FN211
Interest Rates and Their Role in Valuation

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Preview

If you invest in a government bond, do you face any risks?

- No, the government bond is default-protected by the government
- Yes, you still face some other risks…

If yes, what risk are you confronting? And are there any differences between investing in 1-yr bond and 10-yr bond?

Present Value Introduction

- Different debt instruments have very different streams of cash payments to the holder (known as **cash flows**), with very different timing
- All else being equal, debt instruments are evaluated against one another based on the *amount* of each cash flow and the *timing* of each cash flow
- This evaluation, where the analysis of the amount and timing of a debt instrument's cash flows lead to its *yield to maturity or interest rate*, is called *present value* analysis

Present Value

- The concept of **present value** (or **present discounted value**) is based on the commonsense notion that a dollar of cash flow paid to you one year from now is less valuable to you than a dollar paid to you today

- The term present value (PV) can be extended to mean the PV of a single cash flow or the *sum* of a sequence or group of cash flows

PV Applications

There are four basic types of credit instruments which incorporate present value concepts:

1. Simple Loan (สินเชื่อแบบง่าย)
2. Fixed Payment Loan (สินเชื่อที่มีการจ่ายชำระคืนเป็นงวดๆ)
3. Coupon Bond (ตราสารหนี้ที่ระบุดอกเบี้ย)
4. Discount Bond (ตราสารหนี้ที่มีราคาซื้อขายต่ำกว่ามูลค่าหน้าตั๋ว)

PV Concept: Simple Loan Terms

- *Loan Principal (เงินต้น):* the amount of funds the lender provides to the borrower
- *Maturity Date (วันครบกำหนดไถ่ถอน):* the date the loan must be repaid; the *Loan Term* is from initiation to maturity date
- *Interest Payment (ดอกเบี้ยจ่าย):* the cash amount that the borrower must pay the lender for the use of the loan principal
- *Simple Interest Rate (อัตราดอกเบี้ยแบบง่าย):*

PV Concept: Simple Loan

Simple loan of $100

Year: 0	1	2	3	n
$100	$110	$121	$133	$100\times(1+i)^n$

PV of future $1 =

Yield to Maturity: Loans

Yield to maturity (อัตราผลตอบแทนจนครบอายุการไถ่ถอน) = interest rate that equates today's value with present value of all future payments

1. Simple Loan Interest Rate ($i = 10\%$)

$$\$100 = \frac{\$110}{(1 + i)}$$

$$i =$$

Simple PV: Example

What is the PV of $250 to be paid in 2 years if the interest rate is 15%?

PV Concept: Fixed-Payment Loan Terms

- *Simple Loans (สินเชื่อแบบง่าย)* require payment of one amount which equals the loan principal plus the interest

- *Fixed-Payment Loans* (สินเชื่อที่มีการจ่ายชำระคืนเป็นงวดๆ) are loans where the loan principal and interest are repaid in several payments, often monthly, in equal dollar amounts over the loan term
  - *Installment (เงินผ่อน) Loans,* such as ……

Yield to Maturity: Loans

2. Fixed Payment Loan can be written in the following equation…

Yield to Maturity: Bonds

3. **Coupon Bond** (ตราสารหนี้ที่ระบุอัตราดอกเบี้ย) **(Coupon rate = 10% = *C*/*F*)**

$$P = \frac{\$100}{(1+i)} + \frac{\$100}{(1+i)^2} + \frac{\$100}{(1+i)^3} + \cdots + \frac{\$100}{(1+i)^{10}} + \frac{\$1000}{(1+i)^{10}}$$

$$P = \frac{C}{(1+i)} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \cdots + \frac{C}{(1+i)^n} + \frac{F}{(1+i)^n}$$

Consol (or perpetuity): Fixed coupon payments of $*C* forever

$$P = \frac{C}{i} \qquad i = \frac{C}{P}$$

Yield to Maturity: Bonds

4 One-Year Discount Bond
($P = \$900$, $F = \$1000$)

$$\$900 = \frac{\$1000}{(1 + i)}$$

Relationship Between Price & YTM

Yields to Maturity on a 10% Coupon Rate Bond Maturing in 10 Years
(Face Value = $1,000)

Price of Bond ($)	Yield to Maturity (%)
1,200	7.13
1,100	8.48
1,000	10.00
900	11.75
800	13.81

- What are interesting facts from the table?

