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EE325 Introductory Econometrics, Semester 1/2019 (Section 046402)

Due Date: Thursday 27th February 2020 by 09.30 via Assignment Submission in Moodle.

Instruction: Do all questions with your own handwriting and your own attempt.

Use 4 decimal places for numerical answers

1. In Table 1. X_i is total econometrics exam point (total points are 100) and Y_i is GPA of each BE student.

Table 1

Student	Y_i	X_i
1	2.8	63
2	3.4	72
3	3.0	78
4	3.5	81
5	3.6	87
6	3.0	75
7	2.7	75
8	3.7	90

1.1 Now consider the two-variable model $Y_i = \beta_1 + \beta_2 X_i + u_i,\ u_i \sim NIID(0, \sigma^2)$
Use OLS to find the estimator of $\beta_1 and \beta_2$. Interpret the regression.

1.2 Find $\hat{Y}_i$ and $\hat{u}_i$ and show that $\sum_{i=1}^{n} \hat{u}_i \approx 0$

1.3 Find $var(\hat{u}_i), var(\hat{\beta}_1), and\ var(\hat{\beta}_2)$

①

Student	Y_i	X_i	Y_iX_i	X_i^2
1	2.8	63	176.4	3969
2	3.4	72	244.8	5184
3	3.0	78	234	6084
4	3.5	81	283.5	6561
5	3.6	87	313.2	7569
6	3.0	75	225	5625
7	2.7	75	202.5	5625
8	3.7	90	333	8100
Sum	25.7	621	2012.4	48717
Mean	3.2125	77.625	251.55	6089.625

1.1

$$\hat{\beta}_2 = \frac{\Sigma x_i y_i}{\Sigma x_i^2} = \frac{n\Sigma X_iY_i - \Sigma X_i \Sigma Y_i}{n\Sigma X_i^2 - (\Sigma X_i)^2}$$

$$= \frac{8(2012.4) - (621)(25.7)}{8(48717) - (621)^2}$$

$$= \frac{16099.2 - 15959.7}{389736 - 385641}$$

$$= \frac{139.5}{4095} = 0.0341$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$$

$$= 3.2125 - (0.0341)(77.625)$$

$$= 3.2125 - 2.6470$$

$$= 0.5655$$

1.2

Student	$\hat{Y}_i$	$\hat{u}_i$
1	2.7106	0.0894
2	3.0202	0.3798
3	3.2266	-0.2266
4	3.3298	0.1702
5	3.5362	0.0638
6	3.1234	-0.1234
7	3.1234	-0.4234
8	3.6394	0.0606

$$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$$

$$\hat{Y}_i = 0.5655 + (0.0341) X_i$$

$$\hat{Y}_1 = 0.5655 + (0.0341)63 = 0.5655 + 2.1483 = 2.7138$$

$$\hat{Y}_2 = 0.5655 + (0.0341)72 = 0.5655 + 2.4552 = 3.0207$$

$$\hat{Y}_3 = 0.5655 + (0.0341)78 = 0.5655 + 2.6598 = 3.2253$$

$$\hat{Y}_4 = 0.5655 + (0.0341)81 = 0.5655 + 2.7621 = 3.3276$$

$$\hat{Y}_5 = 0.5655 + (0.0341)87 = 0.5655 + 2.9667 = 3.5322$$

$$\hat{Y}_6 = 0.5655 + (0.0341)75 = 0.5655 + 2.5575 = 3.123$$

$$\hat{Y}_7 = 0.5655 + (0.0341)75 = 0.5655 + 2.5575 = 3.123$$

$$\hat{Y}_8 = 0.5655 + (0.0341)90 = 0.5655 + 3.069 = 3.6345$$

$$Y_i = \hat{Y}_i + \hat{u}_i$$

$$\hat{u}_i = Y_i - \hat{Y}_i$$

$$\hat{u}_1 = 2.8 - 2.7138 = 0.0862$$

$$\hat{u}_2 = 3.4 - 3.0207 = 0.3793$$

$$\hat{u}_3 = 3.0 - 3.2253 = -0.2253$$

$$\hat{u}_4 = 3.5 - 3.3276 = 0.1724$$

$$\hat{u}_5 = 3.6 - 3.5322 = 0.0678$$

$$\hat{u}_6 = 3.0 - 3.123 = -0.123$$

$$\hat{u}_7 = 2.7 - 3.123 = -0.423$$

$$\hat{u}_8 = 3.7 - 3.6345 = 0.0655$$

$$\sum_{i=1}^{n} \hat{u}_i = 0.0862 + 0.3793 + (-0.2253) + 0.1724 + 0.0678 + (-0.123) + (-0.423) + 0.0655$$

