000
SSR Residual |  198.311477   350  .566604221           R-squared     =  0.5971
-------------+------------------------------           Adj R-squared =  0.5948
SST    Total |  492.175535   352  1.39822595           Root MSE      =  .75273

------------------------------------------------------------------------------
  log_salary |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       years |    .071318    .012505     5.70   0.000     .0467236    .0959124
     gamesyr |   .0201745   .0013429    15.02   0.000     .0175334    .0228156
       _cons |    11.2238    .108312   103.62   0.000     11.01078    11.43683
------------------------------------------------------------------------------
```

Now, our H_0 and H_a becomes

$$F = \frac{(SSR_r - SSR_{ur})/q}{SSR_{ur}/(n-k-1)} = \frac{(198.311 - 183.186)/3}{(183.186)/(353-5-1)} \approx 9.55$$



reject at 5%

let's use 5% level of sig

Since $F \approx 9.55 > 2.6$ we reject H_0

8 How the Hypothesis Testing is done in Practice

1. Check the values of $t - statistic$ reported by the statistical software (i.e. STATA, SPSS, SAS)

   $\Rightarrow$ These $t - statistics$ are to test $H_0 : \beta_i = 0$

   $\Rightarrow$ If the d.f. > 30, then when $t \ > 1.96$, we can reject H_0

   $\Rightarrow$ **When** $t \ >$ **1.96**, we can say that β_i is **statistically significant** at 5% level. (value of $\beta_i \neq 0$)

   $\Rightarrow$ **When** $t \ <$ **1.96** we can say that β_i is **not statistically significant** at 5% level.

   $\Rightarrow$ If $t \ < 1.96$ we can drop x_i from the model

   $\Rightarrow$ After we drop x_i, we estimate the new regression function and obtain a new set of $\hat{\beta}$.

2. We can also perform other hypothesis testings of interest.

   e.g. $H_0 : \beta_i = \beta_j$

   or $\quad H_0 : \beta_i = 5 \quad$ etc.

   or perform an F-test for testing multiple linear restrictions

3. Usually, in economics, the estimation results are reported using this form

Dependent Variable: log(*salary*)			
Independent Variables	**(1)**	**(2)**	**(3)**
log(*sales*)	.224 (.027)	.158 (.040)	.188 (.040)
log(*mktval*)	——	.112 (.050)	.100 (.049)
profmarg	——	−.0023 (.0022)	−.0022 (.0021)
ceoten	——	——	.0171 (.0055)
comten	——	——	−.0092 (.0033)
intercept	4.94 (0.20)	4.62 0.25)	4.57 (0.25)
Observations *R*-squared	177 .281	177 .304	177 .353

Multiple Regression Analysis : Further Issues

1 Data scaling on OLS statistics

When we change the unit of measurment of a variable, the value of estimators would chance accordingly. For example

$$\widehat{bweght} = \widehat{\beta}_0 + \widehat{\beta}_1 cigs + \widehat{\beta}_2 fa\min c,$$

where
$bwght =$ child birth weight, in grams.
$cigs =$ number of cigarettes smoked by the mother while pregnant, per day.
$fa\min c =$ annual family income, in thousands of dollars.

What if we use bweight in kilograms?