Current Yield

$$i_c = \frac{C}{P}$$

- Current yield (CY หรือ อัตราผลตอบแทนปัจจุบัน) is just an approximation for YTM - easier to calculate.
- Properties:
  1. If a bond's price is near par and has a long maturity, then CY is a good approximation
  2. A change in the current yield always signals change in same direction as yield to maturity

Distinction Between Real and Nominal Interest Rates

- **Real interest rate** (อัตราดอกเบี้ยที่แท้จริง)
  1. Interest rate that is adjusted for expected changes in the price level

$$i_r = i - \pi^e$$

  2. Real interest rate more accurately reflects true cost of borrowing
  3. When the real rate is low, there are _____ incentives to borrow and _____ to lend

Distinction Between Real and Nominal Interest Rates

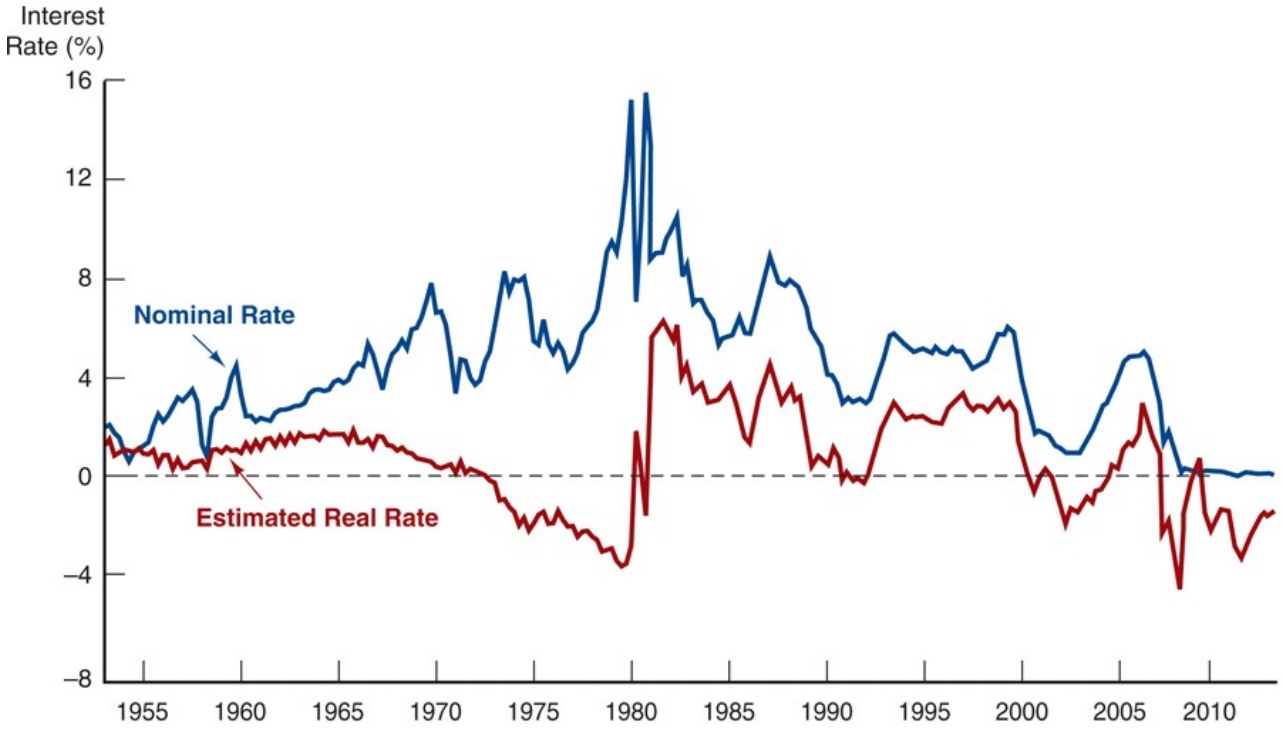
- Real interest rate

$$i_r = i - \pi^e$$

We usually refer to this rate as the *ex ante* real rate of interest because it is adjusted for the *expected* level of inflation. After the fact, we can calculate the *ex post* real rate based on the observed level of inflation.