$$= -0.0001$$

$$\approx 0$$

1.3 mean of $\hat{u}_i = \bar{\hat{u}}_i = \frac{\sum_{i=1}^{n} \hat{u}_i}{n} = 0$

$$\text{var}(\hat{u}_i) = \sigma^2 = \frac{\sum_{i=1}^{n}(\hat{u}_i - \bar{\hat{u}}_i)^2}{n-2} = \frac{\sum_{i=1}^{n}(\hat{u}_i)^2}{n-2}$$

$$= \frac{0.0862^2 + 0.3793^2 + (-0.2253)^2 + 0.1724^2 + 0.0678^2 + (-0.123)^2 + (-0.423)^2 + 0.0655^2}{8-2}$$

$$= \frac{0.0074 + 0.1439 + 0.0508 + 0.0297 + 0.0046 + 0.0151 + 0.1789 + 0.0043}{6}$$

$$= \frac{0.4347}{6}$$

$$= 0.0725$$

Student	$x_i = X_i - \bar{X}$	$y_i = Y_i - \bar{Y}$	x_i^2
1	-14.625	-0.4125	213.890625
2	-5.625	0.1875	31.640625
3	0.375	-0.2125	0.140625
4	3.375	0.2875	11.390625
5	9.375	0.3875	87.890625
6	-2.625	-0.2125	6.890625
7	-2.625	-0.5125	6.890625
8	12.375	0.4875	153.140625
Sum	0	0	511.875
Mean	0	0	63.984375

$$\text{var}(\hat{\beta}_1) = \frac{\sum X_i^2}{n \sum x_i^2} \sigma^2$$

$$= \frac{48717}{8(511.875)} (0.0725)$$

$$= 0.8625$$

$$\text{var}(\hat{\beta}_2) = \frac{\sigma^2}{\sum x_i^2}$$

$$= \frac{0.0725}{511.875}$$

$$= 0.000142$$

②

i	X_i	Y_i	X_iY_i	X_i^2
1	10	0	0	100
2	12	2	24	144
3	14	5	70	196
4	16	6	96	256
5	18	7	126	324
6	22	10	220	484
7	24	10	240	576
8	26	15	390	676
9	28	16	448	784
10	30	20	600	900
Sum	200	91	2214	4440
Mean	20	9.1	221.4	444

2.1 $\hat{\beta}_2 = \frac{\Sigma x_i y_i}{\Sigma x_i^2} = \frac{n\Sigma X_iY_i - \Sigma X_i \Sigma Y_i}{n\Sigma X_i^2 - (\Sigma X_i)^2}$

$= \frac{10(2214) - (200)(91)}{10(4440) - (200)^2}$

$= \frac{22140 - 18200}{44400 - 40000}$

$= \frac{3940}{4400} = 0.89\dot{5}\dot{4} \approx 0.8955$

$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$

$= 9.1 - (0.8955)(20)$

$= 9.1 - 17.91$

$= -8.81$

$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$

$\hat{Y}_i = -8.81 + (0.8955) X_i$

$\hat{Y}_1 = -8.81 + (0.8955)10 = -8.81 + 8.955 = 0.145$

$\hat{Y}_2 = -8.81 + (0.8955)12 = -8.81 + 10.746 = 1.936$

$\hat{Y}_3 = -8.81 + (0.8955)14 = -8.81 + 12.537 = 3.727$

$\hat{Y}_4 = -8.81 + (0.8955)16 = -8.81 + 14.328 = 5.518$

$\hat{Y}_5 = -8.81 + (0.8955)18 = -8.81 + 16.119 = 7.309$

$\hat{Y}_6 = -8.81 + (0.8955)22 = -8.81 + 19.701 = 10.891$

$\hat{Y}_7 = -8.81 + (0.8955)24 = -8.81 + 21.492 = 12.682$

$\hat{Y}_8 = -8.81 + (0.8955)26 = -8.81 + 23.283 = 14.473$

$\hat{Y}_9 = -8.81 + (0.8955)28 = -8.81 + 25.074 = 16.264$

$\hat{Y}_{10} = -8.81 + (0.8955)30 = -8.81 + 26.865 = 18.055$

2.2

i	$\hat{Y}_i$	$\hat{u}_i$
1	0.145	-0.145
2	1.936	0.064
3	3.727	1.273
4	5.518	0.482
5	7.309	-0.309
6	10.891	-0.891
7	12.682	-2.682
8	14.473	0.527
9	16.264	-0.264
10	18.055	1.945