$$\widehat{bweight}_{Kg} = \frac{\widehat{bweight}_g}{1,000} = \frac{\hat{\beta}_0}{1,000} + \frac{\hat{\beta}_1}{1,000} cigs + \frac{\hat{\beta}_2}{1,000} faminc$$

$$= \hat{\alpha}_0 + \hat{\alpha}_1 cigs + \hat{\alpha}_2 faminc$$

$$\rightarrow \hat{\alpha}_0 = \frac{\hat{\beta}_0}{1,000},\ \hat{\alpha}_1 = \frac{\hat{\beta}_1}{1000},\ \hat{\alpha}_2 = \frac{\hat{\beta}_2}{1000}$$

What if we use faminc in USD (instead of 1000 USD)

$$bweight_g = \hat{\beta}_0 + \hat{\beta}_1 cigs + \hat{\beta}_2 faminc_{USD}$$

$$\rightarrow \hat{\theta}_2 = \frac{\hat{\beta}_2}{1,000}$$

2 More on functional forms

- Logarithmic Functional Form

$$\log(y) = \beta_0 + \beta_1 \log(x_1) + \beta_2 x_2 + u$$

- $\beta_1 = \dfrac{d\log(y)}{d\log(x)} = \dfrac{\frac{1}{y}dy}{\frac{1}{x_1}dx_1} = \dfrac{\frac{1}{y}\Delta y}{\frac{1}{x_1}\Delta x_1} = \dfrac{100\times\frac{1}{y}\Delta y}{100\times\frac{1}{x_1}\Delta x_1} = \dfrac{\%y}{\%x}$

- $\beta_2 = \dfrac{d\log(y)}{dx_2} = \dfrac{\frac{1}{y}dy}{dx} = \dfrac{\frac{1}{y}\Delta y}{\Delta x_2}$

if we want the upper term to be % change, then

$$100\beta_2 = \dfrac{100\frac{1}{y}\Delta y}{\Delta x_2} = \dfrac{\%\Delta y}{\Delta x_2}$$

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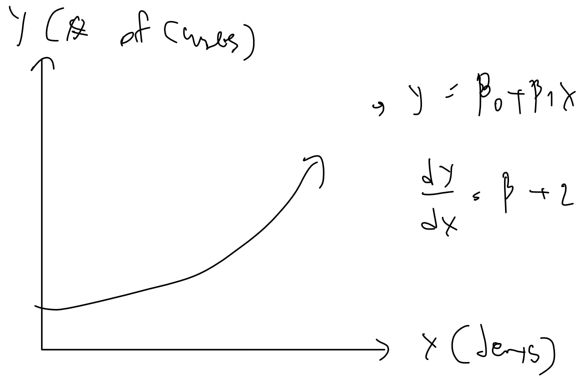
- Models with Quadratics



$\Rightarrow y = \beta_0 + \beta_1 x + \beta_2 x^2 + u$

$\dfrac{dy}{dx} = \beta + 2\beta_2 x$

Decreasing returns

$\Rightarrow Y = \beta_0 + \beta_1 X + \beta_2 X^2 + u$

$\dfrac{dy}{dx} = \beta_1 + 2\beta_2 x$

$mc = 10$

$\pi = (p - mc)q$

$\pi = (100 - q - 10)\, q$

Example : Effects of Pullution on Housing Prices

$$\log(price) = \beta_0 + \beta_1 \log(nox) + \beta_2 \log(dist) + \beta_3 rooms + \beta_4 room^2 + \beta_5 stratio + u$$

$F_{0.1} = 0.9022$

where

price = housing price
nox = level of pollution
dist = distance from downtown
rooms = number of rooms
stratio = average student per teacher ratio

The estimation result is given by

regress lprice lnox dist rooms rooms_sq stratio

Source	SS	df	MS
Model	51.4933152	5	10.298663
Residual	33.0889098	500	.06617782
Total	84.582225	505	.167489554

Number of obs	=	506
F(5, 500)	=	155.62
Prob > F	=	0.0000
R-squared	=	0.6088
Adj R-squared	=	0.6049
Root MSE	=	.25725

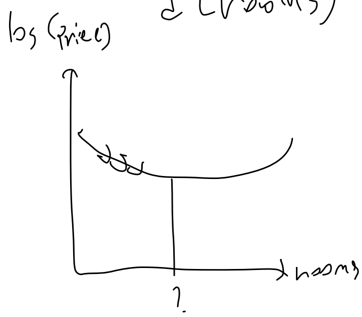
log(price)

lprice	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
lnox	-.9767545	.0995938	-9.81	0.000	-1.172429	-.7810806
dist	-.0321972	.0094013	-3.42	0.001	-.050668	-.0137264
rooms	-.5528032	.1612965	-3.43	0.001	-.8697056	-.2359007
rooms_sq	.0624697	.0124867	5.00	0.000	.0379368	.0870025
stratio	-.0486667	.0058131	-8.37	0.000	-.0600879	-.0372455
_cons	13.59154	.5650901	24.05	0.000	12.4813	14.70178

log(nox

Consider the effect of "room"

$$\frac{d\log(price)}{d(rooms)} = \beta_3 + 2\beta_4 rooms = -0.553 + 2(0.062)\,rooms$$



$$0 = -0.553 + 2(0.062)\,rooms$$

$rooms = 4.4$

answer: at 4.4 rooms or more.

