Lecture 3
Portfolio Theory - Capital Allocation

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FN 312 – INVESTMENTS

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Portfolio Theory

- Portfolio Theory dates back to the late 1950s and the seminal work of Harry Markowitz and is still heavily relied upon today by portfolio managers
- We want to understand the characteristics of portfolios formed from combining assets
- Given our understanding of portfolio characteristics, how does an individual investor form optimal portfolios
- How does this theory apply to the economy or capital markets (investors in aggregate)?
- Is this theory consistent with behavior we observe in financial markets?

Portfolio Preliminaries

- A portfolio is a collection of investment assets which can be updated or rebalanced by buying and selling securities
- Portfolio weights indicate the fraction of the portfolio's total value held in each asset

By definition, portfolio weights must sum to 1

Initially, we will assume the weights are non-negative, but later we can relax these assumptions. Negative portfolio weights allow us to deal with borrowing and short-selling assets

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Data Needed for Portfolio Calculations

$E(r_i)$ Expected returns for all assets i

$Var(r_i)$ Variances (or standard deviations) of return for all assets i

$Cov(r_i, r_j)$ Covariances of returns for all pairs of assets i and j

Where do we obtain this data?

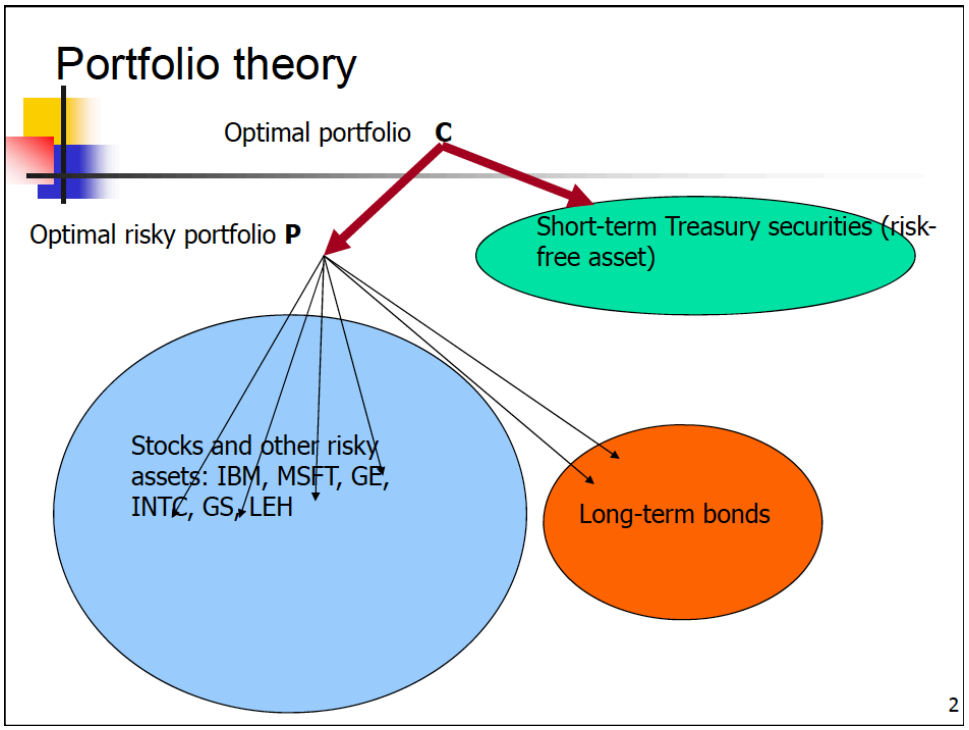
- Estimate them from historical sample data using statistical techniques (sample statistics). This is the most common approach

Portfolio Construction

In constructing their portfolios, individual investors or portfolio managers seek to achieve the best possible trade-off between risk and return

1) Capital allocation - how you allocate your funds in broad asset classes eg. the risk free asset and the risky assets

2) Security selection -how you allocate your money among individual securities eg. IBM stock, Microsoft stock ie. it involves the valuation of particular securities



Capital Allocation

- Capital allocation decision is concerned with how you allocate your funds in broad asset classes eg. the risk free asset and the risky asset
- In making the capital allocation decision, assume that you have already selected the (optimal) risky portfolio (risky asset)
- Assume that the expected return of the risky portfolio is $E(r_p)$ and the standard deviation of the return of the risky asset is σ_p
- Then the expected return of the portfolio is

Capital Allocation (cont.)

- What about the variance of the portfolio?

- The standard deviation of the return of the portfolio is

The standard deviation of the portfolio is proportional to both the standard deviation of the risky asset and the proportion invested in it.

