EE 320 Introductory Mathematical Economics

Semester 1/2017

Assignment# 5

<u>Question 1</u> Output Maximization

A firm's production function is given by

$$Q = x^2 - 2xy + y^2$$

where x and y are the two inputs, and the per unit prices for x and y are \$10 and \$20, respectively. Suppose that the firm's total cost of this production is fixed at \$5200.

a. Show that the isoquant derived from the above production function is convex.
b. Based on the Lagrangian function, write down the first-order conditions for this output maximization problem, and explain the first-order conditions in terms of isoquant and isocost.
c. From the results in part (b), find the input levels that maximize output, and determine the maximum output level as well as the Lagrange multiplier.
d. Suppose now that the firm's total cost increases to 5201 baht. What happens to the optimal production levels? How does this change in the optimal production levels relate to the Lagrange multiplier in part (c)?

<u>Question 2</u> Constrained Optimization with 4 variables and 2 constraints

Consider a constrained optimization problem with the objective function

$$z = f(x_1, x_2, x_3, x_4)$$

Subject to the two constraints:

$$g(x_1, x_2, x_3, x_4) = c$$
$$h(x_1, x_2, x_3, x_4) = d$$

a. Write down the Lagrangian function and the corresponding first-order conditions.
b. Write down the bordered Hessian matrix for this constrained optimization problem.
c. Using the notations from the lecture, indicate the second-order conditions for a maximum <u>and</u> a minimum of z.

<u>Question 3</u> Integration

Suppose the demand curve for a product is given by $Q_d = 20 - 4P$ and the supply curve is given by $Q_s = -4 + 2P$.

a. Use integral to calculate the consumer and producer surplus at the equilibrium price and quantity.
b. Suppose the government imposes a *25 %* ad varolem tax on the consumer. Derive the new demand curve, and use integral to calculate the consumer and producer surplus at the new equilibrium price and quantity. Discuss the changes in both consumer and producer surplus. Is there any deadweight loss to the society?

<u>Question 4</u> Cost function under the multi-plant problem.

A company has *three* plants that it can use for producing a product. Given the level of output chosen in each of the three plants, the operating cost incurred under each plant can be given,

$$c_1(x) = 200 + \tfrac{1}{100}x^2 \,,\; c_2(y) = 200 + y + \tfrac{1}{300}y^2 \;,\; c_3(z) = 200 + 10z \;,$$

where x, y and z are the amount of output chosen in each plant, respectively.

a) Suppose that the total level of production required is equal to Q units. Calculate the optimal plant size that minimizes the total cost. Determine the minimized level of cost.
b) Confirm your result with the second-order derivative test when Q = 2000.
c) How does the minimized level of cost respond to the change in total level of production (Q)?

Question 5

Consider a utility maximization problem where household chooses for the optimal combination of good 1 (x) and good 2 (y). Suppose that p_x and p_y are the prices per unit of good 1 and good 2, respectively. Assume that the household has the budget equal to M, and the utility function is given by $U(x,y) = \sqrt{x} + y$. Consider the following problem

a) Derive for the bundle of consumption that maximizes household's utility, i.e. demand for good 1 and demand for good 2.
b) State the condition under which both types of good are chosen.
c) Derive the maximized level of utility.

Question 6

Suppose that a monopolist faces with a marginal revenue function given by

$$MR(q) = 25 - 2q,$$

and the marginal cost function given by,

$$MC(q) = 37 - 9q + q^2,$$

where q is the unit of output. Assume that fixed cost is \$7. Consider the following problem.

a) Derive the revenue function and infer the functional form of the market demand curve.
b) Derive the total cost function.
c) Determine the level of production that maximizes profit and determine the level of maximized profit.
d) Calculate the consumer surplus and producer under the monopoly equilibrium.
e) If the monopolist were to act as a perfectly competitive firm, calculate the market equilibrium, and corresponding level of consumer's and producer's surplus.