at ~5 rooms or more

What wold be the % change in price when the number of room increases from 5 to 6?

$$\frac{d\log(price)}{d\,rooms} = -0.553 + 2(0.062)\cdot rooms$$

$$\frac{100\cdot\frac{1}{price}\,dprice}{d\,rooms} = 100 \times (0.067) = 6.7\% \text{ increase}$$

What about % in price when # rooms increases from 5 to 7

$$\%\Delta price = 100(-0.553 + 2(0.062)\cdot 6) = 19.7\%$$

3 Models with Interaction Terms

Consider

$$price = \beta_0 + \beta_1 sqrft + \beta_2 bdrms + \beta_3 sqrft \times bdrms + \beta_4 bthrms + u$$

where
$price$ = housing price
$sqrft$ = house size (square feet)
$bdrms$ = number of bedrooms
$bthrms$ = number of bathrooms

$$\frac{d\, price}{d\, bdrms} = \beta_2 + \beta_3 sqrft$$

if $\beta_3 > 0$ then, an additional bedroom would increase price more for a larger house!

4 More on the Goodness-of-Fit and Selection of Regressors

- Adding more regressors ALWAYS improve fit

Using adjusted R-squared to choose between non-nested models (one model is not a subset of another).

Consider Model 1

$$\begin{aligned}\widehat{salary} &= \begin{array}{lll} 830.63 & +0.0163 sales & +19.63 roe \\ (223.90) & (0.0089) & (11.08) \end{array} \\ n &= 209, \quad R^2 = 0.029, \ \bar{R}^2 = 0.020\end{aligned}$$

Consider Model 2

$$\begin{aligned}\log(\widehat{salary}) &= \begin{array}{lll} 4.36 & +0.2751 \log(sales) & +0.0179 roe \\ (0.29) & (0.033) & (0.004) \end{array} \\ n &= 209, \quad R^2 = 0.282, \ \bar{R}^2 = 0.275\end{aligned}$$

Multiple Regression Analysis with Qualitative Information:

1 Outline

- Describing qualitative information
- Using a single dummy independent variable
- Using dummy variables for multiple categories
- Interactions involving dummy variables
- A binary dependent variable (Y variable): The linear probability model

2 Describing Qualitative Information

- "Female" and "Married" are qualitative variable.
- We arbitarily assign a dummy variable to decribe them.

$$famale = \begin{cases} 1 & \text{if female} \\ 0 & \text{otherwise (or if male)} \end{cases}$$
$$married = \begin{cases} 1 & \text{if married} \\ 0 & \text{otherwise (of if single)} \end{cases}$$

TABLE 7.1

A Partial Listing of the Data in WAGE1.RAW

person	*wage*	*educ*	*exper*	*female*	*married*
1	3.10	11	2	1	0
2	3.24	12	22	1	1
3	3.00	11	2	0	0
4	6.00	8	44	0	1
5	5.30	12	7	0	1
. . .	. . .	. . .	. . .	. . .	. . .