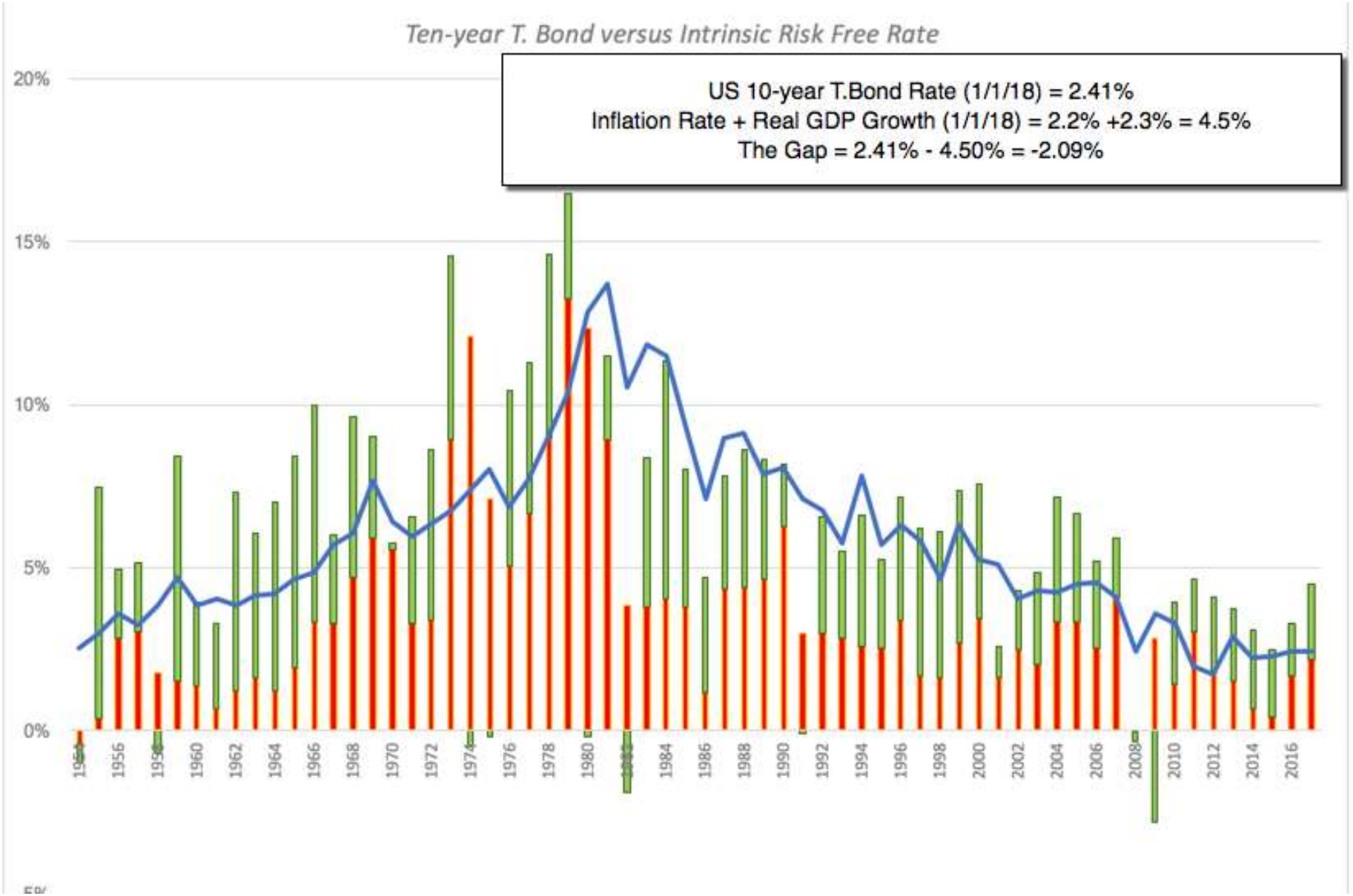
U.S. Real and Nominal Interest Rates

Real and Nominal Interest Rates
(Three-Month Treasury Bill), 1953–2013



Sources: Nominal rates from Federal Reserve Bank of St. Louis FRED database, http://research.stlouisfed.org/fred2/. The real rate is constructed using the procedure outlined in Frederic S. Mishkin, "The Real Interest Rate: An Empirical Investigation," *Carnegie–Rochester Conference Series on Public Policy* 15 (1981): 151–200. This involves estimating expected inflation as a function of past interest rates, inflation, and time trends and then subtracting the expected inflation measure from the nominal interest rate.