Capital Allocation Line (CAL)

The CAL represents the investment opportunity combinations of the risk-free asset and a risky asset

Slope of CAL

- The slope of the CAL measures the excess return being earned per unit of volatility (Sharpe ratio)

Investor Preferences

- How should you make your capital allocation decision? How much should invest in risky versus risk-free assets?
- This decision depends on investor preferences

Example: Choose between

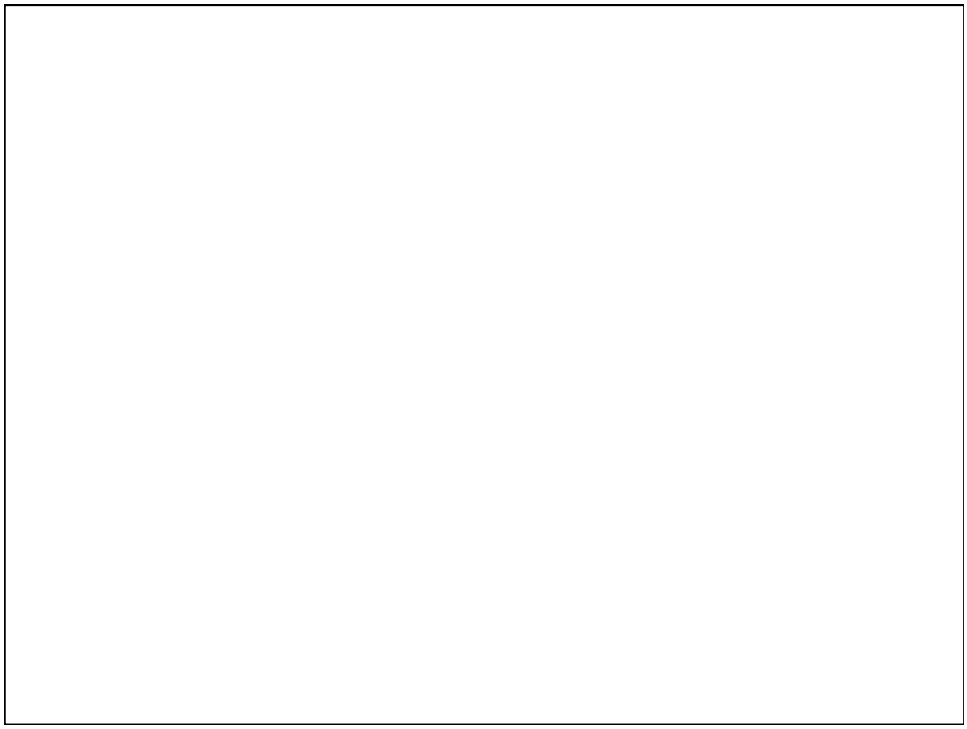
- Security A: certain investment that returns 5%
- Security B: uncertain investment with expected return of 5% and stdev = 5%
- Security C: uncertain investment with expected return 10% and stdev = 5%