525	11.56	16	5	0	1
526	3.50	14	5	1	0

3 Models with a single dummy independent variable

Consider

$$wage = \beta_0 + \delta_0 female + \beta_1 educ + u.$$

where

$$famale = \begin{cases} 1 & \text{if female} \\ 0 & \text{otherwise (or if male)} \end{cases}$$

In this case, the δ_0 notation is used to highlight the interpretation of the parameters multiplying dummy variables. In other cases, we can use any notation that is the most convenient.

① $E(wage|female, educ) = E(\beta_0 + \delta_0 female + \beta_1 educ + u | female, educ)$

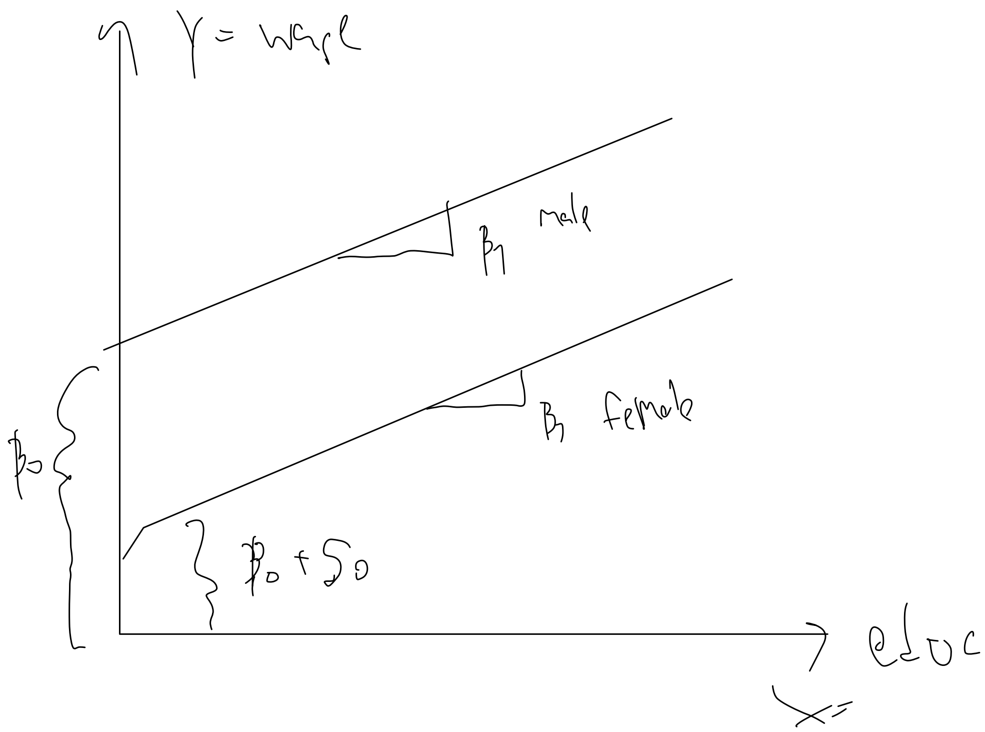
$= \beta_0 + \delta_0 female + \beta_1 educ + E(u|female, educ)$

$= \beta_0 + \delta_0 female + \beta_1 educ$

Thus

② ♀ : $E(wage|female=1, educ) = \beta_0 + \delta_0(1) + \beta_1 educ = \beta_0 + \delta_0 + \beta_1 educ$

♂ : $E(wage|female=0, educ)$ $\beta_0 + \delta_0(0) + \beta_1 educ = \beta_0 + \beta_1 educ$



4 It is not possible to include all of the dummy alternatives in the same model

- If we include all alternatives of a dummy variable in the same model, we will face the "perfect collinearity" problem.

$wage = \beta_0 X_0 + \delta_0 female + \beta_1 educ + \delta_1 male + u$

$(x_1) \quad (x_2) \quad (x_3)$

Intercept x_1

For example:

$$1 = female + male$$
$$female = male + 1$$

id	female	male
1	1	0
2	1	0
3	0	1
4	0	1
⋮	0	1
⋮	0	1
	1	0
99		0

or

$$1 = winter + spring + summer + fall$$
$$winter = 1 - spring - summer - fall$$

$$winter = 1 - spring - summer - fall$$

$winter = \begin{cases} 1 & \text{if winter} \\ 0 & \text{if otherwise} \end{cases}$

$spring = \begin{cases} 1 & \text{if spring} \\ 0 & \text{otherwise} \end{cases}$

etc.