Some Perspective on Interest Rates



Distinction Between Interest Rates and Returns

- Rate of Return: we can decompose returns into two pieces:

$$\text{Return} = \frac{C + P_{t+1} - P_t}{P_t} = \text{i}_\text{c} + \text{g}$$

$$\text{where } i_c = \frac{C}{P_t} = \text{current yield, and}$$

$$g = \frac{P_{t+1} - P_t}{P_t} - \text{capital gains}$$

Key Facts about the Relationship Between Rates and Returns

One-Year Returns on Different-Maturity 10% Coupon Rate Bonds When Interest Rates Rise from 10% to 20%

(1) Years to Maturity When Bond Is Purchased	(2) Initial Current Yield (%)	(3) Initial Price ($)	(4) Price Next Year* ($)	(5) Rate of Capital Gain (%)	(6) Rate of Return (2 + 5) (%)
30	10	1,000	503	–49.7	–39.7
20	10	1,000	516	–48.4	–38.4
10	10	1,000	597	–40.3	–30.3
5	10	1,000	741	–25.9	–15.9
2	10	1,000	917	–8.3	+ 1.7
1	10	1,000	1,000	0.0	+10.0

*Calculated with a financial calculator using Equation 3.

Sample of current coupon rates and yields on government bonds
http://www.bloomberg.com/markets/iyc.html

Maturity and the Volatility of Bond Returns

- Key findings from the table

Reinvestment Risk

- Occurs if investor holds a series of short bonds over long holding period
- *i* at which reinvest uncertain
- Gain from *i* ↑, lose when *i* ↓

Calculating Duration $i = 10\%$, 10-Year 10% Coupon Bond

(1) Year	(2) Cash Payments (Zero-Coupon Bonds) ($)	(3) Present Value (*PV*) of Cash Payments (i = 10%) ($)	(4) Weights (% of total *PV* = *PV*/$1,000) (%)	(5) Weighted Maturity (1 × 4)/100 (years)
1	100	90.91	9.091	0.09091
2	100	82.64	8.264	0.16528
3	100	75.13	7.513	0.22539
4	100	68.30	6.830	0.27320
5	100	62.09	6.209	0.31045
6	100	56.44	5.644	0.33864
7	100	51.32	5.132	0.35924
8	100	46.65	4.665	0.37320
9	100	42.41	4.241	0.38169
10	100	38.55	3.855	0.38550
10	1,000	385.54	38.554	3.85500
Total		1,000.00	100.000	6.75850

Calculating Duration $i = 20\%$, 10-Year 10% Coupon Bond

(1) Year	(2) Cash Payments (Zero-Coupon Bonds) ($)	(3) Present Value (*PV*) of Cash Payments (i = 20%) ($)	(4) Weights (% of total *PV* = *PV*/ $580.76) (%)	(5) Weighted Maturity (1 × 4)/100 (years)
1	100	83.33	14.348	0.14348
2	100	69.44	11.957	0.23914
3	100	57.87	9.965	0.29895
4	100	48.23	8.305	0.33220
5	100	40.19	6.920	0.34600
6	100	33.49	5.767	0.34602
7	100	27.91	4.806	0.33642
8	100	23.26	4.005	0.32040
9	100	19.38	3.337	0.30033
10	100	16.15	2.781	0.27810
10	1,000	161.51	27.808	2.78100
Total		580.76	100.000	5.72204

Formula for Duration

$$DUR = \sum_{t=1}^{n} t \frac{CP_t}{(1+i)^t} \Bigg/ \sum_{t=1}^{n} \frac{CP_t}{(1+i)^t}$$

- Key facts about duration

Duration and Interest-Rate Risk

$$\%\Delta P \approx -DUR \times \frac{\Delta i}{1+i}$$

- $i \uparrow$ 10% to 11%:
  - — From the previous table, for 10% coupon bond

Duration and Interest-Rate Risk (cont.)

- $i \uparrow$ 10% to 11%:
  - For 20% coupon bond, $DUR = 5.72$ years

Duration and Interest-Rate Risk (cont.)

- The greater the duration of a security, the greater the % change in the market value of the security for a given change in interest rates

- Therefore, the greater the duration of a security, the greater its interest-rate risk