$Y_i = \hat{Y}_i + \hat{u}_i$

$\hat{u}_i = Y_i - \hat{Y}_i$

$\hat{u}_1 = 0 - 0.145 = -0.145$

$\hat{u}_2 = 2 - 1.936 = 0.064$

$\hat{u}_3 = 5 - 3.727 = 1.273$

$\hat{u}_4 = 6 - 5.518 = 0.482$

$\hat{u}_5 = 7 - 7.309 = -0.309$

$\hat{u}_6 = 10 - 10.891 = -0.891$

$\hat{u}_7 = 10 - 12.682 = -2.682$

$\hat{u}_8 = 15 - 14.473 = 0.527$

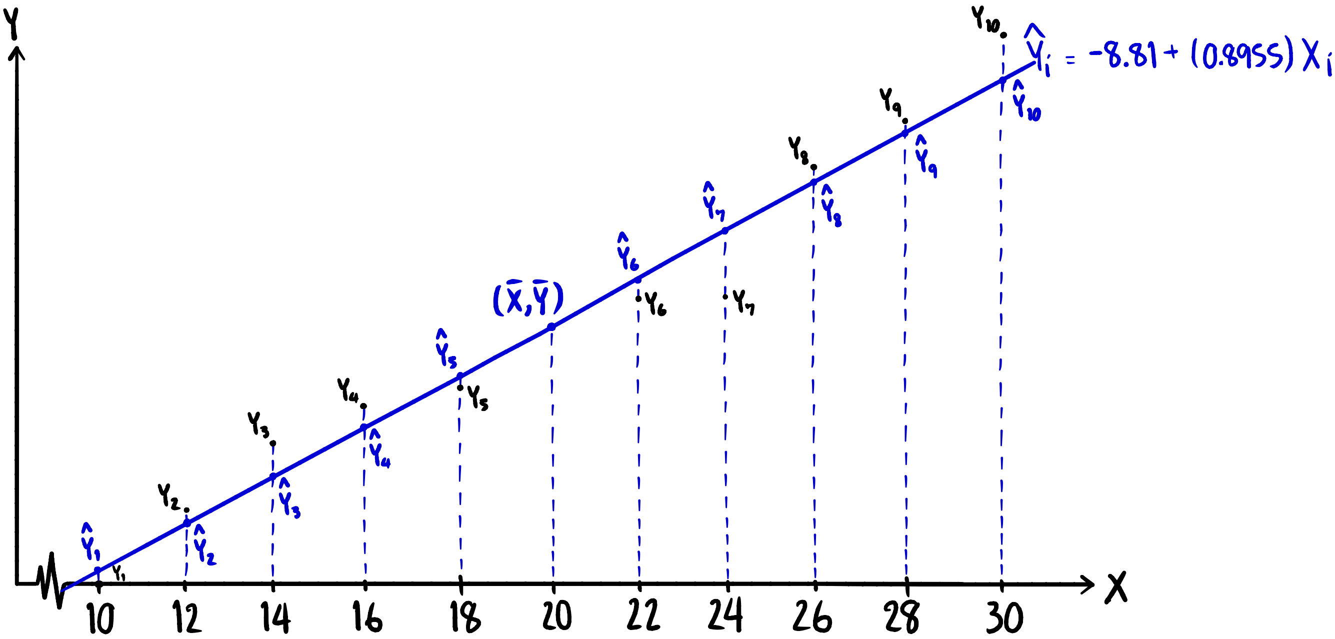
$\hat{u}_9 = 16 - 16.264 = -0.264$

$\hat{u}_{10} = 20 - 18.055 = 1.945$

$\sum_{i=1}^{n} \hat{u}_i = (-0.145) + 0.064 + 1.273 + 0.482 + (-0.309) + (-0.891) + (-2.682) + 0.527 + (-0.264) + 1.945$

$= 0$

2.3



$$\text{At } \bar{X},\ \hat{Y}_i = -8.81 + (0.8955)20$$
$$= -8.81 + 17.91$$
$$= 9.1$$