id	winter	spring	summer	fall
1	1	0	0	0
2	0	0	1	0
3	1	0	0	0
⋮	0	1	0	0
	0	⋮	0	0

- At least one alternative has to be dropped. We treat the dropped alternative as the "BASE GROUP" or "BASELINE" or "BENCHMARK GROUP".

```
. regress lwage female male married educ exper
note: male omitted because of collinearity

      Source |       SS           df       MS      Number of obs   =       526
-------------+----------------------------------   F(4, 521)       =     75.27
       Model |  54.3265253         4  13.5816313   Prob > F        =    0.0000
    Residual |  94.0032262       521  .180428457   R-squared       =    0.3663
-------------+----------------------------------   Adj R-squared   =    0.3614
       Total |  148.329751       525  .28253286    Root MSE        =    .42477

------------------------------------------------------------------------------
       lwage |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      female |  -.3251146   .0377061    -8.62   0.000    -.3991892    -.25104
        male |          0  (omitted)
     married |   .1380145   .0411197     3.36   0.001     .0572338    .2187953
        educ |   .0872644   .0071554    12.20   0.000     .0732075    .1013213
       exper |   .0076213   .0015314     4.98   0.000     .0046129    .0106297
       _cons |   .4690918   .1040575     4.51   0.000      .264668    .6735156
------------------------------------------------------------------------------
```

5 Using dummy variables for multiple categories

Case 1 We can use many dummy variables in the same model

Consider a model which includes 2 dummy variables– $female$ and $married$.

$$\log(wage) = \beta_0 + \delta_0 female + \delta_1 married + \beta_1 educ + \beta_2 \exp er + \beta_3 \exp er^2 + \beta_4 tenure + \beta_5 tenure^2 + u.$$

regress lwage female married educ exper expersq tenure tenursq

Source	SS	df	MS
Model	65.6482326	7	9.37831895
Residual	82.6815188	518	.159616832
Total	148.329751	525	.28253286

Number of obs =	526
F(7, 518) =	58.76
Prob > F =	0.0000
R-squared =	0.4426
Adj R-squared =	0.4351
Root MSE =	.39952

lwage	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
female	-.2901838	.0361121	-8.04	0.000	-.3611279	-.2192396
married	.0529219	.0407561	1.30	0.195	-.0271456	.1329894
educ	.0791547	.0068003	11.64	0.000	.0657952	.0925143
exper	.0269535	.0053258	5.06	0.000	.0164907	.0374163
expersq	-.0005399	.0001122	-4.81	0.000	-.0007603	-.0003196
tenure	.0312962	.0068482	4.57	0.000	.0178426	.0447499
tenursq	-.0005744	.0002347	-2.45	0.015	-.0010355	-.0001134
_cons	.4177837	.0988662	4.23	0.000	.2235557	.6120116

Comments:

$$\frac{d\log(wage)}{d\,female} = \frac{\frac{1}{wage}\,d\,wage}{d\,female} = -0.29$$

female workers are expected to earn less than males workers by 29.02%, holding other factors the same.

$$100\,\frac{\frac{1}{wage}\,d\,wage}{d\,female} = 100(-0.29)$$

$$\frac{\%\,\Delta\,wage}{d\,female} = 29.02\%$$

Consider a model which includes dummy variables for each gender/marital status combination– $marrmale$, $marrfem$ and $singfem$.