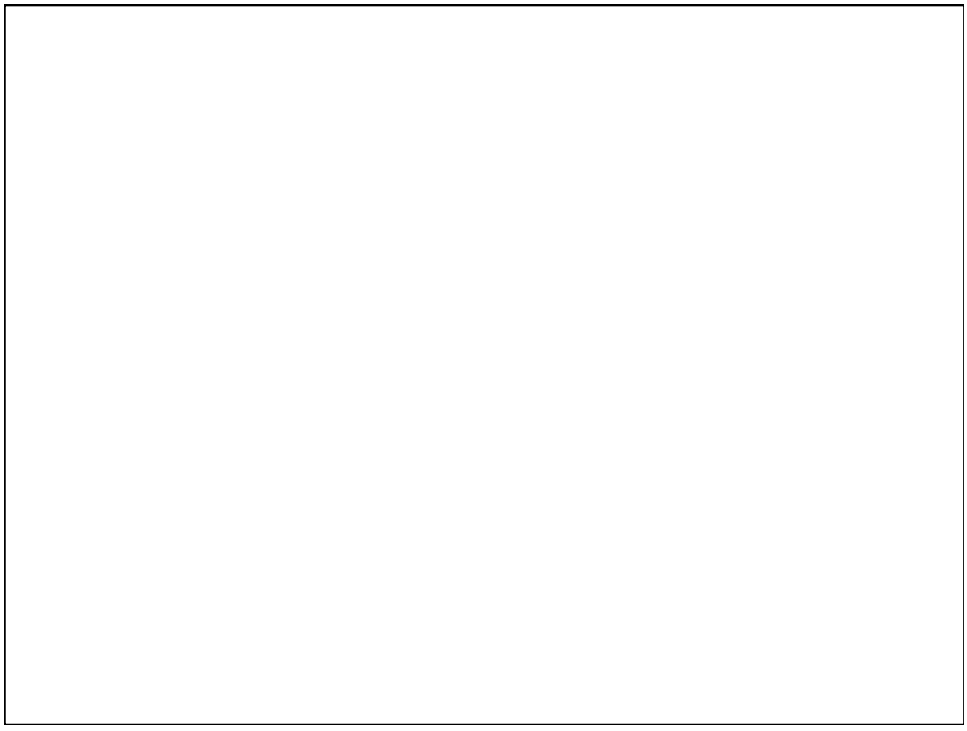
Risk Aversion

- Greater levels of risk aversion leads to larger proportions of the risk free asset
- Lower levels of risk aversion lead to larger proportions of the portfolio of risky assets
- Different investors will choose different positions in the risky asset.
- How to quantify this?

How to compare investments?

- The risk premium an investor requires depends on his/her preference
- To compare investment opportunities an investor assign a welfare, or a utility score to competing investments
- A utility is a cardinal number and increases with more attractive risk-return profiles
- What should be part of the utility function?
  - Investors like higher expected returns and lower risks
  - Put the risk-return tradeoff in one utility function since having two objectives is not convenient to work with





Utility Function

- A common one used in portfolio theory is

$$U = E[\tilde{r}] - 0.5\, A\, \sigma^2$$

Parameter measuring sensitivity to risk

- A, the risk aversion parameter, is positive for risk averse investors.
- The higher the A is, the more risk averse an investor is.
- Experiments show that A is positive and generally under 10.

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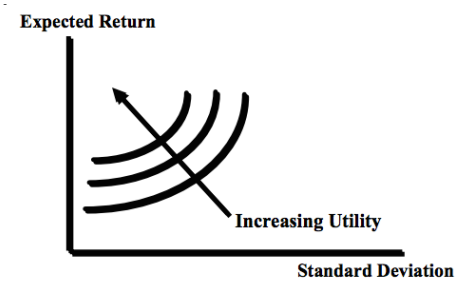
- What should be the value of A (the coefficient of risk aversion) if investors are risk neutral ie. do not care about risk? What if they like risk?

- The level of risk aversion determines whether an investor chooses to invest in risky assets and how much he/she chooses to invest
- What determines the risk aversion level of an investor?
  - Current wealth
  - Future income
  - Personality
  - Investment horizon
- How is risk aversion determined in practice?
  - One way is to use survey methods to assess the relative level of risk aversion of an investor compared to that of an average investor which is approximately 3.0

Indifference Curve



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Utility and Risk Aversion

- A steeper indifference curve indicates that for an increase of one unit of risk (measured by the standard deviation) the investor needs to be compensated with additional expected return to stay indifferent

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Optimal Capital Allocation

Optimal Capital Allocation

- Recall that a portfolio with y as the risky asset and (1-y) as the risk-free asset has expected return:

- The variance of the portfolio return is:

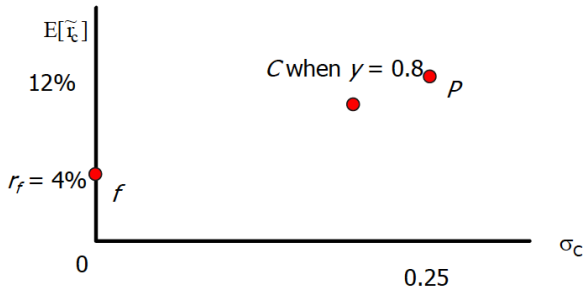
- The investor attempts to maximize her utility level, by choosing the best allocation to the risky asset, y.

Optimal Allocation

- Other things being equal, the higher the expected return of the risky asset, the higher the optimal allocation in the risky asset
- Other things being equal, the higher the variance of the return of the risky asset, the lower the optimal allocation in the risky asset
- Other things being equal, the more risk averse the investor is, the lower the optimal allocation in the risky asset

An Example

- Find possible portfolios with different fraction invested in the risky portfolio



- What happens if we choose to put 80% in the risky portfolio and 20% in the risk free rate. What will be the expected return and standard deviation of this new portfolio C?

Optimal Asset Allocation

- To choose the optimal y, we maximize utility given all the complete choices of the complete portfolio

Optimal Asset Allocation

- Asset allocation for $100,000 between

  1. The riskfree that returns 4%

  2. A diversified portfolio P that has an expected return of 12% and a standard deviation of returns of 25%
- If your risk aversion level is 2 how much do you put in the risky asset?

Optimal Asset Allocation

- It was determined that P has expected return of 12% and standard deviation of 25%
- What is expected return and standard deviation of the optimal portfolio C?

Homework 1

- BKM Ch 1 #14, 15
- BKM Ch 2 #12, 22
- BKM Ch5 # 6, 10
- BKM Ch 6 #6-9, 13-19