$\hat{Y}_i = \bar{Y}$ $\therefore$ $(\bar{X},\bar{Y})$ is on the regression line

2.4 If $X_i = 18$, $\hat{Y}_i = 7.309$ (from the question 2.2)

2.5 $\bar{\hat{u}}_i = \dfrac{\sum_{i=1}^{n} \hat{u}_i}{n} = 0$

$$\text{var}(\hat{u}_i) = \sigma^2 = \frac{\sum_{i=1}^{n}(\hat{u}_i - \bar{\hat{u}}_i)^2}{n-2} = \frac{\sum_{i=1}^{n}(\hat{u}_i)^2}{n-2}$$

$$= \frac{(-0.145)^2 + 0.064^2 + 1.273^2 + 0.482^2 + (-0.309)^2 + (-0.891)^2 + (-2.682)^2 + 0.527^2 + (-0.264)^2 + 1.945^2}{10-2}$$

$$= \frac{0.0210 + 0.0041 + 1.6205 + 0.2323 + 0.0955 + 0.7939 + 7.1931 + 0.2777 + 0.0697 + 3.7830}{8}$$

$$= \frac{14.0908}{8}$$

$$= 1.76135$$

i	$x_i = X_i - \bar{X}$	$y_i = Y_i - \bar{Y}$	x_i^2
1	-10	-9.1	100
2	-8	-7.1	64
3	-6	-4.1	36
4	-4	-5.1	16
5	-2	-2.1	4
6	2	0.9	4
7	4	0.9	16
8	6	5.9	36
9	8	6.9	64
10	10	10.9	100
Sum	0	0	440
Mean	0	0	44

$$\text{var}(\hat{\beta}_1) = \frac{\sum X_i^2}{n\sum x_i^2}\sigma^2$$

$$= \frac{4440}{10(440)}(1.76135)$$

$$= 1.777$$

$$\text{var}(\hat{\beta}_2) = \frac{\sigma^2}{\sum x_i^2}$$

$$= \frac{1.76135}{440}$$

$$= 0.0040$$

③ $$\hat{\beta}_1 = \frac{\sum X_i^2 \sum Y_i - \sum X_i \sum X_i Y_i}{n\sum X_i^2 - (\sum X_i)^2}$$

$$= \bar{Y} - \hat{\beta}_2 \bar{X}$$

Find the $E(\hat{\beta}_1)$

$$E(\hat{\beta}_1) = E[\bar{Y} - \hat{\beta}_2 \bar{X}]$$

$$= E(\bar{Y}) - \bar{X}E(\hat{\beta}_2) \qquad (\text{Note: } E(\hat{\beta}_2) = \beta_2)$$

$$= \beta_1 + \beta_2 \bar{X} - \bar{X}\beta_2$$

$$E(\hat{\beta}_1) = \beta_1$$

Therefore, $\hat{\beta}_1$ is an unbiased estimator of the true β_1

Assumption 1 : $Y_i = \beta_1 + \beta_2 X_i + u_i$
Assumption 8 : Variability in X values
Assumption 9 : The regression model is correctly specified
Assumption 10 : There is no perfect multicollinearity

2. Data is listed in the table

X_i	Y_i
10	0
12	2
14	5
16	6
18	7
22	10
24	10
26	15
28	16
30	20

2.1 From the simple regression model $Y_i = \beta_1 + \beta_2 X_i + u_i, u_i \sim NIID(0, \sigma^2)$

Find estimators of β_1 and β_2 from the OLS method and interpret the meaning.

2.2 Find the value of $\hat{Y}_i$ and $\hat{u}_i$. Show that $\sum \hat{u}_i \approx 0$

2.3 Plot graph and draw regression line. Does the line pass $(\bar{X}, \bar{Y})$?

2.4 If $X_i = 18$, what is the predicted Y?

2.5 Find $var(\hat{u}_i)$, $var(\hat{\beta}_1), var(\hat{\beta}_2)$

3. Consider the below regression function: consider the two-variable model

$\boldsymbol{Y_i = \beta_1 + \beta_2 X_i + u_i}$, $\boldsymbol{u_i \sim NIID(0, \sigma^2)}$

Find an OLS estimator of β_1. Then, provide a proof that this is an unbiased estimator. Please state the assumption(s) of CLRM when used (pages 66-75 in Gujarati).

"Practice makes Perfect."