$$\log(wage) = \beta_0 + \delta_0 marrmale + \delta_1 marrfem + \delta_3 singfem + \beta_1 educ + \beta_2 exper + \beta_3 exper^2 + \beta_4 tenure + \beta_5 tenure^2 + u. \quad (8.1)$$

regress lwage marrmale marrfem singfem educ exper expersq tenure tenursq

Source	SS	df	MS
Model	68.3617623	8	8.54522029
Residual	79.9679891	517	.154676961
Total	148.329751	525	.28253286

Number of obs =	526
F(8, 517) =	55.25
Prob > F =	0.0000
R-squared =	0.4609
Adj R-squared =	0.4525
Root MSE =	.39329

lwage	Coef.	Std. Err.	t	P>|t|	[95% Conf. Interval]	
marrmale	.2126757	.0553572	3.84	0.000	.103923	.3214284
marrfem	-.1982676	.0578355	-3.43	0.001	-.311889	-.0846462
singfem	-.1103502	.0557421	-1.98	0.048	-.219859	-.0008414
educ	.0789103	.0066945	11.79	0.000	.0657585	.092062
exper	.0268006	.0052428	5.11	0.000	.0165007	.0371005
expersq	-.0005352	.0001104	-4.85	0.000	-.0007522	-.0003183
tenure	.0290875	.006762	4.30	0.000	.0158031	.0423719
tenursq	-.0005331	.0002312	-2.31	0.022	-.0009874	-.0000789
_cons	.3213781	.100009	3.21	0.001	.1249041	.5178521

Comments: This regression is not the same as the previous one. It uses "single male" as the base group.

- δ_0 measure the expected diff in wage of married compared with single males, holding other factors constant
- δ_1 measures the expected diff in wage of married
- δ_2 ~> same reasoning

Case 2 We can use dummy variables to represent multiple categories of a variable Consider the relationship between law school rankings and starting salaries

$$\log(salary) = \beta_0 + \delta_0 top10 + \delta_1 r11\_25 + \delta_3 r26\_40 + \delta_4 r41\_60 + \beta_1 LSAT + \beta_2 GPA + \beta_3 \log(libvol) + \beta_4 \log(cost) + u.$$

where $top10$, $r11\_25$, $r26\_40$, $r41\_60$ would be equal to 1 when the variable $rank$ falls into the appropriate range.
** Rank below 60 would be the base case.

```
. regress lsalary top10 r11_25 r26_40 r41_60 LSAT GPA llibvol lcost

      Source |       SS       df       MS              Number of obs =     136
-------------+------------------------------           F(  8,   127) =  120.15
       Model |  9.16538532     8  1.14567316           Prob > F      =  0.0000
    Residual |   1.2109665   127  .009535169           R-squared     =  0.8833
-------------+------------------------------           Adj R-squared =  0.8759
       Total |  10.3763518   135  .076861865           Root MSE      =  .09765

------------------------------------------------------------------------------
     lsalary |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       top10 |   .5393428    .053542    10.07   0.000     .4333927    .6452928
      r11_25 |   .4716199   .0390921    12.06   0.000     .3942637     .548976
      r26_40 |   .2790977   .0346972     8.04   0.000     .2104383    .3477571
      r41_60 |    .182382   .0283098     6.44   0.000      .126362     .238402
        LSAT |   .0060482   .0034919     1.73   0.086    -.0008616     .012958
         GPA |   .1305893   .0818678     1.60   0.113    -.0314122    .2925908
     llibvol |   .0725522   .0289213     2.51   0.013     .0153221    .1297824
       lcost |   .0249169   .0283224     0.88   0.381     -.031128    .0809619
       _cons |   8.363103   .4457314    18.76   0.000     7.481081    9.245125
------------------------------------------------------------------------------
```

Comments:

Rank	top 10	$r_{11}-25$	$r_{26}-40$	etc.
1	1	0	0	
2	1	0	0	
3	1	0	0	
⋮	1	0	0	
10	1	0	0	
11	0	1	0	
12	0	1	0	
⋮		1	0	
25		1	0	
26		0	1	
⋮		0	1	
40		0	1	

1) δ_0 measures the difference in expected log(salary) of a law-school graduate from a top 10 university compared to expected log(salary) of those who graduated from the school)